POSSIBLE ASYMPTOTIC BEHAVIOR OF ELASTIC SCATTERING

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UNTIL recently, elastic scattering at high energies s and at small transferred momenta t (region of the diffraction peak) was described exclusively on the basis of the classical picture of diffraction from a black body, in which the invariant scattering amplitude A (s, t) has the form

$$A(s, t) = s f(t). \tag{1}$$

Recently, owing to the progress in the study of the analytic properties of the scattering amplitudes,[1] it became possible to approach the asymptotic behavior of the scattering amplitude in a somewhat less phenomenological fashion. It turned out here[2] that an asymptotic behavior such as (1) cannot be reconciled in simple fashion with the unitarity and analycity conditions. It was simple to reconcile diffraction with a slowly decreasing cross section, such as from a grey sphere. At the present time it is not clear whether a classical diffraction pattern is produced at all in the scattering of high-energy particles.

In this connection, we wish to discuss in the present note a different type of asymptotic behavior, which in spite of having a few unusual features is theoretically feasible and does not contradict the experimental values. Before we formulate this asymptotic behavior [see (4a) and (4b) below], let us see how it is deduced from the analytic properties of A (s, t). We consider the scattering of identical spinless particles with mass μ, which are the lightest in theory.

The physical region corresponds to $t < 0$. If we continue A (s, t) into the region $t > 4\mu^2$, then A (s, t) can be regarded as the scattering amplitude of particles having a squared energy t in the c.m.s. and having a nonphysical squared momentum transfer s. The asymptotic behavior at large values of s and at the values of t referred to here was investigated in [2], where it was shown that it cannot have the form sf (t). Regge[3] has shown that in nonrelativistic theory, at large momentum transfers, A (s, t) behaves as $s^{l(t)}$, where $l(t)$ is the position of the pole of the partial wave f_l as a function of the momentum l in the complex plane. He also showed that the poles are determined by the possible bound and resonant states. When $t < 4\mu^2$, these poles go to the real axis of the complex l plane and $l(t)$ decreases with t. In a future more detailed paper we shall show that in field theory the amplitudes of the partial waves are also analytic functions of l, and that the asymptotic behavior of A (s, t) as $s \to \infty$ is determined by the nearest singularities of f_l on the side of the larger l. If we now assume that the nearest singularity in the l plane is a simple pole, we arrive at the same asymptotic behavior of $s^{l(t)}$ with real l for $t < 4\mu^2$ as in the nonrelativistic theory. In the nonrelativistic theory, however, there is only one channel (t, the energy). In the relativistic energy, on the other hand, the region $t < 0$ is a new physical region (where s is the energy) which is of principal interest to us. In this region, A (s, t) should satisfy the unitarity conditions. As shown by Frautschi,[4] it follows from the unitarity condition and from the Mandelstam representation that

$$|A(s, 0)| < Cs \ln^2 s. \tag{2}$$

This means that $l(0) \leq 1$. If we assume that the strongest possible interaction is realized, then $l(0) = 1$. If we write for small t

$$l(t) = 1 + \gamma t, \tag{3}$$

then we obtain

$$A(s, t) \sim s e^{\gamma t \xi}, \qquad \xi = \ln s \tag{4}$$

and consequently, A (s, t) decreases rapidly with increasing t at high energies, so that the significant interval of t is of the order of $-1/\xi$ (it is readily shown that $\gamma > 0$). If we take crossing symmetry into account we obtain the following expressions for the imaginary (A_1) and real (D) parts of A (s, t) when $t \sim -1/\xi$:

$$A_1(s, t) = Cse^{\gamma t \xi}, \tag{4a}$$

$$D(s, t) = Cs(-\gamma t) e^{\gamma t \xi}. \tag{4b}$$

Let us list the main properties of such an asymptotic scattering behavior: 1) the total interaction cross section is constant at high energies, so that $A_1(s, 0) = Cs$; 2) the elastic-scattering cross section tends to zero as $1/\xi$; 3) the scattering amplitude in the significant region becomes pure imaginary because of the factor $-\gamma t$ in (4b); 4) the region of momentum transfer significant for the elastic scattering decreases with increasing energy: $\sqrt{-t} \sim \xi^{-1/2}$; 5) the amplitudes of the partial waves

$$a_l(s) = \frac{1}{2}\int_{-1}^{1} P_l(z)\, A(s, t)\, dz$$

as functions of l (or, what is more convenient, as functions of the impact parameter $\rho = 1/p$) behave in the following fashion:

$$a_l(s) \equiv a(\rho, s) = \begin{cases} \dfrac{C}{\gamma\xi} e^{-\rho^2/\gamma\xi}\left[i + \dfrac{1}{\xi}\right], & \rho \ll \gamma\xi \\ \sim s^{l(4\mu^2)} e^{-2\mu\rho}, & \rho \gg \gamma\xi \end{cases},$$

i.e., an important role is played in the scattering by impact parameters $\rho \sim (\gamma\xi)^{1/2}$ for which $a(\rho, s) \sim 1/\xi$. This behavior means that the particles become grey with respect to high-energy interaction, but increase in size, so that the total cross section remains constant.

If we assume, as was done above, that the asymptotic behavior is determined by the position of the pole of the partial wave as a function of l, for example for $\pi\pi$-scattering, then, inasmuch as the partial waves of different processes are related by the unitarity condition, it can be shown that in general the processes with partial waves that have poles for the same value of l will be $N + \overline{N} \rightarrow 2\pi$, $N + \overline{N} \rightarrow N + \overline{N}$, etc, and consequently the behavior of the amplitudes of the reactions $\pi + N \rightarrow \pi + N$, $N + N \rightarrow N + N$ etc. will be similar to (4a) and (4b), with the same value of γ as for $\pi\pi$ scattering.

Chu and Frautschi [5] expressed in several articles the conviction that the asymptotic behavior of large momentum transfers in the nonphysical region should have common features in relativistic and nonrelativistic theory. It was emphasized, in particular, that the condition that the total cross section be constant, which calls for a pole at $l = 1$, is indeed the condition for the interaction force to have a maximum. These authors have disregarded, however, the fact that $l(t)$ is an analytic function of t, a consequence of which is the extraordinary nature of the described scattering picture.

Ordinary diffraction arises only if the partial waves have a pole at $l = 1$ when $t < 4\mu^2$, or have no pole when $t > 4\mu^2$. As will be shown in a more detailed article, the partial wave must have for this purpose complicated analytic properties, which have no nonrelativistic analogue.

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[1] S. Mandelstam, Phys. Rev. **112**, 1344 (1958).
[2] V. N. Gribov, Nucl. Phys. **22**, 246 (1961).
[3] T. Regge, Nuovo cimento **14**, 952 (1959); **18**, 957 (1960).
[4] Froissart. Preprint.
[5] G. Chew and S. Frautschi, Phys. Rev. Lett. **5**, 12 (1960).

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120

Contents of Coming Issues

SOVIET PHYSICS JETP VOLUME 14, NUMBER 3 MARCH, 1962

CONTENTS