ANOMALOUS DOPPLER EFFECT IN A PLASMA

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Excitation of electromagnetic waves in a plasma by a superluminal ion beam is considered by taking into account the motion of the ions in the plasma. The geophysical significance of this problem is pointed out.

THE radiation of an oscillator moving in a magnetoactive plasma with superluminal velocity has been investigated by a number of authors[1-6] and the anomalous Doppler effect arising from such motion has been considered. In the present work, additional account is taken of the motion of the ions of the plasma, and certain geophysical applications of the results are indicated.

1. Let an ion of mass M_1 move in a plasma along the external magnetic field H with a velocity u. Inasmuch as it simultaneously rotates about the lines of force of the field H, it can be regarded as an oscillator with a set of natural frequencies $\omega_s = s\Omega_1$ ($s = 1, 2, \ldots$; Ω_1 is the Larmor frequency of the ion). Substituting in the formula for the Doppler effect

$$\omega = \frac{\Omega_1}{(un/c)\cos\theta - 1}$$

the well known[7] expression for the index of refraction of the extraordinary wave (with account of the ions), we find the following expression for the dimensionless frequency $\eta = \omega/\Omega$ for the case $\omega < \omega_H < \omega_{0,e}$ and $\theta = 0$ (i.e., for propagation along the field):

$$u/v_A = (1 + Q/\eta)\sqrt{(1+\eta)(1-\alpha\eta)}, \qquad (1)$$

where m — mass of the electron, M — mass of the plasma ion, $\alpha = m/M$, $Q = M/M_1$, N — concentration of ions in the plasma, $v_A = H/\sqrt{4\pi NM}$ — the Alfvén velocity.

The approximate values of the roots of Eq. (1) are:

$$\eta_1 = v_A/u, \qquad \eta_2 = (u/v_A)^2, \qquad \eta_3 = 1/\alpha - (u/v_A)^2.$$

2. The plot of the right side of Eq. (1) is shown in the drawing (curve 1). From this plot, we obtain the following results:

1) When $u > (1/2)\, v_A \sqrt{M/m}$, the ion excites a wave of only one frequency F_1.

2) When $\frac{1}{2} v_A \sqrt{M/m} > u > 2.6 v_A$, the ion excites waves in the three frequency ranges F_1, F_2 and F_3.

3) When $u < 2.6 v_A$, the ion excites (in the case of superluminal motion) a wave only in the region of gyromagnetic resonance of the electrons ($\omega \sim \omega_H$). When $\omega \sim \omega_H$ it is also necessary to take into account the collision-free resonance absorption and to compare the decrement associated with it with the increment brought about by the ion (or by the ion beam).

The essentially new effect, which follows from Eq. (1) and the figure, is the excitation by the ion of electromagnetic waves with a frequency less than the ion Larmor frequency, i.e., the excitation of magnetohydrodynamic waves. It is also of interest that the ion excites electronic plasma vibrations as well (the range F_3).*

3. For waves propagating at an angle θ to the field H, the condition for excitation is similar to (1):

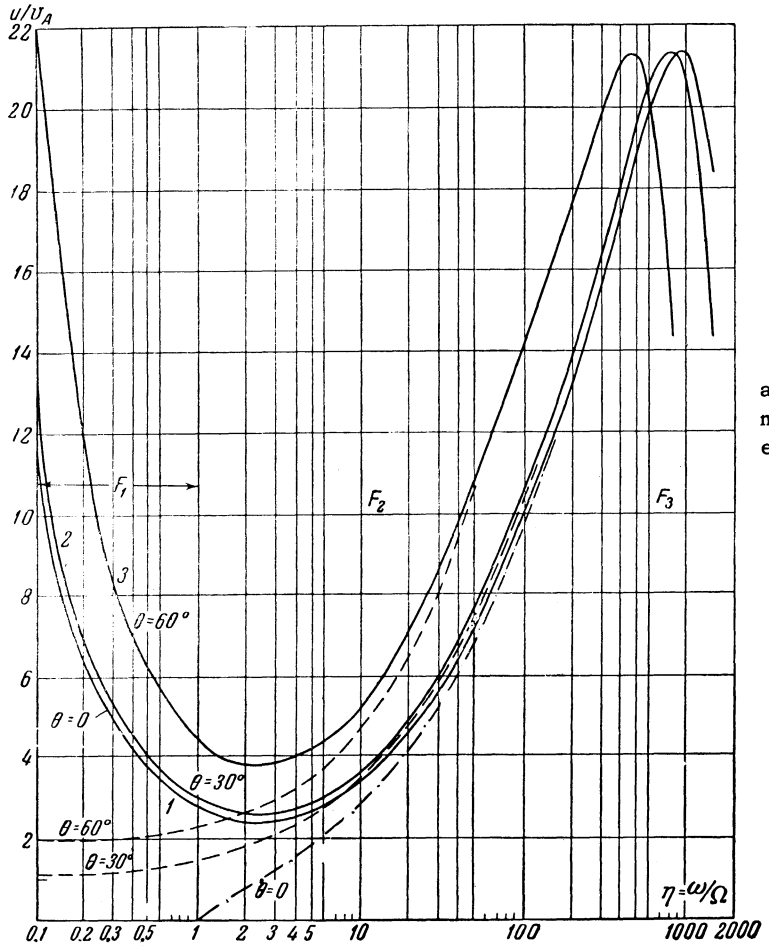
$$\frac{u}{v_A} = \left(1 + \frac{Q}{\eta}\right)\frac{1}{n(\theta, \eta)\cos\theta}\frac{c}{v_A}, \qquad (2)$$

but the dependence of the index of refraction n on θ and η has a much more complicated form.[7] The right side of Eq. (2) is shown in the figure by the curves 2 and 3 ($\theta = 30°$ and $\theta = 60°$).

There exist three cones of radiation, corresponding to the three frequency ranges F_1, F_2, and F_3. In the first of these (F_1) the minimum frequency is propagated along the field, and the radiated frequency increases with increase in angle θ ($\eta_{max} \approx \pm 1$). In the second cone (F_2), the frequency is inversely proportional to the angle. For the third cone (F_3), the elementary theory developed here is insufficient; as was shown above, one must also take into account the spatial dispersion in the plasma.

For Q = 0, Eq. (2) is the condition for Cerenkov radiation (the corresponding curves are shown as dashed lines in the figure).

*The correlation predicted in the present paper between the radiation in the ranges F_1, F_2 and F_3 was discovered experimentally in;[8] the radiation frequencies agree with Eq. (1).



Condition for radiation: solid curves – for the anomalous Doppler effect, dash-dot curve – for the normal Doppler effect, dotted curve – for the Cerenkov effect.

In the range of frequencies $\alpha\eta^2 < 1\ (\omega < \sqrt{\omega_H\Omega}\,)$ the following approximate expression for n was used in constructing the curves of the drawing:*

$$n = \left\{\frac{1 + \cos^2\theta + [(1+\cos^2\theta)^2 - 4\cos^2\theta\,(1-\eta^2)]^{1/2}}{2\,(1-\eta^2)\cos^2\theta}\right\}^{1/2}\frac{c}{v_A}. \quad (3)$$

Obviously the low-frequency vibrations also excite relativistic particles, for example, relativistic protons of the internal radiation belt. For relativistic particles, the term Q/η in (1), (2) must be replaced by $(Q/\eta)(1 - u^2/c^2)^{1/2}$.

4. The sun periodically emits streams of protons; these are the solar corpuscular streams. On the other hand, it is known that the magnetic field of the earth undergoes vibrations of small amplitude—the so-called short period variations with frequencies of 10^{-2}-10 cps. In the present article we advance the hypothesis that one of the possible mechanisms of the short period variations is the radiation of the protons of the solar corpuscular streams, due to their Larmor rotation around the lines of force of the earth's magnetic field (cyclotron instability of the stream in its superluminal motion in the earth's plasma). The vibrations with frequencies of the order of a fraction of a cycle per second correspond to the magnetohydrodynamic branch of the radiation (the range F_1 in the drawing).*

MacArthur[10] and Murcray and Pope[11] explained the radiation of the solar corpuscular streams as being an incoherent cyclotron radiation along the field ($\theta = 0$) of ions moving in the earth's plasma. In contrast with our assumption, the forward motion of the ions is assumed by them in this case to be subluminal, $u < c/n$, and consequently the condition (1) is replaced by the expression

$$u/v_A = (1 - Q/\eta)\sqrt{(1+\eta)(1-\alpha\eta)},$$

*Equation (3) was obtained independently by V. D. Shafranov.

*For the Alfvén wave [which corresponds to the plus sign in front of the root in (3)], the condition for cyclotron excitation is similar to (2): when $\eta \ll 1$, we have $\eta \approx v_A/u$ ($\theta \neq 0$), i.e., waves of a single frequency are generated at all angles to the field. It is also important that the group velocity of the Alfvén waves is parallel to the magnetic field for any θ. As a result, the amplitude of the Alfvén wave at the earth's surface is much larger than the amplitude of the oppositely polarized magnetoacoustic wave. The polarization observed in[9] can be explained in this manner.

which is shown in the drawing by the dash-dot curve. But it follows from the general expression obtained by Éidman[4] for the radiation of a particle moving in a gyrotropic medium (supplemented by account of the motion of the plasma ions) that when u < c/n the ion does not radiate along the field: the Doppler-effect equation for the frequency has no roots for the ordinary wave, while there are roots for the extraordinary wave (they were found by MacArthur[10]), but the intensity of the radiation is equal to zero at $\theta = 0$.

5. We now consider the radiation not of an isolated particle but of an unbounded ion beam. Assuming for the beam a shifted Maxwellian distribution:

$$f(v) = \mathrm{const}\cdot \exp\{-S^{-2}[v_x^2 + v_y^2 + (v_z - u)^2]\},$$

where $S = (2\kappa T)^{1/2}M^{-1/2}$ — thermal velocity, we calculate the index of refraction at $\theta = 0$ from the general formula for the permittivity of a hot plasma (see, for example,[12]):

$$n^2 = 1 + \frac{i\sqrt{\pi}}{(\omega + i\gamma)^2 k}\sum_{l=1}^{4}\frac{\omega_{0l}^2}{S_l}(\omega + i\gamma - ku_l)\,W(p_l), \quad (4)$$

where k — wave number, γ — increment ($E, H \sim e^{\gamma t}e^{i(kz-\omega t)}$),

$$p_l = \frac{\omega \mp \omega_{H,l} + i\gamma - ku_l}{kS_l},$$

$$W(p) = e^{-p^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^p e^{t^2}dt\right).$$

The sum in (4) is carried out over the electrons and ions of the plasma and the beam ($l = 1, 2, 3, 4$).

Assuming the beam to be hot and the plasma cold, we find from the equation $k^2 = (\omega + i\gamma)^2 n^2/c^2$ the following expression for the increment:

$$\frac{\gamma}{\omega} = -\frac{\omega - ku}{kS}\frac{\omega_0^2}{\omega^2}\frac{1}{2\varepsilon_{\mathrm{pl}} + \omega\partial\varepsilon_{\mathrm{pl}}/\partial\omega}.$$

Equation (5) is obtained for $|p_4| \ll 1$ (that is, $\omega - ku \pm \Omega_1 = 0$, cyclotron instability), for low beam density ($\gamma/kS \ll 1$), and for not too high a temperature ($\omega/kS > 1$). In (5),

$$\varepsilon_{\mathrm{pl}} = (c/v_A)^2[(1+\eta)(1-\alpha\eta)]^{-1}$$

is the permittivity of the cold plasma, ω_0 — plasma frequency of the beam.

In the case of an anomalous Doppler effect ($\omega - ku + \Omega_1 = 0$, superluminal motion), it follows from (5) that $\gamma > 0$, the beam is unstable, the amplitude of the wave increases. Conversely, for the normal Doppler effect (i.e., for $\omega - ku - \Omega_1 = 0$, subluminal motion), it follows from (5) that $\gamma < 0$, i.e., in this case the buildup is replaced by damping.

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