INTERACTION WITH THE SURFACE OF NUCLEI CONTAINING A NUCLEON IN EXCESS OF A CLOSED SHELL

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Nuclei with doubly filled shells with one added nucleon or with one nucleon missing are investigated. Single-particle nucleon energies due only to the central part of the self-consistent potential are singled out by taking into account the interaction with the nuclear surface vibrations. Surface interaction parameters are derived, which explain the experimentally established quadrupole moments and the E2 transition probabilities. The applicability of the perturbation method to light nuclei is discussed.

THE shell potential in which nucleons move in the nucleus is usually determined by investigating nuclei having doubly-filled shells with one nucleon removed or added.[1,2] Thus, for example, Sliv and Volchok[1] have determined the parameters of the potential in the Woods-Saxon form. They noted that the nucleon is acted on not only by the central part of the potential but by an additional part connected with the deformation of the potential surface. However, a hypothesis has been advanced that account of this interaction results in a small level shift (on the order of 100 kev), which does not influence the choice of the parameters and whose influence on the E2 transitions can be determined by perturbation theory.

A check of these assumptions for O^{17}, Pb^{207}, Pb^{209}, and Bi^{209}, reported in the present paper, has shown that the interaction due to the deformation of the potential surface is not small in the case of heavy nuclei, and is particularly large for light nuclei. A more accurate calculation shows that perturbation theory cannot be used to account for the quadrupole moments and for the transition probabilities with the determined parameters of the interaction with the surface, not only in the region of light nuclei, but also for certain heavy nuclei (Pb^{207}). We have obtained the true single-particle nucleon excitation energies, due only to the central potential of the self-consistent nuclear field.

THE NUCLEUS O^{17}

The nucleus O^{17} was chosen for research in the region of light nuclei, since it has been relatively thoroughly investigated; the quadrupole moment and probability of E2 transition ($1/2 \rightarrow 5/2$) have been measured for this nucleus.

The motion of the external neutron and of the surface of the nucleus are described by a Hamiltonian

$$\hat{H} = \hat{H}_C(r) + V_S(\alpha, \mathbf{r}) + \hat{H}_S(\alpha). \tag{1}$$

Here $H_C(r)$ is the central part of the self-consistent potential[1]:

$$H_C = \frac{\nabla^2}{2M} + V(r) - \lambda \left(\frac{\hbar}{2Mc}\right)^2 \frac{1s}{r}\frac{dV}{dr}, \tag{2}$$

where $V(r)$ is the Woods-Saxon potential

$$V(r) = V_0/(1 + e^{\alpha(r-r_0)}); \tag{3}$$

$V_S(\alpha, r)$ is the quadrupole terms of the expansion of the self-consistent potential in Legendre polynomials.

V_S can be regarded as an interaction between the nucleon and the oscillations of the potential surface:

$$V_S(\alpha, \mathbf{r}) = -k(r)\sum_{\mu}\hat{\alpha}_{2\mu}Y_{2\mu}(\theta, \varphi), \tag{4}$$

where $\alpha_{2\mu}$ are the parameters of the nuclear deformation[3]:

$$\alpha_{2\mu} = \sqrt{\hbar\omega/2C}\,(b_\mu + (-1)^\mu b^*_{-\mu}), \tag{5}$$

b^*_μ and b_μ are the creation and annihilation operators for vibrational phonons with spin 2. Then the Hamiltonian H_S, describing the collective oscillations of the surface, is

$$H_S = \hbar\omega\sum_{\mu}(1/2 + b^*_\mu b_\mu). \tag{6}$$

For small deformations of the potential surface we have

$$k(r) = r\,\partial V / \partial r. \quad (7)$$

Calculation shows that independently of the states of the particles, the matrix elements $(nl \mid k(r) \mid n'l')$ are approximately constant and have a value

$$(nl \mid k(r) \mid n'l') = \int_0^\infty R_{nl}(r)\, R_{n'l'}(r)\, r^3 \frac{\partial V}{\partial r}\, dr \approx (-1)^{n+n'} k, \quad (8)$$

where $R_{nl}(r)$ are the radial functions in the potential (2). For heavy nuclei in the lead region $k \approx 36$—40 Mev, while for O^{17} we have $k \approx 30$—35 Mev. If we assume $k = 40$ Mev, as is customarily done, the constant C in (5) assumes the meaning of the effective surface tension.

The eigenfunctions and the eigenvalues of the operators $H_S(\alpha)$ and $H_C(r)$ are known. An eigenfunction of the Hamiltonian (1) is a superposition of the product of the eigenfunctions of Hamiltonians (2) and (4), with different particle angular momentum j and angular momentum of the surface R; only the total momentum of the level I is conserved, with

$$\psi_I = \sum_{jNR} c^I_{jNR} \mid j; NR; I \rangle. \quad (9)$$

The term V_S of (4) is determined by the diagonalization of the energy matrices in the three-phonon approximation. The nuclear energy levels and the coefficients c^I_{jNR} will be functions of two parameters, namely C and the phonon energy $\hbar\omega$. The matrix element of the operator V_S is

$$\left(j; NR; J \mid -k\sum_\mu \alpha_{2\mu} Y_{2\mu} \mid j';\ N'R'; I\right)$$
$$= -(-1)^{n+n'} \cdot 40 \sqrt{\hbar\omega / 2C}\, \delta_{N,N'\pm 1} (-1)^{j+R'-J}$$
$$\times (j \| Y^2 \| j')\, (NR \| b^2 \| N'R')\, W(jRj'R'; I2). \quad (10)$$

The matrix elements $(NR \| b^2 \| N'R')$ given here are taken from the paper by Raz [4] and $W(jRj'R'; I2)$ are Racah coefficients.

Tables I and II list the coefficients c^I_{jNR} for C = 100 Mev with $\hbar\omega$ ranging from 1 to 10 Mev. When $\hbar\omega$ is 1 and 2 Mev, the first excited level with $I = 1/2$ is the level with a single phonon; but when $\hbar\omega$ has values from 3 to 10 Mev, the admixture of levels with phonons is sufficiently large. Since the coefficients of mixture c^I_{jNR} depend relatively weakly on the choice of the single-particle level energy $E_{2S\,1/2}$, we can calculate the quadrupole moment and the reduced probability of the E2 transition $1/2 \rightarrow 5/2$ without specifically fixing this energy. The quadrupole moment of the external neutron is determined by the collective part of the operator Q [3]:

$$Q_S = 3(5\pi)^{-1/2} Z R_0^2 \langle \alpha_{20} \rangle_{M=I}. \quad (11)$$

The matrix element of the operator α_{20} has the form

$$(j; NR; II \mid \alpha_{20} \mid j'; N'R'; II) = C^{II}_{I20} \delta_{jj'} (-1)^{j-R-I} \sqrt{\hbar\omega / 2C}$$
$$\times (NR \| b^2 \| N'R') \sqrt{2I+1}\, W(RIR'I; j2). \quad (12)$$

Let us calculate now the transition probability. By reduced transition probability we mean

$$B(E2) = T(E2) / (\Delta E)^5, \quad (13)$$

where T(E2) is the total probability of γ transition per second; ΔE is measured in Mev. Then

$$B_{exp}(E2) = 0.79 \cdot 10^{10} \text{sec}^{-1} \cdot \text{Mev}^{-5}.$$

Since diagonalization causes the wave function of state (9) to be a mixture of states with different j and different number of phonons, $B_{E2}(I \rightarrow I')$ has the form

$$B(E2) = \frac{4\pi}{75\,\hbar\,(\hbar c)^5} \left[\frac{3ZeR_0^2}{4\pi}\right]^2 \frac{\hbar\omega}{C} (2I'+1)$$
$$\times \left\{ \sum_{jNR, j'N'R'} c^I_{jNR}\, c^{I'}_{j'N'R'} (-1)^{j-I'} (NR \| b^2 \| N'R') \right.$$
$$\left. \times W(RIR'I'; j2) \right\}^2, \quad (14)$$

Table I. Mixture coefficients c^I_{jNR} for C = 100 Mev at various values of $\hbar\omega$ for the level $I = 5/2$

	$\hbar\omega$, Mev					
	1	2	3	5	7	10
$d_{5/2}$; 00; 5/2	0.762	0.830	0.866	0.906	0.926	0.945
$d_{5/2}$; 12; 5/2	−0.524	−0.443	−0.392	−0.327	−0.287	−0.248
$d_{5/2}$; 20; 5/2	0.168	0.116	0.090	0.062	0.048	0.035
$d_{5/2}$; 22; 5/2	0.029	0.033	0.031	0.025	0.021	0.016
$d_{5/2}$; 24; 5/2	0.186	0.140	0.113	0.082	0.064	0.048
$d_{5/2}$; 30; 5/2	−0.025	−0.017	−0.012	−0.007	−0.005	−0.003
$d_{5/2}$; 32; 5/2	−0.044	−0.023	−0.015	−0.008	−0.006	−0.003
$d_{5/2}$; 34; 5/2	−0.012	−0.009	−0.007	−0.004	−0.003	−0.002
$s_{1/2}$; 12; 5/2	−0.243	−0.253	−0.255	−0.240	−0.222	−0.201
$s_{1/2}$; 22; 5/2	0.124	0.100	0.081	0.059	0.046	0.035
$s_{1/2}$; 32; 5/2	−0.046	−0.031	−0.022	−0.013	−0.009	−0.006

Table II. Mixture coefficients c^I_{jNR} for C = 100 Mev at various values of $\hbar\omega$ for the level I = 1/2

	$\hbar\omega$, MeV					
	1	2	3	5	7	10
$d_{5/2}$; 12; 1/2	0.762	0.719	—0.661	—0.564	—0.495	—0.423
$d_{5/2}$; 22; 1/2	—0.385	—0.276	0.211	0.140	0.103	0.074
$d_{5/2}$; 32; 1/2	0.158	0.097	—0.065	—0.036	—0.023	—0.014
$s_{1/2}$; 00; 1/2	—0.442	—0.603	0.701	0.805	0.859	0.901
$s_{1/2}$; 20; 1/2	—0.210	—0.176	0.143	0.101	0.077	0.056
$s_{1/2}$; 30; 1/2	0.076	0.050	—0.035	—0.019	—0.013	—0.008

where

$$R_0 = 1{,}2\, A^{1/3} \cdot 10^{-13} \text{ cm}.$$

As can be seen from Figs. 1 and 2, the quadrupole moment Q and particularly the reduced transition probability B(E2) are very sensitive to changes in C and depend little on $\hbar\omega$. We can state immediately that the values 50 and 300 Mev are not suitable for C, and that C ranges from 100 to 200 Mev. The value of $\hbar\omega$ is determined quite stringently by the probability of transition from the second excited level. But this probability has not been measured experimentally, and we can therefore use for the choice of $\hbar\omega$ the calculations of the energy levels and transition probabilities for the neighboring nucleus O^{18}, for which $\hbar\omega$ = 2—5 Mev. It is seen from Figs. 1—4 that when $\hbar\omega < 7$ Mev perturbation theory, according to which Q and B(E2) are independent of $\hbar\omega$,[5] is inapplicable. Choosing $\hbar\omega \approx 4$ Mev with C ≈ 130 Mev, we find that Q and B(E2) are close to the experimental values:

$$Q_{exp} = -0{,}26 \cdot 10^{-25} \text{ cm}^2,$$

$$B(E2)_{exp} = 0{,}79 \cdot 10^{10} \text{ sec}^{-1} \text{ Mev}^{-5}.$$

Calculations yield

$$Q\,(\hbar\omega = 3;\ C = 120) = -0.3 \cdot 10^{-25} \text{ cm}^2,$$
$$B\,(E2)\,(\hbar\omega = 3;\ C = 120) = 0.65 \cdot 10^{10} \text{ sec}^{-1} \cdot \text{Mev}^{-5};$$
$$Q\,(\hbar\omega = 4;\ C = 130) = -0.29 \cdot 10^{-25} \text{ cm}^2,$$
$$B\,(E2)\,(\hbar\omega = 4;\ C = 130) = 0.70 \cdot 10^{10} \text{ sec}^{-1} \cdot \text{Mev}^{-5}.$$

Having determined $\hbar\omega$ and C, we must take account of the level shifts due to the interaction with the surface. Whereas in perturbation theory these shifts are independent of the state in the second approximation, a more accurate calculation shows that the interaction is much stronger for I = 1/2 than for I = 5/2. Thus, for C = 100 Mev, the level I = 1/2 drops by almost 2.5 Mev, while the level I = 5/2 drops by 1 Mev (see Fig. 3). Therefore, when the interaction with the surface is taken into account, these levels are closer together than in the single-particle case. Thus, the experimentally measured position of the level I = 1/2 is much lower than its single-particle value. With $\hbar\omega$ = 4 Mev and C = 130 Mev, the distance between levels agrees with the experimental value (0.87 Mev) at a single-particle energy $E_{s\,1/2}$ = 1.8 Mev.

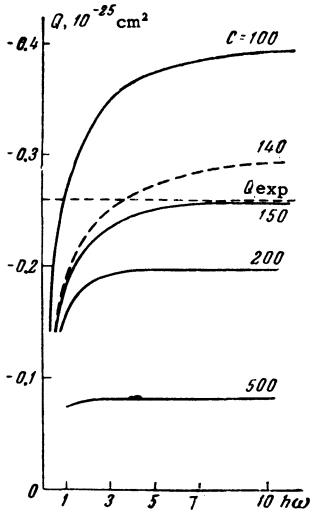

FIG. 1. Quadrupole moment of O^{17} for various C as a function of $\hbar\omega$.

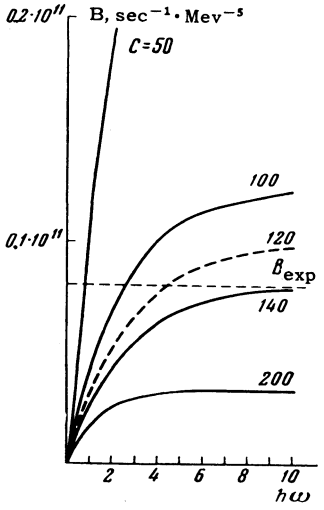

FIG. 2. Reduced probability of E2 transition (1/2 → 5/2) in O^{17} for various C as a function of $\hbar\omega$.

Table III. Comparison of calculated and experimental data* for $\hbar\omega = 3$ Mev and C = 1000 Mev

Nucleus	E, Mev	E_{exp}, Mev	Q, 10^{-24} cm²		T(E2), 10^{10} sec^{-1}	
			theory	expt.	theory	expt.
Pb^{207}	0.48 ($I = 5/2$)	0.57	—	—	0.9	0.9
Pb^{209}	0.76 ($I = 11/2$)	0.75	—0.26	—	—	—
Bi^{209}	0.93 ($I = 7/2$)	0.91	—0.46	—0.4	0.6	—

*E – calculated single-particle energy, E_{exp} – experimental location of excited levels.

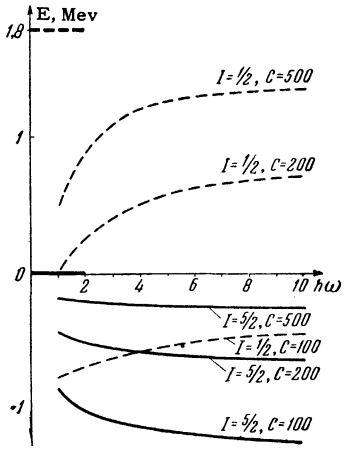

FIG. 3. Change in energy levels of O^{17} due to interaction with the surface, as a function of $\hbar\omega$. The single-particle energies are shown by the heavy lines (levels with $I = 5/2$ are shown solid, levels with $I = 1/2$ dotted).

THE NUCLEI Pb^{207}, Pb^{209}, AND Bi^{209}

The nuclei Pb^{207}, Pb^{209}, and Bi^{209} were investigated analogously. The parameters of the surface interaction were assumed to be approximately the same for these nuclei, inasmuch as the nuclei are close.

From the experimentally known quadrupole moment of Bi^{209} and of the probability of the E2 transition in Pb^{207} ($5/2 \rightarrow 1/2$) we can determine $\hbar\omega$ and C.

Table III lists the calculated data, from which it is seen that when $\hbar\omega = 3$ Mev and C = 1000 Mev, the calculated values of Q and T(E2) are close to the experimental ones. If we take C = 2000 Mev, then the quadrupole moment of Bi^{209} is also close to the experimental value $Q = 0.35 \times 10^{10}$ sec^{-1}. It is obvious that for Pb^{209} and Bi^{209} the value of C lies between 1000 and 2000 Mev. The transition probability in Pb^{207} with C = 200 Mev is found to be too low, $T(E2) = 0.25 \times 10^{10}$ sec^{-1}; it is consequently better to take C = 1000 Mev for Pb^{207}.

Choosing the interaction parameters in this fashion, we can calculate the level shifts in these nuclei. The interaction is most appreciable for Pb^{207} (see Fig. 5). For Bi^{209} and Pb^{209}, the distance between levels remains practically unchanged. In the region of heavy nuclei, calculation by perturbation theory does not introduce appreciable errors in the quadrupole moment and in the transition probability from the first level. For example, assuming C = 1000 Mev, we obtain in the second perturbation-theory approximation T(E2) = 0.93 $\times 10^{10}$ sec^{-1} for Pb^{207} and $Q = -0.50 \times 10^{-24}$ cm² for Bi^{209}. When C = 2000 Mev, $Q = -0.35 \times 10^{-24}$ cm² for Bi^{209}.

However, if the probability of transition from the next levels is taken into account, the results

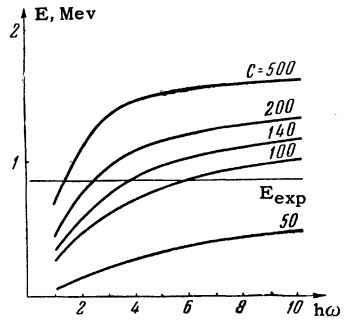

FIG. 4. Location of level $I = 1/2$ in O^{17} as a function of $\hbar\omega$ with $E_{2s1/2} = 1.8$ Mev.

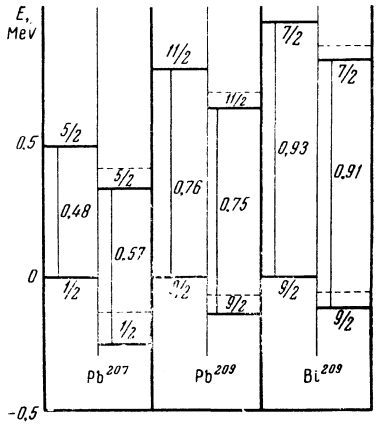

FIG. 5. Account of the interaction with the surface for heavy nuclei. Left column – single particle energies, right column – level shift due to interaction for $\hbar\omega = 3$ Mev, C = 1000 Mev; the dashed lines show the levels shifts for $\hbar\omega = 3$ Mev and C = 2000 Mev.

of perturbation theory are no longer correct,[6] for they can differ by several orders of magnitude from those calculated by the diagonalization method. For levels with small j (as for example in Pb^{207}) the determination of the correct level shifts calls also for a more accurate account of the interaction with the surface.

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[1] L. A. Sliv and B. A. Volchok, JETP **36**, 539 (1959), Soviet Phys. JETP **9**, 374 (1959).

[2] J. Blomqvist and S. Wahlhorn, Ark. Fysik **16**, 545 (1960).

[3] A. Bohr and B. Mottelson, Mat.-Fys. Medd. Dan. Vid. Selsk. **27**, No. 16, (1953).

[4] B. J. Raz, Phys. Rev. **114**, 1116 (1959).

[5] B. J. Raz, Phys. Rev. **120**, 169 (1960).

[6] Guman, Sliv, and Sogomonova, JETP **40**, 341 (1961), Soviet Phys. JETP **13**, 232 (1961).

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