EFFECT OF ZERO-POINT SHAPE VIBRATIONS OF HEAVY NUCLEI ON ALPHA-DECAY PROBABILITIES

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The alpha decay theory previously developed for nonspherical nuclei is used to determine a new shape parameter, the amplitude of the zero-point vibration of the quadrupole deformation. It is shown that different experiments actually measure different deformation values. In particular, the difference between the quadrupole deformation determined from alpha decay and the equilibrium quadrupole deformation can be expressed in terms of the mean square fluctuation of the ground state deformation of the nucleus. This mean square fluctuation is determined from the experimental data for 21 even-even nuclei. It is shown that the amplitude of zero-point quadrupole deformation vibrations has a sharp maximum at the boundary between spherical and nonspherical nuclei.

MEASUREMENTS of various physical nuclear properties that depend on the quadrupole deformation α_2 are frequently dealt with at present. There are zero-point vibrations of this deformation in the ground state of the internal nuclear motion. In fact, according to quantum mechanics [1] the deformation distribution function has the form

$$f(\alpha_2) = (2\pi\overline{x^2})^{-1/2}\exp\{-x^2/2\overline{x^2}\}, \qquad (1)$$

where $x = \alpha_2 - \overline{\alpha}_2$ with $\overline{\alpha}_2$ the equilibrium deformation. There is therefore a question as to which values of the quadrupole deformation are involved in the results of such measurements.

In many experiments (Coulomb excitation, lifetimes of excited states, etc.) the measured quantity is the internal quadrupole moment of the nonspherical nucleus

$$Q_0 = \tfrac{6}{5} Z R_0^2 \alpha_2. \qquad (2)$$

Multiplying by (1) and integrating over α_2 gives again Eq. (2) but with α_2 replaced by $\overline{\alpha}_2$. Thus, it is the equilibrium deformation $\overline{\alpha}_2$ that is involved in the measurements of the electric quadrupole moment of a nonspherical nucleus.

The situation is different in the theory of alpha decay.[2] According to Eq. (2.18) of [2], the probability amplitude for emission of an alpha particle with angular momentum l depends on the quadrupole deformation of the daughter nucleus through a factor $X_l(\beta)$. The true probability amplitude is proportional to the integral

$$\int X_l(\beta) f(\alpha_2)\, d\alpha_2.$$

With the aid of Eq. (2.23) of [2], it is not difficult to see that $X_l(\beta) = e^{\beta} F_l(\beta)$, where $F_l(\beta)$ is a slowly varying function (in the region of deformation values of interest it is sufficient to retain the first few terms in the asymptotic expansion of F_l in inverse powers of β).

Thus, the dependence of the probability amplitude on the deformation is essentially determined by the factor $\exp(b\alpha_2)$, where $b \equiv \beta/\alpha_2$ is given by Eq. (2.23) of [2]. We transform the integrand so as to reveal the position of its maximum:

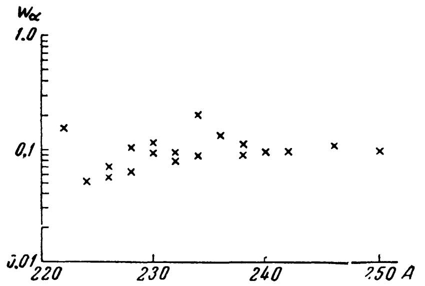
$$e^{b\alpha_2} f(\alpha_2) = \exp\{-b^2\overline{x^2}/2 + b\alpha_2^{(\alpha)}\}(2\pi\overline{x^2})^{-1/2}\exp\{-x_\alpha^2/2\overline{x^2}\}; \qquad (3)$$

$$x_\alpha = \alpha_2 - \alpha_2^{(\alpha)}, \quad \alpha_2^{(\alpha)} = \overline{\alpha}_2 + b\overline{x^2}. \qquad (4)$$

We see that the exponential dependence of the alpha decay probability amplitude on the deformation shifts the maximum of the integrand to the point $\alpha_2 = \alpha_2^{(\alpha)}$. Consequently, it is not the equilibrium deformation $\overline{\alpha}_2$ that enters everywhere in the formulas of the theory of alpha decay,[2] but another deformation $\alpha_2^{(\alpha)}$ defined by Eq. (4).

Since the lifetimes of the rotational 2^+ states in many heavy even-even nuclei have been measured by Bell et al,[3] it is possible to use Eq. (4) to compute the corresponding zero-point vibration amplitudes $(\overline{x^2})^{1/2}$. The results are listed in the table. The deformation $\alpha_2^{(\alpha)}$ was determined from the alpha decay fine structure and Eq. (2.22) [2], by the procedure described in [4]. The values of $\overline{\alpha}_2$ are taken from [3]. The first lines of the table show nuclei near the boundary between the regions of spherical and nonspherical nuclei. The nucleus Ra^{222} is apparently of an intermediate type, and the theory of nonspherical nuclei is not fully ap-

Nucleus	$\alpha_2^{(\alpha)}$	$\bar{\alpha}_2$	$(\overline{x^2})^{1/2}$	$(\overline{x^2})^{1/2}/\bar{\alpha}_2$	Nucleus	$\alpha_2^{(\alpha)}$	$\bar{\alpha}_2$	$(\overline{x^2})^{1/2}$	$(\overline{x^2})^{1/2}/\bar{\alpha}_2$
Ra^{222}	0.18	0.12	0.07	0.6	Th^{234}	0.17	0.15	0.04	0.3
Ra^{224}	0.24	0.11	0.10	0.9	U^{230}	0.15	0.15	~0	~0
Ra^{226}	0.22	0.12	0.08	0.7	U^{232}	0.22	0.16	0.07	0.4
Ra^{228}	0.21	0.13	0.08	0.6	U^{234}	0.20	0.16	0.06	0.4
Th^{226}	0.23	0.14	0.08	0.6	U^{236}	0.17	0.17	~0	~0
Th^{228}	0.21	0.14	0.07	0.5	U^{238}	0.16	0.17	~0	~0
Th^{230}	0.19	0.15	0.06	0.4	Pu^{238}	0.19	0.17	0.04	0.2
Th^{232}	0.19	0.15	0.05	0.3	Pu^{240}	0.17	0.17	~0	~0



plicable to it.* As is evident from the last column of the table, the quantum fluctuations of the quadrupole deformation are quite significant near the boundary of the region of stability of nonspherical shape.

An independent estimate of the zero-point vibration amplitude can be obtained from the probabilities w_α for formation of alpha particles within a nucleus. These probabilities are determined from the observed half-lives.[4,5] It is easy to see that the effect of the zero-point deformation vibrations is to renormalize these internal formation probabilities. Integrating both sides of Eq. (3) over α_2 and squaring, we obtain

$$w_\alpha = \tilde{w}_\alpha \exp(-b^2\overline{x^2}), \quad (\overline{x^2})^{1/2} = b^{-1}\ln^{1/2}(\tilde{w}_\alpha / w_\alpha), \qquad (5)$$

where $\tilde{w}_\alpha$ is the true internal formation probability and w_α is the experimentally observed renormalized internal formation probability, which is determined from the experimental data by the procedure described in references 4 and 5. The values of w_α for twenty even-even nuclei are plotted against mass number in the figure. The general trend of the internal formation probabilities reflects some increase of the renormalization logarithm $\ln(w_\alpha/\tilde{w}_\alpha)$ with decreasing mean square quadrupole deformation fluctuation. Unfortunately, the internal formation probability fluctuates from nucleus to nucleus, so that we had to average $\ln w_\alpha$ over several nuclei in order to use Eq. (5). In order to estimate the zero-point vibration amplitude $(\overline{x^2})^{1/2}$ we averaged $\ln w_\alpha$ over the five nuclei $Ra^{224,226,228}$ and $Th^{226,228}$ (for the reason mentioned above, the extreme left point of the figure, corresponding to Ra^{222}, was omitted from consideration) and, assuming that there is no renormalization in the other fourteen, we obtained a corresponding average of $\ln \tilde{w}_\alpha$. Substitution in Eq. (5) gave $(\overline{x^2})^{1/2} = 0.05$. The fact that this estimate is somewhat lower than the values in the table is probably due to the neglect of the zero-point vibrations in the region A > 228.

In conclusion, we note that the zero-point deformation vibration amplitude also rises sharply if the nonspherical shape stability boundary is approached from the region of spherical nuclei. This is easily seen from the experimental data on electric quadrupole $2^+ \rightarrow 0^+$ transition probabilities for isotopes of radon [3] and polonium.[6] The quadrupole vibrations of spherical nuclei are fivefold degenerate.[6] If, of the five degrees of freedom, we consider just the one corresponding to the oscillation which does not destroy the axial symmetry of the nuclear shape with respect to some direction in space, then the amplitude of the zero-point vibrations of the corresponding deformation is determined from the relation

$$w = \frac{3}{125}\frac{e^2\omega^5}{\hbar c^5} Z^2 R_0^4 \overline{x^2}, \qquad (6)$$

where w is the probability per unit time for emission of the nuclear gamma from the 2^+ first excited state, $\hbar\omega$ is the energy of the gamma, and $\overline{x^2} \equiv \overline{\alpha_2^2}$ is the mean square quadrupole deformation fluctuation in the ground state of the nucleus and is actually the measured quantity in this case.

The mean square fluctuations $(\overline{\alpha_2^2})^{1/2}$ calculated with an assumed value $R_0 = 1.2\,A^{1/3}$ Fermi units, are:

Nucleus	Po^{212}	Po^{214}	Em^{218}	Em^{220}	Em^{222}
$(\overline{\alpha_2^2})^{1/2}$:	0.015	0.021	>0.025	0.035	0.039

It is evident that large quantum fluctuations of the quadrupole deformation are a characteristic feature of the nuclear "phase transition" corresponding to a change in the symmetry of the equilibrium shape.

*In this connection, see below, and also the seventh footnote of [4].

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[6] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

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