DETERMINATION OF THE HYPERFINE SPLITTING ENERGY OF THE 1s MUONIUM STATE

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The hyperfine splitting of the ground state of muonium is estimated on the basis of the available experimental data on the dependence of μ^+ polarization in nuclear emulsion on the magnetic field.

THE aim of the present work is to obtain an estimate of the hyperfine splitting in the ls state of muonium from the existing data on the dependence of the polarization of positive muons in nuclear emulsion on the strength of the applied magnetic field.[1-3]

It is well known that the main mechanism for depolarization of positive muons in matter is the formation of muonium atoms (μ^+, e^-) in the ls state. An external magnetic field that is strong enough to break the connection between the magnetic moments of the muon and electron eliminates the depolarization, which arises from the hyperfine splitting in this state. The theoretical treatment[4,5] of this problem shows that the polarization P depends on the applied field H according to

$$P = 3a = P_0\left(\frac{1}{2} + \frac{x^2}{2(1+x^2)}\right), \qquad x = \frac{H}{H_0}. \tag{1}$$

Here a is the asymmetry coefficient of the spatial distribution of electrons in $\mu^+ \to e^+$ decay, P_0 is the muon polarization at the instant of decay, and H_0 characterizes the average field in the first electronic Bohr orbit in muonium due to the muon magnetic moment. H_0 is related to the hyperfine splitting ΔE of the ls state:

$$H_0 = \Delta E / (-2\mu_e + 2\mu_\mu) \tag{2}$$

where μ_e and μ_μ are the magnetic moments of the electron and muon respectively. ΔE is given by the well-known Fermi formula

$$\Delta E \cong 32\mu_e\mu_\mu / 3a_B^3, \tag{3}$$

where a_B is the radius of the first Bohr orbit in muonium. If the magnetic moment values $\mu_e = e\hbar/2m_e c$ and $\mu_\mu = e\hbar/2m_\mu c$ are substituted into (2) and (3), we obtain $H_0 = 1580$ gauss. Taking into account radiative corrections and the effect of the reduced mass changes this value to $H_0 = 1588$ gauss.

Actually, when muons are stopped in emulsion[1,2] or other substances[7] the growth of polarization with magnetic field does not obey Eq. (1) because of the charge exchange which occurs when a nearly-spent muon which is still sufficiently fast loses and captures electrons several (n) times. Each new electron capture further "dilutes" the polarization; therefore, for a given value of the field H the value of the polarization P is less than that predicted by Eq. (1). Ferrell, Lee, and Pal[8] obtained a formula which takes into account the charge exchange near the end of the muon path:

$$P = P_0\left[1 - \frac{1}{2}\left(\frac{1}{1+\tau^{-2}+x^2}\right)\right]^n. \tag{4}$$

Here $\tau/2$ is the average lifetime of the muonium (in units of $\hbar/\Delta E = 3.6 \times 10^{-11}$ sec) in each capture. If only one capture occurs ($n = 1$, $\tau = \infty$), this formula becomes the same as Eq. (1).

Rather accurate measurements have been made[1-3] of the asymmetry parameter $a = P/3$ in nuclear emulsion placed in a magnetic field. All of the experiments were performed by nearly identical methods and similar criteria for selecting events were used. The consistent group of data thus obtained is shown in Table I. Assuming the depolarization mechanism described by Eq. (3), we can use the data of Table I to evaluate the depolarization parameters n and τ and the hyperfine splitting ΔE.

The problem consists of finding the minimum of the function

$$\chi^2 = \Sigma(\Delta a_i)^2/\sigma_i^2$$

under simultaneous variation of the parameters τ, n, and H_0, i.e., of solving the system of equations

$$\partial\chi^2/\partial\tau = \partial\chi^2/\partial n = \partial\chi^2/\partial H_0 = 0. \tag{5}$$

In the analysis of the data it is necessary to take into account the fact that some of the muons

Table I

Reference	H, gauss	− a	Reference	H, gauss	− a
[1]	0	0.095±0.009	[1]	2500	0.206±0.020
[1]	54	0.117±0.020	[1]	3500	0.185±0.020
[1]	110	0.110±0.020	[1]	5100	0.196±0.020
[1]	206	0.093±0.020	[1]	6300	0.265±0.021
[2]	405	0.114±0.021	[2]	10000	0.263±0.012
[1]	420	0.142±0.020	[1]	14000	0.262±0.028
[1]	680	0.149±0.017	[1]	17000	0.280±0.014
[2]	805	0.096±0.020	[2]	20000	0.272±0.023
[1]	1300	0.142±0.017	[2, 3]	25000	0.290±0.013
[2]	1610	0.178±0.020	[2]	27000	0.284±0.016
[1]	1900	0.155±0.022	[2]	35000	0.290±0.013
[2]	2370	0.184±0.020			

stop in the gelatin, where the depolarization is either small or absent, and some stop in the AgBr crystals which are responsible for most of the depolarization.[9] If we let f be the ratio of the number of stoppings in the gelatin to the total number of muon stoppings in the emulsion, then we obtain

$$P = 3a = P_0\left[f + (1-f)\left(1 - \frac{0.5}{1-\tau^{-2}+x^2}\right)^n\right] \quad (6)$$

which is to be used in the analysis of the data in Table I. As the most likely value of f we choose f = 0.32,[1] which is obtained as the ratio of the asymmetry coefficient a(0) for H = 0 to the asymmetry coefficient $a(\infty)$ obtained by averaging the data in Table I for $H \geq 17$ kilogauss: f = (0.095 ± 0.009)/(0.292 ± 0.007). P_0 in (6) is taken to be $P_0 = 3a(\infty) = 0.88 \pm 0.02$.

The solution of (5) obtained on an electronic computer by the method of "gradient descent" is $\tau = 1.22$, n = 6.90, and $H_0 = 1560$ gauss, with $\chi^2 = 28$. Since the number of degrees of freedom is 22, the result for χ^2 is rather good.

The weak links in this determination of H_0 and thence, via Eq. (2), the hyperfine splitting ΔE in the ls state of muonium are first, the use of a specific mechanism for the dilution of polarization which occurs in the slowing-down process, and second, the value of f the fraction of stoppings in the gelatin. It can be shown, however, that all values of f in a wide range give the same value of H_0.

Table II shows the solutions we obtained for n, τ, and H_0 for values of f in the interval 0—0.5.

We see that a radical change in f leads to noticeable changes in the parameters n and τ but does not change the value of H_0 nor, consequently, the value of ΔE. The statistical accuracy of our determination of ΔE, as evaluated by the usual method of constructing the error matrix,[10] is about 10%.

Table II

f	χ^2	n	τ	H_0
0	26.2	5.0	0.73	1560
0.20	26.8	6.1	0.88	1560
0.33	28.2	6.9	1.22	1560
0.40	32.1	7.4	2.50	1560
0.50	65.0	12.1	2.58	1560

Thus, the value we obtain for the hyperfine splitting of the ls state of muonium agrees, within the limits of error, with the theoretical value.

Measurements analogous in principle to these were carried out by Prepost, Hughes, and Ziock[11] on muons stopping in argon. Their experiment, performed with electronic counters, showed that the observed increase of polarization with increasing field was consistent with the theoretical value of ΔE. Their measurements were not sufficiently accurate to allow them to make a quantitative evaluation of ΔE.

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