SINGLE PARTICLE EXCITED STATES AND THE MODEL DESCRIPTION OF LIGHT NUCLEI

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We show that the location and reduced widths of the positive parity levels in $C^{13}-N^{13}$ are in good agreement with the unified model. The reason for the small separation between the ground state of $C^{13}-N^{13}$ and the closest single-particle levels with positive parity is the large nuclear deformation. The general features of the giant dipole resonance are in good agreement with the picture of a strong deformation in the light nuclei. The suggestion is made that in O^{17} the outer nucleon moves in the field of a highly nonspherical core with tetrahedral symmetry. Possibly the most generally suitable description of the collective properties of light nuclei is given by the α model, where in many cases ($C^{13}-N^{13}$, $Mg^{25}-Al^{25}$, etc.) the description of the single-particle levels in the framework of the α model may be close to the description on the unified model.

FROM an analysis of the single-particle excited states of p-shell nuclei, in particular $C^{13}-N^{13}$, Lane[1,2] proposed the hypothesis of a weak coupling of the 2s and 1d nucleons with the rest of the nucleus (the 1p core). We want to point out various objections to this point of view.

In $C^{13}-N^{13}$ the separation between the single-particle level of the p shell (ground state) and the nearest single-particle levels of the 1d—2s shell is 3—4 Mev, which is approximately one fourth of the value obtained from electron scattering by C^{12} (where $\hbar\omega = 16$ Mev was used[3] in describing C^{12} by means of harmonic oscillator functions on the shell model). A similar situation exists for the other well-investigated $Mg^{25}-Al^{25}$ nucleus. It is known[4] to have a sizable deformation ($\delta = +0.3-0.4$, where the notation is that of Nilsson[5]) and therefore the distance between the ground state, in which the odd nucleon is in the 1d—2s shell, and the nearest levels of the 1f—2p shell is ~ 4 Mev, even though $\hbar\omega_0^0 = 13$ Mev, as is indicated by the electron scattering data.[3] It is natural to attempt an explanation of the features of the levels of C^{13} on a similar basis.

A comparison of the wave functions for C^{12} on the shell model and on the unified model shows that the description of C^{12} on the unified model is very close to the shell model description if the nucleus is highly oblate.[6] Taking $\hbar\omega_0^0 = 16$ Mev (from electron scattering[3]) and $C = -2$ Mev (from the spin-orbit splitting in $O^{17}-F^{17}$ and $Ca^{41}-Sc^{41}$)[7,8] and using Nilsson's results,[5] we get the correct positions of all the known positive-parity levels of $C^{13}-N^{13}$ as well as their reduced nucleon widths for a value of $\delta \approx -0.6$. In the computations the moment of inertia was found from the position of the 2^+ level of C^{12}. Corrections for coupling of the motion of the external nucleon to the rotation amount to 0.2—0.3 Mev, so that they were omitted. The results for $\delta = -0.65$ are shown in the table. The ground state of $C^{13}-N^{13}$ is orbit 4.

In addition to the reduced widths θ^2 given in the table, θ^2 was computed for the s-nucleon decay of the 6.91 (7.64) Mev level of C^{13}(N^{13}) to the 2^+ level of C^{12}. The computation gave $\theta^2 = 0.35$, in approximate agreement with experiment.[9] Such a picture also gives completely satisfactory agreement with the general features of the giant resonance dipole absorption of γ quanta in Li^7, $C^{12}-C^{13}$ and $Mg^{24}-Mg^{25}$, including the absolute position of the maximum, which is calculated by the method of ref. 10: for Li^7, in accordance with the work of Inopin et al[11] and Kurath and Pičman,[6] large values of the deformation are used.

Although the computations were done with wave functions containing "superfluous" coordinates, a comparison of a whole series of specific results with the results of computations made with exact functions shows that they are in good agreement, if, as one should have anticipated, the Bohr-Mottelson functions are taken with a somewhat smaller deformation parameter than the exact functions. However, it remains a mystery why in $O^{17}-F^{17}$ the lowest single-particle levels of the

Positive parity levels of C^{13}

Orbits	5	6	6	6	7	8	9	11
$\Omega\pi$	$5/2^+$	$1/2^+$	$1/2^+$	$1/2^+$	$3/2^+$	$3/2^+$	$1/2^+$	$1/2^+$
J	$5/2$	$1/2$	$3/2$	$5/2$	$3/2$	$3/2$	$1/2$	$1/2$
E_{exc}(Mev) theor.	3.5	3.0	8,7	6,7	7.5	13	15	23
E_{exc}(Mev) exp.	3,85	3.09	7.64	6.86	8.3			
θ^2 theor.	0,35	0,35	0.04	0.08	0,30			
θ^2 exp.	0,2	0.5	0.04	0.01	0.5			

1f—2p shell also appear, starting with an excitation of 4—5 Mev. For example, the $3/2^-$ level at 4.55 (4.4) Mev in O^{17} (F^{17}) has a reduced width which is approximately the same as that of the $1d_{5/2}$ and $1d_{3/2}$ levels.[1,7,12,13]

It seems to us that the only description of all the above facts from a single viewpoint is the description by means of the α model. The α model immediately shows that in Be^9 the even-even core in whose field the odd nucleon moves, is highly prolate, whereas it is highly oblate in C^{13}. Moreover the α model predicts that the deviation from spherical shape is much greater for Ne^{21} than for Mg^{25}. In fact, for Ne^{21}, $\delta = 0.5—0.6$, if the 4.81 Mev level is the ground level of orbit 7 (on the basis of the experimental data for Ca^{41},[8] we set D = 0 for the 1f—2p shell). The success of the unified model in describing Be^9, C^{13}, Mg^{25} and other nuclei may be related to the fact that for these nuclei it is a good approximation to replace the α core by an ellipsoid. Only for the nuclei near O^{16} will this approximation be unsuccessful (tetrahedral nuclei), and here even for a crude description one needs a different mathematical apparatus from that of the unified model (cubic harmonics). The small separation of the different shells in O^{17} shows that there is a marked deviation of the core from spherical shape, i.e., there is a strong coupling of the nucleon with the deformed (tetrahedral) core. Another indication of the strong coupling is the fact that the reduced widths of all the single-particle levels of O^{17} — F^{17}, excepting possibly the $2s_{1/2}$ level, are two to three times smaller than the Wigner limit. (This same situation is also characteristic for the unified model when there is strong coupling of the nucleon to the core.[14])

In conclusion we emphasize that the experimental data show that the nonadiabatic coupling of the external nucleon to the core vibrations plays an important part in electromagnetic transitions (whereas it has little effect on level positions). Thus in O^{17} the quadrupole moment corresponds to an "effective charge" of the neutron of ~ 0.5;[15] one can draw a similar conclusion for C^{13} on the basis of recent results of Wilkinson et al.[16] for the intensity of the 3.85 Mev → 3.09 Mev E2 transition in C^{13}. The appearance of the α model vibrations (T = 0) in this case is indicated by the fact that E1 transitions between the low-lying states of C^{13} — N^{13} seem to be quite strongly inhibited (by a factor $\sim 10^{-2}$ compared to a Weisskopf unit[2]). This is explained qualitatively by the selection rules for the asymptotic quantum numbers in the unified model,[2] although quantitatively the unified model gives a very much stronger inhibition (10^{-4} — 10^{-5}).[2] Such a reduction of the inhibition is not surprising in view of the large effect of the nonadiabaticity on the radiation widths, and the differences between the wave functions of the α model and the unified model.

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