ABSORPTION OF ELECTROMAGNETIC WAVES IN PLASMA

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The absorption coefficient of electromagnetic waves in a plasma is calculated for $e^2/\hbar v \ll 1$. Application of the temperature diagram technique makes it possible to take systematic account of collective effects and to obtain exact values of the factors in the Coulomb logarithm.

1. As is well known, the Coulomb scattering cross section diverges logarithmically. This leads to the necessity of taking into account the screening, which is generally dynamic.[1,2] The kinetic plasma coefficients contain the Coulomb logarithm $\ln(r_{max}/r_{min})$, where r_{max} and r_{min} are respectively the maximum and minimum impact parameters of the collision. The order of magnitude of these quantities can be established from simple physical considerations. For sufficiently low frequencies, the Debye radius can be used for r_{max}.[3] For frequencies greatly exceeding the plasma frequency, the role of r_{max} is played by the path traversed by the electrons in a single period.[4] The value of the parameter r_{min} depends on the plasma temperature. For high temperatures, when this distance becomes less than the de Broglie wavelength of the electrons, this wavelength can be used for r_{min}.[5] All these considerations permit us to determine the expression under the logarithm with accuracy up to a factor of the order of unity.

The purpose of the present paper is a rigorous calculation of the absorption coefficient of electromagnetic waves in a high-temperature plasma. An exact value is obtained for the expression under the logarithm for all frequencies of the field that are much higher than the collision frequency (including frequencies of the order of the plasma frequency). A plasma is considered with singly-ionized positive ions.

2. For the determination of the absorption coefficient, one must compute the real part of the electron conductivity of the plasma. We shall start out from the general expression for the conductivity[2] in the absence of a magnetic field:

$$\sigma_{\mu\nu}(\mathbf{k},\omega)=\left(\frac{e\hbar}{m}\right)^2\int p_\mu G_{pp'}(\mathbf{k},\omega)\,p'_\nu\,\frac{d^3p\,d^3p'}{(2\pi)^6}, \qquad (1)$$

where $\mathbf{k}$ is the wave vector of the electromagnetic field, ω is its frequency, $\mathbf{p}$ and $\mathbf{p}'$ are the wave vectors of the electrons, while e and m are the charge and mass of the electron. The function $G_{pp'}(\mathbf{k},\omega)$ is determined by the relations

$$G_{pp'}(\mathbf{k},\omega)=\int_0^\infty \exp[(i\omega-\nu)\tau]\,\tilde G_{pp'}(\mathbf{k},\tau)\,d\tau, \qquad \nu\to+0, \qquad (2)$$

$$\tilde G_{pp'}(\mathbf{k},\tau)=\int_0^{1/T} d\lambda\,\langle A_{p',k}(\tau-i\hbar\lambda)\,A_{p,-k}(0)\rangle, \qquad (3)$$

where

$$A_{p,k}(z)=\exp(-H'z/i\hbar)\,a^+_{p-k/2}a_{p+k/2}\exp(H'z/i\hbar).$$

$H'=H-\mu N$; H is the Hamiltonian of the system, μ is the chemical potential, T is the temperature in energy units, $\langle\ldots\rangle$ denotes the statistical average.

For calculation of the function $G_{pp'}(\mathbf{k},\omega)$, we use the diagram technique suggested by Abrikosov, Gor'kov, and Dzyaloshinskii.[6] Performing integration by parts in Eq. (2), we get

$$G_{pp'}(\mathbf{k},\omega)=\{K^R_{pp'}(\mathbf{k},\omega)-K_{pp'}(\mathbf{k},0)\}/i\omega, \qquad (4)$$

where

$$K^R_{pp'}(\mathbf{k},\omega)=\int_{-\infty}^{\infty} e^{i\omega\tau}\tilde K^R_{pp'}(\mathbf{k},\tau)\,d\tau, \qquad (5)$$

$$K_{pp'}(\mathbf{k},\omega_n)=\frac{1}{2}\int_{-1/T}^{1/T} e^{\hbar\omega_n\lambda}\,\tilde K_{pp'}(\mathbf{k},\lambda)\,d\lambda,$$

$$\hbar\omega_n=2\pi inT, \qquad n=0,\pm1\ldots, \qquad (6)$$

$$\tilde K^R_{pp'}(\mathbf{k},\tau)=\theta(\tau)\,\partial\tilde G_{pp'}(\mathbf{k},\tau)/\partial\tau=(i/\hbar)\langle[A_{p',k}(\tau),\ A_{p,-k}(0)]\rangle\,\theta(\tau), \qquad (7)$$

$$\tilde K_{pp'}(\mathbf{k},\lambda)=\langle T_\lambda(A_{p',k})_\lambda(A_{p,-k})_0\rangle; \qquad (A)_\lambda=e^{\lambda H'}Ae^{-\lambda H'}. \qquad (8)$$

It can be stated[6] that the function $K^R_{pp'}(\mathbf{k},\omega)$ is the analytic continuation of the function $K_{pp'}(\mathbf{k},\omega_n)$ in the variable ω from discrete points of the imaginary axis $(n>0)$ to the real axis.

3. The function $K_{pp'}(\mathbf{k},\omega_n)$ is the sum of graphs of which some are shown in Fig. 1. In cor-

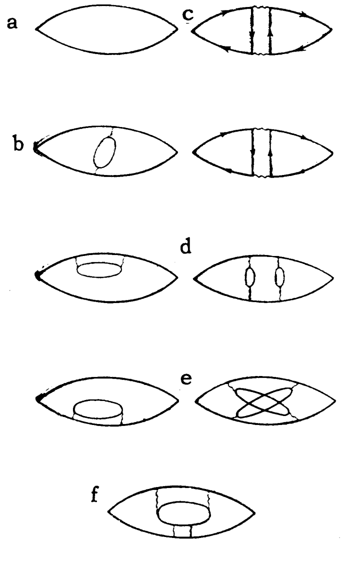

FIG. 1

respondence with the rules of application of the diagram technique,[6] to each electron line on the graph there corresponds the quantity

$$\frac{1}{2}\int_{-1/T}^{1/T} e^{\lambda e_n}\langle T_\lambda (a_q)_\lambda (a_q^+)_0\rangle\, d\lambda = -G_q(e_n) = -(e_n - \varepsilon_q)^{-1}, \tag{9}$$

where $e_n = (2n+1)\pi i T$, $\epsilon_q = \hbar^2 q^2/2m - \mu$. The interaction matrix element $U_\gamma = 4\pi e^2/\gamma^2$ corresponds to the wavy line; a minus sign corresponds to each closed loop.

Account of the interaction of the electrons with the ions leads to the appearance of graphs similar to those shown in Fig. 1, on which internal loops are formed by the ion lines. The ion lines will be henceforth drawn dotted. One must integrate over each internal wave vector $\mathbf{q}$ in the diagram, so that $(2\pi)^{-3}\int d^3q$ corresponds to it. Summation is carried out over each internal imaginary "energy" α_n so that $T\Sigma_n$ corresponds to it.

Without account of the interaction between the plasma particles the conductivity is determined by only one graph, shown in Fig. 1a. If the interaction potential were short-range, then we could limit ourselves in the calculation of the absorption coefficient to the graphs shown in Figs. 1b, c and the graphs which are obtained by substitution of the internal electron loop for the ion (see Fig. 1b). It can be expected[2] that graphs of the type shown in Fig. 1d make a small contribution to the conductivity if $\omega/\nu \gg 1$ (ν is the collision frequency); graphs of the type shown in Fig. 1e are small if the gas condition holds: $e^2 n^{1/3}/T \ll 1$. Finally, one can neglect graphs of the type shown in Fig. 1f if $e^2/\hbar v \ll 1$ (v is the thermal velocity of the electron).

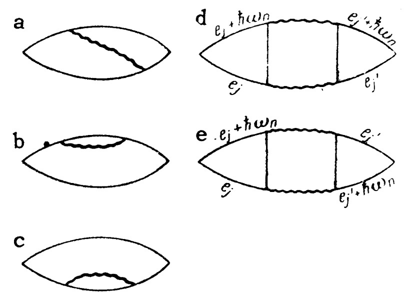

FIG. 2

Because of the presence of the Coulomb divergence the interaction must be renormalized. The renormalization reduces to the result[1,2] that we must calculate the graphs shown in Fig. 2 in place of those shown in Figs. 1b and c. The thick wavy line satisfies the equation shown in Fig. 3.

This equation corresponds to the function

$$D_\gamma(\alpha_m) = \{P_\gamma(\alpha_m) + P'_\gamma(\alpha_m) - U_\gamma^{-1}\}^{-1}, \tag{10}$$

$$P_\gamma(\alpha_m) = (2\pi)^{-3}\int d^3q\,(n_{\mathbf{q}+\gamma/2} - n_{\mathbf{q}-\gamma/2})(\varepsilon_{\mathbf{q}+\gamma/2} - \varepsilon_{\mathbf{q}-\gamma/2} - \alpha_m)^{-1}, \tag{11}$$

$$P'_\gamma(\alpha_m) = (2\pi)^{-3}\int d^3q\,(n'_{\mathbf{q}+\gamma/2} - n'_{\mathbf{q}-\gamma/2})(\varepsilon'_{\mathbf{q}+\gamma/2} - \varepsilon'_{\mathbf{q}-\gamma/2} - \alpha_m)^{-1}; \tag{12}$$

where $\alpha_m = 2m\pi i T$, $n_q = \exp(-\epsilon_q/T)$; the primed quantities n'_q and ϵ'_q are obtained from the unprimed ones by replacing the electron mass by the ion mass.

4. We proceed to the calculation of the graphs of Fig. 2. We shall consider the case in which the wavelength of the electromagnetic wave is much greater than the maximum impact parameter. Under these conditions, we can neglect spatial dispersion.*

The expressions corresponding to the graphs in Figs. 2a, b, c have the form

$$K_{pp'}^{(a)}(\omega_n) = T\sum_m D_{p-p'}(\alpha_m)\,T\sum_j G_p(e_j + \hbar\omega_n)\,G_p(e_j) \times G_{p'}(e_j + \alpha_m + \hbar\omega_n)\,G_{p'}(e_j + \alpha_m),$$

$$K_{pp'}^{(b)}(\omega_n) = \int\frac{d^3\gamma}{(2\pi)^3}\,T\sum_m D_\gamma(\alpha_m)\,T\sum_j [G_p(e_j + \hbar\omega_n)]^2 \times G_{p+\gamma}(e_j + \alpha_m + \hbar\omega_n)\,G_p(e_j)\,(2\pi)^3\,\delta(\mathbf{p} - \mathbf{p}').$$

$$K_{pp'}^{(c)}(\omega_n) = \int\frac{d^3\gamma}{(2\pi)^3}\,T\sum_m D_\gamma(\alpha_m)\,T\sum_j G_p(e_j + \hbar\omega_n)\,[G_p(e_j)]^2 \times G_{p+\gamma}(e_j + \alpha_m)\,(2\pi)^3\,\delta(\mathbf{p} - \mathbf{p}').$$

Carrying out the summation over j and adding all three expressions, we get

*Account of spatial dispersion may prove to be essential in an analysis of the surface impedance in the case $c/\omega_0 < v/\omega$ (ω_0 is the plasma frequency, v is the thermal velocity).

FIG. 3

$$(2\pi)^{-6}\int d^3p\,d^3p'\,p_\mu p'_\nu\,[K^{(a)}_{pp'}(\omega_n)+K^{(b)}_{pp'}(\omega_n)+K^{(c)}_{pp'}(\omega_n)]$$
$$=(2\pi)^{-3}(\hbar\omega_n)^{-2}\int\gamma_\mu\gamma_\nu d^3\gamma\,T\sum_m D_\gamma(\alpha_m)\,[P_\gamma(\alpha_m+\hbar\omega_n)-P_\gamma(\alpha_m)]. \qquad (13)$$

Here P_γ is determined by Eq. (11). In similar fashion, after summation over j and j′, the sum of the graphs represented in Figs. 2d and e leads to the expression

$$(2\pi)^{-6}\int d^3p\,d^3p'\,p_\mu p'_\nu\,[K^{(d)}_{pp'}(\omega_n)+K^{(e)}_{pp'}(\omega_n)]$$
$$=(2\pi)^{-3}(\hbar\omega_n)^{-2}\frac{1}{2}\int\gamma_\mu\gamma_\nu d^3\gamma\,T\sum_m D_\gamma(\alpha_m)\,D_\gamma(\alpha_m+\hbar\omega_n)\times[P_\gamma(\alpha_m+\hbar\omega_n)-P_\gamma(\alpha_m)]. \qquad (14)$$

Making use of Eq. (10) [after addition of the expressions (13) and (14)], we get

$$(2\pi)^{-6}\int d^3p\,d^3p'\,p_\mu p'_\nu K^{(2)}_{pp'}(\omega_n)$$
$$=-\int\frac{d^3\gamma}{(2\pi)^3}\frac{\gamma_\mu\gamma_\nu}{2(\hbar\omega_n)^2}T\sum_m D_\gamma(\alpha_m)\,D_\gamma(\alpha_m+\hbar\omega_n)\,\{P'_\gamma(\alpha_m+\hbar\omega_n)-P'_\gamma(\alpha_m)\}\{P_\gamma(\alpha_m+\hbar\omega_n)-P_\gamma(\alpha_m)\}. \qquad (15)$$

Summation over m in Eq. (15), and analytic continuation in ω can be carried out as follows. Let $F(\omega)$ be the analytic continuation of the function

$$T\sum_m\varphi(\alpha_m)\,\psi(\alpha_m+\hbar\omega_n)$$

in the variable ω from discrete points on the imaginary axis ($n > 0$) to the real axis. Further, let the functions $\varphi(\alpha)$ and $\psi(\alpha)$ coincide in the upper half plane with the analytic functions $\varphi^R(\alpha)$ and $\psi^R(\alpha)$, and in the lower half plane with the functions $\varphi^A(\alpha)$ and $\psi^A(\alpha)$, wherein the functions $\varphi^R(\alpha)$ and ψ^R are obtained on the real axis from the functions φ^A and ψ^A by complex conjugation and $\varphi(0)=[\varphi^R(0)+\varphi^A(0)]/2$, $\psi(0)=[\psi^R(0)+\psi^A(0)]/2$. Then the following relation holds:*

$$\operatorname{Im}F(\omega)=\operatorname{sh}\frac{\hbar\omega}{2T}\frac{1}{2\pi}\int_{-\infty}^{\infty}d\alpha\,\frac{\operatorname{Im}\varphi^R(\alpha)}{\operatorname{sh}(\alpha/2\pi)}\,\frac{\operatorname{Im}\psi^R(\alpha+\hbar\omega)}{\operatorname{sh}[(\alpha+\hbar\omega)/2T]}. \qquad (16)$$

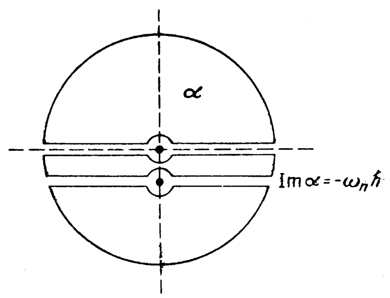

FIG. 4

*sh = sinh.

The integral is taken in the sense of its principal value. The relation (16) follows from the formula

$$T\sum_m\varphi(\alpha_m)\,\psi(\alpha_m+\hbar\omega_n)$$
$$=\frac{1}{4\pi i}\int_C\operatorname{cth}\frac{\alpha}{2T}\,\varphi(\alpha)\,\psi(\alpha+\hbar\omega_n)\,d\alpha$$
$$+T\varphi(0)\,\psi(\hbar\omega_n)+T\varphi(-\hbar\omega_n)\,\psi(0). \qquad (17)^*$$

The contour C consists of the three closed contours shown in Fig. 4, in which the paths along the large circle are infinitely far away. The integrals over the circle of large radius are equal to zero, so that integration over the contour C reduces to integration over the edges of the two cuts in the integrand functions.

The integrals over the small circles surrounding the points $\alpha = 0$ and $\alpha = -\omega_n$ exactly cancel out the last two terms on the right side of (17). Thus,

$$T\sum_m\varphi(\alpha_m)\,\psi(\alpha_m+\hbar\omega_n)=\frac{1}{4\pi i}\int_{-\infty}^{\infty}\operatorname{cth}\frac{\alpha}{2T}\{[\varphi^R(\alpha)-\varphi^A(\alpha)]\,\psi^R(\alpha+\hbar\omega_n)+[\psi^R(\alpha)-\psi^A(\alpha)]\,\varphi^A(\alpha-\hbar\omega_n)\,d\alpha,$$

where the integral is taken in the sense of its principal value. Analytic continuation now leads to the replacement of ω_n by ω, and Eq. (16) is obtained without difficulty.

Making use of Eq. (16) and the fact that $K_{pp'}(0)$ is a real quantity, we obtain (in the approximation of interest to us) an expression for the real part of the conductivity:

$$\operatorname{Re}\sigma(\omega)=\frac{e^2(2\pi)^{-4}}{3m^2\omega^3}\operatorname{sh}\frac{\hbar\omega}{2T}\int\gamma^2d^3\gamma\int_{-\infty}^{\infty}d\alpha\left[\operatorname{sh}\frac{\alpha}{2T}\operatorname{sh}\frac{(\alpha+\hbar\omega)}{2T}\right]^{-1}$$
$$\times\{\operatorname{Im}[D^R_\gamma(\alpha)P^R_\gamma(\alpha)]\operatorname{Im}[D^R_\gamma(\alpha+\hbar\omega)\,P'^R_\gamma(\alpha+\hbar\omega)]-\operatorname{Im}[D^R_\gamma(\alpha+\hbar\omega)]\operatorname{Im}[D^R_\gamma(\alpha)\,P^R_\gamma(\alpha)\,P'^R_\gamma(\alpha)]\}. \qquad (18)$$

Here $\sigma(\omega)=\sigma_{xx}(\omega)$. We have made use of the isotropy of the problem and replaced γ_x^2 by $\gamma^2/3$ under the integral. The index R denotes the functions obtained from (10) — (12) by replacement of α_m by $\alpha + i\delta$; $\delta \to +0$.

Let us introduce the dimensionless quantities

$$x=\left(\frac{m}{T}\right)^{1/2}\frac{\alpha}{\hbar\gamma},\quad \tau=\frac{\gamma}{\varkappa},\quad \eta^2=\frac{\hbar^2\varkappa^2}{2mT},\quad \rho^2=\frac{m}{M},\quad \omega^*=\frac{\omega}{\omega_0},$$

*cth = coth.

where $\kappa = (4\pi n_0 e^2/T)^{1/2}$ is the reciprocal of the Debye radius, n_0 is the concentration of electrons (ions), $\omega_0 = (4\pi n_0 e^2/m)^{1/2}$ is the electron plasma frequency.

After reduction of similar terms in the curly brackets and integration over the angles, we write the formula (18) in the following form:

$$\mathrm{Re}\,\sigma(\omega) = \frac{n_0e^2}{m\omega^2}\frac{2T}{\hbar\omega}\,\mathrm{sh}\frac{\hbar\omega}{2T}\cdot\frac{2}{3}\pi n_0\left(\frac{e^2}{T}\right)^2\left(\frac{8T}{\pi m}\right)^{1/2}(Q + Q' + Q''); \tag{19}$$

$$Q = (2\pi)^{-1/2}\frac{1}{\rho}\int_0^\infty \tau^3 d\tau \int_{-\infty}^{\infty} dx\left|D_\tau(x)\,D_\tau\left(x+\frac{\omega^*}{\tau}\right)\right|^2\Phi(x,\tau)$$
$$\times\exp\{-\eta^2\tau^2(1+\rho^2)/4 - x^2/2\rho^2 - (x+\omega^*/\tau)^2/2\}, \tag{20}$$

$$Q' = -(2\pi)^{-1/2}\int_0^\infty \tau^3 d\tau \int_{-\infty}^{\infty} dx\left|D_\tau(x)\,D_\tau\left(x+\frac{\omega^*}{\tau}\right)\right|^2 F'(x,\tau)$$
$$\times\exp\{-\eta^2\tau^2/2 - x^2/2 - (x+\omega^*/\tau)^2/2\}, \tag{21}$$

$$Q'' = -(2\pi)^{-1/2}\frac{1}{\rho^2}\int_0^\infty \tau^3 d\tau \int_{-\infty}^{\infty} dx\left|D_\tau(x)\,D_\tau\left(x+\frac{\omega^*}{\tau}\right)\right|^2 F(x,\tau)$$
$$\times\exp\{-\eta^2\tau^2\rho^2/2 - x^2/2\rho^2 - (x+\omega^*/\tau)^2/2\rho^2\}. \tag{22}$$

Here the following nondimensional functions have been introduced:

$$|D_\tau(x)|^{-2} = [\tau^2 + \varphi(x) + \varphi(x/\rho)]^2 + [\psi(x) + \psi(x/\rho)]^2,$$
$$\Phi(x,\tau) = \{\tau^2 + \varphi[\rho^{-1}(x+\omega^*/\tau)] + \varphi(x)\}^2 + \{\psi[\rho^{-1}(x+\omega^*/\tau)] + \psi(x)\}^2,$$
$$F(x,\tau) = \varphi(x)[\varphi(x+\omega^*/\tau) - \varphi(x)] + \psi(x)[\psi(x+\omega^*/\tau) - \psi(x)],$$
$$F'(x,\tau) = \varphi(x/\rho)\{\varphi[\rho^{-1}(x+\omega^*/\tau)] - \varphi(x/\rho)\} + \psi(x/\rho)\{\psi[\rho^{-1}(x+\omega^*/\tau)] - \psi(x/\rho)\},$$
$$\varphi(x) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} e^{-y^2/2}\frac{y\,dy}{y-x} = 1 - xe^{-x^2/2}\int_0^x e^{t^2/2}dt,$$
$$\psi(x) = (\pi/2)^{1/2}xe^{-x^2/2}. \tag{23}$$

The functions $\varphi(x)$ and $\psi(x)$ are obtained by reduction of the real and imaginary parts, respectively, of the function P^R to nondimensional form. In this case we have used the fact that under the assumptions made in the work the quantity η is a small parameter.

We note that, in spite of the smallness of η, it is impossible for it to approach zero in the exponent of the term Q, for then a logarithmic divergence arises in the integral over τ at the upper limit. We can, however assume $\eta = 0$ in the terms Q' and Q''.

5. Further simplifications are possible if we take it into account that $m/M \equiv \rho^2 \ll 1$ in the case of the electron-ion plasma under discussion. It is not difficult to prove that this makes it possible to neglect the terms Q' and Q''. In addition, we can write the quantity $\Phi(x,\tau)\,|D_\tau(x+\omega^*/\tau)|^2$ in the term Q in the form

$$(\tau^2+1)^2\{[\tau^2+\varphi(\omega^*/\tau)]^2 + [\psi(\omega^*/\tau)]^2\}^{-1}.$$

Taking it into account that values of x of order ρ are significant in the integral, it is easy to see that when $\omega^*/\tau \gg \rho$ this substitution is exact. On the other hand, for $\omega^*/\tau \ll 1$, both expressions are the same and are equal to unity.

In this way, we get (after substitution of the variable $x/\rho = z$ and simplifications associated with the smallness of ρ)

$$Q = \left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty \tau^3 d\tau\, e^{-\eta^2\tau^2/4}\frac{(\tau^2+1)^2\exp(-\omega^{*2}/2\tau^2)}{[\tau^2+\varphi(\omega^*/\tau)]^2+[\psi(\omega^*/\tau)]^2}$$
$$\times\int_0^\infty e^{-z^2/2}\{[\tau^2+1+\varphi(z)]^2+[\psi(z)]^2\}^{-1}dz. \tag{24}$$

We note that although the ratio of the masses of the electron and ion does not appear in Eq. (24), it would have been incorrect to consider the ions to be at rest from the very beginning, since they in fact take part in the screening of the electronic interaction. Were we to assume that the ions are fixed, then we would obtain an expression [in place of Eq. (24)] which does not contain the functions $\varphi(z)$ and $\psi(z)$ in the denominator under the integral in z.

6. Let us consider some limiting cases. We shall write out the final results for the effective collision frequency, which is connected with the conductivity by the relation

$$\mathrm{Re}\,\sigma(\omega) = (n_0e^2/m\omega^2)\,\nu(\omega).$$

In accord with Eq. (19)

$$\nu(\omega) = \nu_0\frac{2T}{\hbar\omega}\,\mathrm{sh}\frac{\hbar\omega}{2T}\,Q(\omega),$$

where $\nu_0 = \frac{2}{3}\pi(e^2/T)^2 n_0(8T/\pi m)^{1/2}$ and Q is determined by Eq. (24).

1) The frequency of the wave is much larger than the electron plasma frequency ($\omega^* \gg 1$). In this case, the values $\tau \sim \omega^* \gg 1$ are important in the integral over τ. Therefore we can write Eq. (24) in the form

$$Q = \int_0^\infty \frac{d\tau}{\tau}\exp\left[-\eta^2\frac{\tau^2}{4} - \frac{\omega^{*2}}{2\tau^2}\right] = K_0\left(\frac{\eta\omega^*}{\sqrt{2}}\right).$$

Here K_0 is the Bessel function of imaginary argument. Noting that $\eta\omega^*/\sqrt{2} = \hbar\omega/2T$, we get for the collision frequency in the case under discussion:

$$\nu = \nu_0\frac{2T}{\hbar\omega}\,\mathrm{sh}\frac{\hbar\omega}{2T}K_0\left(\frac{\hbar\omega}{2T}\right), \quad \omega \gg \omega_0. \tag{25}$$

Within the framework of this limiting case, we can consider two frequency regions. Formula (25) yields

$$\nu = \nu_0 \ln (4T\,A/\hbar\omega), \quad T \gg \hbar\omega \gg \hbar\omega_0; \tag{26}$$

$$\nu = \nu_0 \sqrt{\pi}\,(T/\hbar\omega)^{1/2}, \quad \hbar\omega \gg T \tag{27}$$

(here $A = e^{-\gamma} = 0.56$, γ is Euler's constant). We note that in the last case the effective collision frequency does not depend on the temperature.

The absorption of electromagnetic radiation for $\omega \gg \omega_0$ was considered in application to semiconductors.[7] If the expression obtained in the papers cited is supplemented by consideration of induced radiation, it becomes equivalent to (25).

2) The frequency of the wave is much less than the plasma frequency, $\omega^* \ll 1$. In this case, we can set $\omega^* = 0$ in Eq. (24). It is convenient to make the substitution $(\eta\tau)^2 = x$. We have

$$Q = (2\pi)^{-1/2} \int_0^\infty e^{-z^2/2}\,dz \int_0^\infty e^{-x/4} \frac{x\,dx}{\{x + \eta^2[1+\varphi(z)]\}^2 + \eta^4\{\psi(z)\}^2}\,. \tag{28}$$

As $\eta \to 0$, this expression diverges logarithmically. To eliminate the logarithmic term we carry out integration over x by parts. We can then set $\eta = 0$ in the integral term. We then obtain*

$$Q = \ln (B_0/\eta);$$

$$\ln B_0 = \ln 2 - \frac{\gamma}{2} - (2\pi)^{-1/2} \int_0^\infty e^{-z^2/2} f(z)\,dz,$$

$$f(z) = \tfrac{1}{2} \ln \{[1+\varphi(z)]^2 + [\psi(z)]^2\} - \frac{1+\varphi(z)}{\psi(z)} \left\{\operatorname{arctg} \frac{1+\varphi(z)}{\psi(z)} - \frac{\pi}{2}\right\}.$$

Numerical calculation yields

$$\ln B_0 = -0.29, \quad B_0 = 0.75.$$

Thus,

$$\nu = \nu_0 \ln [0.75\,(2mT)^{1/2}/\hbar\varkappa]. \tag{29}$$

*arctg = $\tan^{-1}$.

This result could, of course, have been obtained from the kinetic equation for the high temperature plasma.[2] We note that the numerical factor under the logarithm sign is approximately one third the value used by Spitzer.[5]

3) In the intermediate case ($\omega^* \sim 1$) numerical calculation is necessary, which it is convenient to carry out for the difference $Q(\omega) - Q(0)$, since in this case the logarithmic singularity at the upper limit is eliminated as $\eta \to 0$. The result can be written in the following form:

$$\nu = \nu_0 \ln (B_\omega \sqrt{2mT}/\hbar\varkappa). \tag{30}$$

Calculation gives, for $\omega = 0.5\,\omega_0$, ω_0 and $2\omega_0$, the corresponding values of $\ln B_\omega$: -0.29, -0.30, -0.48. As is seen, the value of the factor under the logarithm sign, obtained for $\omega \ll \omega_0$ is shown to be valid in a much wider range of frequencies than one could have expected.

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