EFFECT OF SUPERFLUIDITY OF ATOMIC NUCLEI ON THE STRIPPING AND PICKUP REACTIONS

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Submitted to JETP editor April 8, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 894-897 (September, 1961)

Formulas for stripping and pickup reactions in the forward hemisphere are derived by taking superfluidity of atomic nuclei into account. The nuclear model proposed can be verified by comparing the results with the experimental data.

WHEN the energy of the incoming particles is greater than the Coulomb barrier of the target nucleus, the main contribution to the stripping and pickup reactions is made by the direct and exchange effects,[1] provided there are no isolated resonances of the compound nucleus in the energy region under consideration. This condition is always satisfied in nuclei with $A \sim 40$ at incoming-particle energies on the order of 10 Mev. The direct reaction mechanism[2] is due to interaction between the nucleons initially contained in the incoming particle, and describes the cross section in the forward hemisphere (in the c.m.s.). Among the exchange effects are knock-out and heavy-particle stripping,[3] the latter describing the cross section in the rear hemisphere, while knock-out is estimated[1,4] to make a negligibly small contribution to the cross section of the direct mechanism. In an examination of the cross section in the forward hemisphere it is therefore sufficient to account for the direct reaction mechanism.

Let us consider for simplicity the reactions (d, p), (d, n) and their inverses. Within the framework of the method of distorted waves,[5,6] usually used for a description of the direct interactions, the amplitude of the reaction is

$$S_{i\to\mathbf{k}} = \left\langle \Phi_f \left| \int \varphi_{\mathbf{k}}^{-*}(x_f) V(x_c - x_f) \psi_{\mathbf{K}}^{+}(x_c x_f)\, dx_f \right| \Phi_i \right\rangle. \quad (1)$$

where Φ_f and Φ_i are respectively the wave functions of the product and target nuclei, $\varphi_{\mathbf{k}}^{-}(x_f)$ and $\psi_{\mathbf{K}}^{+}(x_c x_f)$ are the wave function of the outgoing nucleon and of the incoming deuteron with corresponding asymptotic approximations, $\hbar\mathbf{K}$ is the deuteron momentum, and $\hbar\mathbf{k}$ is the nucleon momentum. Expanding the integral in (1) in terms of the eigenfunctions $\varphi_{jm}(x_c)$ of the captured nucleon in the self-consistent field of the nucleus, we obtain

$$\int \varphi_{\mathbf{k}}^{-*}(x_f) V(x_c - x_f) \psi_{\mathbf{K}}^{+}(x_c x_f)\, dx_f = \sum_{jm} \varphi_{jm}(x_c) F(\mathbf{K}\mathbf{k}j; \mu_D \mu_f m), \quad (2)$$

where μ_D and μ_f are spin projections of the deuteron and the nucleon and $F(\mathbf{K}\mathbf{k}j; \mu_D\mu_f m)$ are the matrix elements that determine the angular distribution of the reaction products. They can be calculated by choosing the optical potentials of the deuteron and nucleon and the interaction potential $V(x_c - x_f)$.[6] The problem then reduces to a calculation of the matrix elements $\langle \Phi_f | \varphi_{jm} | \Phi_i \rangle$

Following Belyaev[7,8] we describe the states Φ_f and Φ_i in the independent quasiparticle approximation, the state operators being related to the nucleon operators by the Bogolyubov canonical transformation

$$a_{jm} = u_j \alpha_{jm} + (-1)^{j-m} v_j \alpha_{j-m}^{+}, \qquad u_j^2 = \frac{1}{2}\left[1 + \frac{\varepsilon_j - \lambda}{E_j}\right],$$

$$v_j^2 = \frac{1}{2}\left[1 - \frac{\varepsilon_j - \lambda}{E_j}\right], \qquad E_j = \sqrt{(\varepsilon_j - \lambda)^2 + \Delta_j^2}, \quad (3)$$

where ϵ_j is the energy of independent motion of the nucleons in the field of the nucleus, λ is the chemical potential (which coincides with the Fermi level when $\Delta = 0$), and E_j is the energy of the quasiparticle. The fundamental and low-lying states of the nuclei with odd A contain one quasiparticle, and the number of quasiparticles in the ground and excited states of even-even nuclei is respectively zero and two (collective excitations of the type considered by Belyaev[8] are not taken into account).

We start with an examination of the stripping reaction on a target nucleus with odd A. If the even-even product nucleus is in the ground state, we readily obtain with the aid of (1) — (3)

$$S_{\mathbf{K}j\to\mathbf{k}0} = (-1)^{j-m} v_j F(\mathbf{K}\mathbf{k}j;\ \mu_D \mu_f m),$$

$$\frac{d\sigma(\mathbf{K}j \to \mathbf{k}0)}{d\Omega} = \frac{m_f^2 v_f}{4\pi^2\hbar^4 v_D} \frac{v_j^2}{3(2j+1)} \sum_{m\mu_D\mu_f} |F(\mathbf{K}\mathbf{k}j;\ \mu_D\mu_f m)|^2. \quad (4)$$

In the case when the product nucleus is in the excited state

$$S_{\mathbf{K}j\to\mathbf{k}'j_1j_2J} = u_{j1}C^{JM}_{j_1m_1jm}F(\mathbf{K}\mathbf{k}'j;\ \mu_D\mu_f m_1)\,\delta_{j_2j} - u_{j_2}C^{JM}_{jmj_2m_2}F(\mathbf{K}\mathbf{k}'j;\ \mu_D\mu_f m_2)\delta_{j_1j}, \tag{5}$$

i.e., the state of the quasiparticle in the target-nucleus is not changed by the reaction. In particular, when $j_1 = j_2 = j$, i.e., when the states of the quasiparticles in the product nucleus coincide with the state of the quasiparticle in the target, we have

$$S_{\mathbf{K}j\to\mathbf{k}'j^2J} = \sqrt{2}u_jC^{JM}_{jm_1jm}F(\mathbf{K}\mathbf{k}'j;\ \mu_D\mu_f m_1),$$

$$\frac{d\sigma(\mathbf{K}j\to\mathbf{k}'j^2J)}{d\Omega} = \frac{m_f^2v_f'}{4\pi^2\hbar^4v_D}\frac{2(2J+1)u_j^2}{3(2j+1)^2}\sum_{m\mu_D\mu_f}|F(\mathbf{K}\mathbf{k}'j;\ \mu_D\mu_f m)|^2, \tag{6}$$

where m_f and v_f are the mass and velocity of the outgoing nucleon, and v_D is the velocity of the incoming deuteron. A comparison of (4) and (6) shows that at sufficiently high incident-particle energies the angular distributions coincide and the limit of the ratio of the corresponding intensities is

$$\lim_{v_f'\to v_f}\frac{d\sigma(\mathbf{K}j\to\mathbf{k}'j^2J)}{d\sigma(\mathbf{K}j\to\mathbf{k}0)} = \frac{2(2J+1)}{2j+1}\frac{u_j^2}{v_j^2}. \tag{7}$$

For pickup reactions on odd targets we have

$$S_{\mathbf{k}j\to\mathbf{K}0} = u_jF^*(\mathbf{K}\mathbf{k}j;\ \mu_D\mu_f m),$$

$$\frac{d\sigma(\mathbf{k}j\to\mathbf{K}0)}{d\Omega} = \frac{m_D^2v_D^2}{4\pi^2\hbar^4v_f}\frac{u_j^2}{2(2j+1)}\sum_{m\mu_D\mu_f}|F(\mathbf{K}\mathbf{k}j;\ \mu_D\mu_f m)|^2, \tag{8}$$

$$S_{\mathbf{k}j\to\mathbf{K}'j^2J} = (-1)^{j-m_1}\sqrt{2}v_jC^{JM}_{jm_1jm}F^*(\mathbf{K}'\mathbf{k}j;\ \mu_D\mu_f m_1),$$

$$\frac{d\sigma(\mathbf{k}j\to\mathbf{K}'j^2J)}{d\Omega} = \frac{m_D^2v_D'}{4\pi^2\hbar^4v_f}\frac{(2J+1)v_j^2}{(2j+1)^2}\sum_{m\mu_D\mu_f}|F(\mathbf{K}'\mathbf{k}j;\ \mu_D\mu_f m)|^2, \tag{9}$$

$$\lim_{v_D'\to v_D}\frac{d\sigma(\mathbf{k}j\to\mathbf{K}'j^2J)}{d\sigma(\mathbf{k}j\to\mathbf{K}0)} = \frac{2(2J+1)}{2j+1}\frac{v_j^2}{u_j^2}. \tag{10}$$

Let us consider now the stripping and pickup reactions on even-even targets. Since the corresponding cross sections are connected with (8) and (4) by the principle of detailed balance, we have in the case of stripping

$$\frac{d\sigma(\mathbf{K}0\to\mathbf{k}j)}{d\Omega} = \frac{m_f^2v_f}{4\pi^2\hbar^4v_D}\frac{u_j^2}{3}\sum_{m\mu_D\mu_f}|F(\mathbf{K}\mathbf{k}j;\ \mu_D\mu_f m)|^2, \tag{11}$$

and in the case of pickup

$$\frac{d\sigma(\mathbf{k}0\to\mathbf{K}j)}{d\Omega} = \frac{m_D^2v_D}{4\pi^2\hbar^4v_f}\frac{v_j^2}{2}\sum_{m\mu_D\mu_f}|F(\mathbf{K}\mathbf{k}j;\ \mu_D\mu_f m)|^2. \tag{12}$$

It is interesting to compare (11) and (12) with the predictions of the single-particle model. Since we have when $\Delta = 0$

$$u_j^2 = \begin{cases}1, & \varepsilon_j > \lambda\\ 0, & \varepsilon_j < \lambda\end{cases},\qquad v_j^2 = \begin{cases}0, & \varepsilon_j > \lambda\\ 1, & \varepsilon_j < \lambda\end{cases},$$

only nucleon levels can be excited in stripping reactions, according to the single-particle model, and only hole levels can be excited in pickup reactions. An account of superfluidity, as can be seen from (11) and (12), leads to a nonvanishing probability of excitation of hole levels in stripping reactions and nucleon levels in pickup reactions. We can, however, assume here that levels with $\epsilon_j > \lambda$ are excited in stripping reactions and levels with $\epsilon_j < \lambda$ are excited in pickup reactions.

The derived formulas (4) — (12) are applicable to all direct reactions in which the number of nucleons in the nucleus changes by one. They can therefore be verified experimentally by using any of the foregoing reactions. The influence of the superfluidity of atomic nuclei on electromagnetic transitions and on beta decay was already pointed out by Urin[9] and Grin'[10]. However, since the number of quasiparticles in the initial and final states in these cases is either the same or differs by two, the corresponding matrix elements contain combinations of the canonical transformation such as $u_{j_1}u_{j_2} \pm v_{j_1}v_{j_2}$ or $u_{j_1}v_{j_2} \pm v_{j_1}u_{j_2}$. In the case of stripping and pickup, however, the number of quasiparticles differs by one, and consequently the matrix elements contain only one coefficient [u_j if an increase (decrease) in number of nucleons is accompanied by an increase (decrease) in the number of quasiparticles, and v_j in the opposite case). The reactions considered can therefore be used to check the given model of the nucleus.

It is of interest to set up the following experiments: a) Direct determination of u_j^2 and v_j^2 by measuring the relative intensity of the transitions to the ground and excited states of even-even nuclei in reactions on targets with odd A. In this case the dt or He$^3\alpha$ pickup reactions are more convenient to use than the pd reaction, for the exothermal nature of the former makes it easier to satisfy the condition $v' \to v$, needed to cancel out the matrix elements $F(\mathbf{K}\mathbf{k}j;\ \mu_D\mu_f m)$. b) Determination of the dependence of u_j^2 and v_j^2 on the number of external nucleons, by measuring the intensities of excitation of the states of odd nuclei with given l and j in reactions on groups of even-even isotopes, corresponding to the filling of the same shell.

The author is grateful to S. T. Belyaev, L. A. Sliv, and the staff of the Nuclear Physics Institute of the Moscow State University for a discussion of this work.

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Translated by J. G. Adashko
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