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ON AN UPPER LIMIT ON THE DENSITY OF NEUTRINOS, GRAVITONS, AND BARYONS IN THE UNIVERSE

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To estimate the maximum energy density of neutrinos and other weakly interacting particles in the universe we consider the gravitational action of these particles on the expanding universe. It turns out that the energy density does not exceed $2 \times 10^{-28}\,(\mathrm{g/cm^3}) \cdot c^2 = 2 \times 10^{-7}$ erg/cm^3; this estimate is much more accurate (lower) than that from nuclear physics experiments. The question of stars with large gravitational mass defects is considered, and in this connection an upper limit on the density of nucleons is given.

Present data do not enable us to exclude the possibility that the density of forms of matter that are not readily observed and the density of nucleons in a state in which they are hard to observe is several times the density of the well known easily observed forms of matter and energy in the form of nucleons in ordinary stars and in interstellar gas.

1. As has been shown by Pontecorvo and one of the writers,[1] the determination of the cosmic density of neutrinos is an extremely difficult experiment, owing to the very weak interaction of neutrinos with matter. The experiments of Reines and Cowan on the determination of the antineutrino flux from a pile[2] and the experiments of Davis, who showed that under these conditions reactions requiring neutrinos are absent,[3] are well known. From these experiments it follows in particular that the flux of cosmic ν and $\bar{\nu}$ is smaller than $W = 10^{12} - 5 \times 10^{11}$ cm^{-2} sec^{-1} [4] for ν and $\bar{\nu}$ with energies of the order of 2 — 3 Mev. This flux corresponds to a particle density $W/c = \rho_{\nu,\bar{\nu}} = 20$ cm^{-3} and a mass density $E\rho/c^2 \sim 10^{-25}$ g/cm^3. This quantity is to be compared with the mean density of nucleons in the universe, which is at present believed to be 10^{-29} g/cm^3 (which corresponds to $\sim 10^{-5}$ proton/cm^3).

Thus direct experiment does not exclude the possibility that the density of mass and energy in the form of neutrinos is 10^4 to 10^5 times the density of rest mass-energy in ordinary forms.

As is remarked by Reines,[4] and also in [1], these estimates actually depend on an assumption about the spectrum of the neutrinos. At high energies the interaction cross section increases as E^2, and therefore when recomputed for a different neutrino energy the maximum flux varies $\sim 1/E^2$, and the maximum mass-energy density varies as $1/E$. But even for ν and $\bar{\nu}$ with energies ~ 1 Bev the maximum mass-energy density is still larger than the density of nucleons.

On the other hand, at small energies, less than the threshold of the reactions in question, the efficiency of registration of neutrinos by present methods falls sharply; the threshold is ~ 1.8 Mev for $\bar{\nu} + p \rightarrow n + e^+$ and ~ 1 Mev for $\nu + Cl \rightarrow A + e^-$. If there are two kinds of neutrinos—"electron neutrinos" and "muon neutrinos"[5]—then for the second kind the threshold of the reactions

$$\bar{\nu}_\mu + p = n + \mu^+, \qquad \nu_\mu + n = p + \mu$$

is of the order of 100 Mev. Subthreshold neutrinos can be detected only through the ionization caused by elastic collisions, and therefore for them the estimate of the cosmic density consistent with experiment goes up by several more orders of magnitude.

2. A different approach to the setting of an upper limit on the cosmic density of weakly interacting particles is based on consideration of their gravitational action.* The advantage of this approach is that the estimate does not depend on the shape of the spectrum nor on the interaction cross section, and in principle applies also to all possible as yet unknown weakly interacting fields, and also to the density of high-frequency oscillations of the gravitational field (gravitons).†

*The idea of such an approach is already contained in lectures by Einstein.[6]

†Because of the weakness of the gravitational interaction all of the cross sections of gravitons are so small that there is practically no hope of detecting them (cf., e. g., a calculation by Gandel'man and Pinaev,[7] according to whom the bremsstrahlung of gravitons is a factor 10^{10} smaller than the production of $\nu - \bar{\nu}$ pairs).

The point of the proposed estimate is that in the present known state of the universe (characterized by a definite value of the Hubble constant h in the equation v = hr, where v is the speed and r is the distance) the density ρ of all forms of matter-energy determines the past of the universe. For example, for $\rho \rightarrow 0$ the distance between each pair of distant objects varies linearly with the time and the state of maximum density occurred at the time $T_0 = -1/h$, i.e., approximately 10^{10} years ago.

For nonvanishing ρ the expansion of the universe slows down in the course of time; the average speed of expansion from the time of maximum density to the present time is larger than the experimentally known instantaneous speed, and consequently the past time of maximum density was closer to the present time.*

Let us introduce a critical density value ρ_c. This concept corresponds to the dividing line between the open model and the closed model of the universe; this corresponds physically to qualitatively different predictions about the future of the universe: for $\rho < \rho_c$ the expansion will continue without limit, but for $\rho > \rho_c$ the expansion will be replaced in the course of time by a contraction. The existence of the singularity and the character of the solution for the past are qualitatively independent of the ratio of ρ to ρ_c.

We give here the value of the time interval from the time of the maximum density in the past to the present time as a function of the total density of all forms of matter-energy, in the dimensionless variables $\tau = Th = T/T_0$ as function of $q = \rho/2\rho_c$:

q:	0	0.5	2	5	20	100
τ:	1	0.67	0.47	0.35	0.21	0.10
τ':	1	0.50	0.33	0.24	0.14	0.07

The quantity τ is here calculated for stationary matter with the pressure p equal to zero. For τ' the calculation is made (in accordance with a remark of L. D. Landau) for the case in which the rest mass can be neglected, the particles are moving with the speed of light, and $p = \epsilon/3$, where ϵ is the energy density. The difference between τ and τ' is not large.

We note that the difference in the behavior of the two models (with $p = 0$ and $p = \epsilon/3$) is due to the decrease of the energy of relativistic particles on adiabatic expansion. This concept can be applied in spite of the fact that the neutrino does not interact with any kind of particles and does not "do work." In fact, in the course of time there will pass through a given point neutrinos emitted from more and more distant regions, and thus having suffered stronger and stronger red shifts, or Doppler energy losses. The Doppler energy loss is indeed the physical mechanism that brings about the decrease of the mean energy of the neutrinos as the universe expands.

For reference we present the following formulas:*

$$\tau = k^{-2}\left[(k + k^{-1})\operatorname{arctg} k - 1\right], \qquad k = (2q-1)^{1/2};$$

$$\tau' = [1 + (2q)^{1/2}]^{-1}.$$

We take $T_0 = 10^{10}$ years, and the corresponding critical density is $\rho_c = 2 \times 10^{-29}$ g/cm^3. Assuming that the time T is not less than the geological age of the earth, which is of the order of 4×10^9 years, we find that $\tau > 0.4$. From this we have $q < 5$ and $\rho < 2q\rho_c = 2 \times 10^{-28}$ g/cm^3.

Another possibility for getting an idea of the past of the universe is to consider distant galaxies, since we are actually observing them in a long past state. In the theory of the expanding universe one gets a nonlinear relation of the speed of expansion on the distance, $v = \chi(r)$; the Hubble constant is only the first derivative, $h = \chi'(0)$, and the value of the second derivative $\chi''(0)$ depends on the density. At present $\chi''(0)$ has not been reliably determined, but the upper limit on χ'' also sets a limit on q: $q < 10$; thus two independent estimates of the upper limit are in agreement (cf. [9]).

Finally, we note that one of the methods by which the observed mean density $\bar{\rho} = 10^{-29}$ has been obtained is as follows: from the motion of the stars in a galaxy one determines the mass of the galaxy, and then finds the average density over the entire universe by dividing the average mass M of a galaxy by the average volume V per galaxy in the universe. It is obvious that in this calculation the mass of a galaxy includes the mass of the neutrinos that are inside it. The density of the neutrinos must be the same inside galaxies and in intergalactic space. Therefore still another way of giving an upper limit on the density of neutrinos is to assume that the entire mass of a galaxy, which determines its gravitational field, is the mass of the neutrinos.

We thus have $\rho_\nu \le M/V_0$, where V_0 is the actual volume of a galaxy. From this we get

$$\rho_\nu \leqslant (M/V)(V/V_0) = \bar{\rho}V/V_0 \approx \bar{\rho}/0.1,$$

since on the average the volume of the galaxies makes up about 10 percent of the volume of the universe. Consequently this third estimate also

*The question of the relation between the age of the galaxy and the quantity 1/h has also been considered by Hoyle.[8]

*arctg = $\tan^{-1}$.

agrees in order of magnitude with the preceding ones.

Thus although the density of neutrinos, gravitons, and so on in the universe may indeed exceed the observed mean density of nucleons (10^{-29} g/cm^3), it does not do so by more than a factor 10 to 20.

The supposition that the density of energy in unobservable forms is several times that of ordinary matter is surprising, but nevertheless at present it cannot be rejected. We remind the reader that with all its imperfection the gravitational estimate is still more sensitive by a very large factor than the estimate from nuclear physics experiments.

Digressing from the topic of this paper, we note the striking and as yet unexplained close correspondence between the critical value ρ_c and the observed density $\bar{\rho}$ of ordinary matter (2×10^{-29} and 10^{-29}g/cm^3). This could mean, for example, that the age of the universe is relatively small. In this case all three models behave about the same, and $\rho \sim \rho_c$. The connection between the Hubble constant and the age τ is given by the formula $\tau = 2/3h$. In the estimates given we have started from the hypothesis of the isotropic model of the universe and have rejected the introduction of a cosmological constant into the equations, and also have rejected the hypothesis of the spontaneous creation of energy.

3. Let us now turn to the question of the density of nucleons in the universe. At first glance, a determination of the mean density of mass in the universe also gives, to whatever accuracy the estimate can be made, the mean density of nucleons, or more exactly, of baryons.* If the mass density of neutrinos is comparable with that of nucleons, then the mass density of nucleons is also less than the accepted value of $\sim 10^{-29}$ g/cm^3, which corresponds to a number density of nucleons of $\sim 10^{-5}$ cm^{-3}.

We wish here to call attention to the possibility in principle that the number density of nucleons may be much larger than this value. In principle we can imagine a star which after gravitational collapse is in a state in which its gravitational mass defect ΔM is close to M_0, the sum of the rest masses of the nucleons making up the star. Then its mass will be much smaller than M_0,

$$M = M_0 - \Delta M \ll M_0 = Nm_0$$

and the gravitational field produced by such a star gives a decided underestimate of the number of nucleons in the star, $M/m_0 \ll N$.

The existence of a gravitational mass defect in the general theory of relativity is not open to doubt. For example, when a rarefied cloud of hydrogen atoms is converted into a dense neutron nucleus, a calculation by the classical Newtonian theory of gravitation shows that if the mass of the cloud is large enough the process is energetically favorable; the release of gravitation energy exceeds the loss of energy in the conversion of hydrogen atoms into the heavier neutrons. Such a process must occur with the release of energy to the outside world in the form of light and neutrinos. There is no doubt that when this happens the total mass of the star decreases, in accordance with the energy which it has given up. At the same time the rest mass of the particles composing the star has increased in the transition $H \rightarrow n$. The decrease of the mass of the star (and of its gravitational field at large distances) is a direct result of the gravitational mass defect, that is, of the fact that the mass of a body is less than the sum of the parts composing it by an amount equal to the energy of the gravitational interaction of the parts (divided by c^2).

Thus the existence of the gravitational mass defect is qualitatively obvious, and it can be calculated in an elementary way for weak gravitational fields. In order, however, for the mass defect to be nearly equal to the rest mass, the gravitational potential must be of the order of c^2, and in this region we must use the exact equations of the general theory of relativity. It is also necessary to know the equation of state, i.e., the dependence of the pressure and energy on the density of nucleons. At present the theory of such a state of a star has not been developed. There are only approximate estimates of the minimum number of nucleons necessary for gravitational collapse to be possible; Landau and Lifshitz [12] give for the critical mass a value which is 0.76 times the mass of the sun.

At present it is not known whether there is even one collapsed star in the universe. Nevertheless, in order to estimate the mean density of nucleons in the universe it is necessary to give an upper limit on the number of collapsed stars which have dimmed out and lost their mass, but have kept their original numbers of nucleons (we take the point of view of the unlimited validity of the law of conservation of the number of nucleons, or, more exactly, of the baryon charge). We can make such an estimate if we assume that these stars have arisen by evolution from ordinary hot

*Allowing for the fact that a partial conversion of nucleons into strange particles is possible at ultrahigh densities.[10,11]

stars—accumulations of hydrogen. Then the energy that corresponds to the difference between the initial mass of the star (in a rarefied state) and its final mass (in the collapsed state) must have been removed from the star in some form or other.

Since the during release of the main part of the energy the density of the star is extremely large, the main mechanism is probably the emission of neutrinos and antineutrinos (cf. [7,13-15]). Even independently of the concrete form in which it is released, however, the energy is not lost, and can be detected by its gravitational action. Thus we can use the estimate of the preceding section and draw the conclusion that the mean density in the universe of nucleons in collapsed stars cannot exceed the known value of the density of ordinary nucleons by more than a factor of 10 to 20.

To our earlier stipulations about the assumptions on which our conclusions are based (isotropic model, and so on), we must add one more, namely, that the collapse did not occur too early; otherwise during the time from the collapse to the present the energy of the neutrinos could decrease through the adiabatic expansion of the neutrino gas in the expanding universe. If this were so, our estimate of an upper limit on the number of nucleons in collapsed stars would have to be raised.

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