CORRELATION OF ELECTRON AND POSITRON POLARIZATIONS IN RELATIVISTIC PAIRS

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An electron-positron pair for which the square of the sum of the four-momenta is much larger than m_e^2 consists either of a right-handed electron and a left-handed positron, or else of a left-handed electron and a right-handed positron; here the terms "left-handed" and "right-handed" refer to the longitudinal polarization of the particle. In the center-of-mass system of the pair the projection of the total spin of the pair on the direction of motion of one particle can have only two values, $+1$ and -1, but not 0. Processes in which this rule is violated have a probability which is smaller by a factor $(m_e/E)^2$ as compared with the probability of "allowed" processes.

FOR the proof of the statements made in the abstract, which apply to particles with energies many times the rest mass, we omit the term in the Dirac equation that contains the mass of the electron. It is well known that the Dirac equation then separates into two first-order equations for two two-component spinors. Physically this corresponds to the fact that for zero mass we can speak of right-handed and left-handed electrons as two independent particles which do not get converted into each other. Furthermore each type of particle has its antiparticles: the antiparticles of right-handed electrons are left-handed positrons, and those of left-handed electrons are right-handed positrons. In a recent paper by Sannikov[1] these ideas have been used in a consistent way for the construction of the Green's function and the scattering matrix and for the calculation of scattering and radiation emission of high-energy electrons.

Still earlier, before the discovery of parity nonconservation and the two-component nature of the neutrino, the separation of the equation was used in the concrete problem of the scattering of fast electrons by a nucleus by Yennie, Ravenhall, and Wilson[2]; they showed without calculation that the longitudinal polarization of the electron does not change sign in the scattering.

Let us now examine the qualitative consequences of the separation of the equation for the effect of pair production.

The production of a right-handed (left-handed) electron is necessarily accompanied by the production of a left-handed (right-handed) positron. For $m = 0$ the production of a right-handed electron and a right-handed positron, i.e., the produc-

tion of a particle with the wrong antiparticle, is just as impossible as the direct production of a $\mu^+ e^-$ pair. Thus there is a correlation between the longitudinal polarizations of the two particles of a pair.

Let us consider a pair in the coordinate system in which $\mathbf{P} = \mathbf{p}_e^+ + \mathbf{p}_e^- = 0$. In this coordinate system the momenta of the electron and positron are antiparallel, and consequently the projections of the spins of the electron and positron on the direction of motion, say of the electron, must be either $+\frac{1}{2}$ and $+\frac{1}{2}$ (right-handed electron and left-handed positron), or else $-\frac{1}{2}$ and $-\frac{1}{2}$ (left-handed electron and right-handed positron). Since the projection of the orbital angular momentum on this direction is zero, the projection of the total angular momentum j of the pair is ± 1.

Let us consider the process $\pi^+ + \pi^- \rightarrow \gamma \rightarrow e^+ + e^-$. The projection of the angular momentum j_π on the direction of motion of a π meson is zero. The projection of the angular momentum j_e of the pair on the direction of motion of e is ± 1. Consequently, the direction of motion of the electron cannot coincide with the direction of motion of the meson. In fact, according to a formula derived by Akhiezer and Berestetskiĭ,[3]

$$d\sigma/d\Omega \sim 1 - (1 - m^2/E^2)\cos^2\theta,$$

and for $m = 0$ we have $d\sigma/d\Omega = 0$ at $\theta = 0$.

In the approximation $m = 0$ the decay of a π^0 meson ($j = 0$) into a pair $e^+ + e^-$ is impossible. For m different from zero, Gell-Mann and Feynman[4] propose formulating the theory as a second-order equation for a two-component quantity. For the following argument, however, it is more conven-

ient to deal with two first-order equations for two types of particles, with the mass playing the part of an interaction constant which causes the conversion of one type into the other.[5] Then besides the order of a process in the constant $e^2/\hbar c$ we can speak of its order in the constant m. For example, the decay $\pi^0 \rightarrow e^+ + e^-$ is forbidden in zeroth order in m, but occurs in first order in m. Therefore it is indeed so[6] that the probability of the process $\pi^0 \rightarrow e^+ + e^-$, taken as a fraction of the probability of the normal decay $\pi^0 \rightarrow 2\gamma$, is not of the order $(e^2/\hbar c)^2 = 10^{-4}$, but of the order $(e2/\hbar c)^2 (m_e/m_\pi)^2 = 10^{-8}$. The situation in the decay $\pi^0 \rightarrow e^+ + e^-$ is the same as for the decays $\pi^+ \rightarrow e^+ + \nu$ and $\pi^- \rightarrow e^- + \bar{\nu}$, where because of conservation of angular momentum the charged particles are produced with the "incorrect" sign of the longitudinal polarization.

Pair production in 0-0 transitions of nuclei is allowed for m = 0, since owing to the recoil of the nucleus the angular momentum of the pair as a whole compensates the intrinsic angular momentum of the pair.

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[2] Yennie, Ravenhall, and Wilson, Phys. Rev. **95**, 500 (1954).

[3] A. I. Akhiezer and V. B. Berestetskii, Kvantovaya élektrodinamika (Quantum Electrodynamics), 2nd edition, Fizmatgiz, 1959, page 645.

[4] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[5] Ya. B. Zel'dovich, JETP **40**, 637 (1961), Soviet Phys. JETP **13**, 444 (1961).

[6] S. M. Berman and D. A. Geffen, Nuovo cimento **18**, 1192 (1960).

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