PRODUCTION OF LEPTON PARTICLE PAIRS ON A COULOMB CENTER

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Submitted to JETP editor April 25, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 949-953 (September, 1961)

The cross sections for the processes $\nu + A \rightarrow \nu + A + \mu^+ + \mu^-$ and $\mu + A \rightarrow \mu + A + \nu + \bar{\nu}$ occurring in the Coulomb field of the nucleus are calculated. It is shown that the Weizsäcker-Williams method can be applied to perform the calculations in the case under consideration.

THE present article is devoted to a study of the behavior of the cross section of production of lepton pairs when high-energy leptons are scattered in a Coulomb field. We have examined the processes

$$A + \nu \rightarrow A + \nu + \mu^+ (e^+) + \mu^- (e^-), \quad (1)$$

$$A + \mu (e) \rightarrow A + \mu (e) + \nu + \bar{\nu}. \quad (2)$$

We used in the calculations a method analogous to that of Weizsäcker and Williams [1] (see also [2]). This method enables us to obtain the cross sections of the processes (1) and (2), if the respective cross sections are known for the reactions

$$\gamma + \nu \rightarrow \nu + \mu^+ (e^+) + \mu^- (e^-), \quad (1')$$

$$\gamma + \mu (e) \rightarrow \mu (e) + \nu + \bar{\nu}. \quad (2')$$

We have therefore first calculated the cross sections of processes (1′) and (2′). (The cross section of the process (2′) was calculated earlier,[3] but the numerical coefficients in the formula of [3] are incorrect.)

In Sec. 2 we discuss the possibility of applying a method similar to that of Weizsäcker and Williams to the problem under consideration; our attention was called to this possibility by I. Ya. Pomeranchuk.

In the last section are contained results on the cross sections of production of $\mu^+\mu$ and $\nu\bar{\nu}$ pairs in scattering of a neutrino and muon respectively in the field of the nucleus.

1. CROSS SECTION OF PRODUCTION OF LEPTON PAIRS BY A γ QUANTUM

We consider the process (1′). It is described in the lowest approximation of the weak and electromagnetic interactions by the sum of the two diagrams a and b of Fig. 1. The matrix element of such a process, based on the V-A coupling, has the form

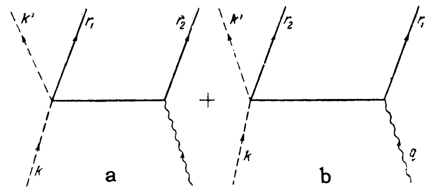

FIG. 1. Feynman diagram for process (1′); k and k′ are the momenta of the incoming and scattered neutrino, q is the momentum of the photon, and r_1 and r_2 are the corresponding momenta of the positron and the electron.

$$M = \frac{Ge}{\sqrt{2}} \frac{1}{\sqrt{2\omega_q}} \bar{u}(r_2) \left[\hat{e} \frac{1}{\hat{r}_2 - \hat{q} - m} \gamma_\alpha (1 + \gamma_5) + \gamma_\alpha (1 + \gamma_5) \frac{1}{-\hat{r}_1 + \hat{q} - m} \hat{e} \right] u(r_1) (\bar{u}(k') \gamma_\alpha (1 + \gamma_5) u(k)), \quad (3)$$

where G is the weak-interaction constant, e is the electric charge, u are spinors, and $\hat{e} = e_\mu \gamma_\mu$.

The calculations lead to the following formula for the differential cross section

$$d\sigma_{\text{ph}} = \frac{4e^2G^2}{(2\pi)^5 (kq)} \left\{ \frac{(r_1q)(kq)(r_2k')}{[(r_1 - q)^2 - m^2]^2} + \frac{(r_2q)(k'q)(r_1k)}{[(r_2 - q)^2 - m^2]^2} + [(r_1 - q)^2 - m^2]^{-1} [(r_2 - q)^2 - m^2]^{-1} [(2r_1r_2 - r_1q - r_2q)(r_1k)(r_2k') - (r_1r_2)(r_1k)(qk') - (r_1r_2)(r_2k')(qk) + (qr_1)(kr_2)(k'r_2) + (qr_2)(r_1k)(r_1k')] \right\} \times \delta^4 (k + q - k' - r_1 - r_2) \frac{d^3r_1}{E_{r_1}} \frac{d^3r_2}{E_{r_2}} \frac{d^3k'}{E_{k'}}. \quad (4)$$

We have left out of this expression the terms proportional to m_e^2 (or m_μ^2). Inasmuch as the energy of the particles participating in the reaction is much greater than their masses, we shall henceforth neglect the particle mass where possible.

Integrating (4) we obtain the cross section for the process (1′):

$$\sigma_{\text{ph}} = \frac{2}{9\pi^2} \alpha G^2 w^2 \left(\ln \frac{w}{m} - \frac{19}{12} \right), \quad (5)$$

where w is the total energy in the c.m.s. of the photon and the neutrino, m is the electron (muon), mass, and $\alpha = 1/137$.

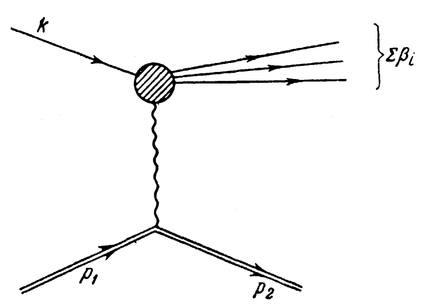

FIG. 2. Feynman diagram for the multiple production of particles in the Coulomb field of the nucleus. p_1 and p_2 are the initial and final momenta of the nucleus, k is the momentum of the incoming particle, and $\Sigma\beta_1$ is the totality of created particles with a summary momentum Q.

For the cross section of the process (2′) we obtain the expression

$$\sigma'_{\mathrm{ph}} = \frac{1}{18\pi^2}\alpha G^2 w^2\left(\ln\frac{w}{m} - \frac{55}{48}\right). \tag{6}$$

2. PROCEDURE FOR USING THE PHOTON CROSS SECTIONS FOR PROCESSES IN A COULOMB FIELD

An invariant formulation of the method was proposed by Gribov, Kolkunov, Okun', and Shekhter.[4] The multiple production of particles in the Coulomb field of a nucleus is described in the lowest order by the diagram of Fig. 2. If we consider a nucleus with zero spin,* we obtain for the reaction cross section

$$\sigma = \frac{F^2(q)}{q^4} Z^2 e^2 \frac{\Phi_{\mu\nu}(p_1+p_2)_\mu (p_1+p_2)_\nu}{4\sqrt{(kp_1)^2 - k^2 p_1^2}} \frac{d^3 p_2}{(2\pi)^3 E_2}. \tag{7}$$

Here F(q) is the electromagnetic form of the nucleus, and $\Phi_{\mu\nu}$ is a tensor describing the upper vertex of the diagram (Fig. 2). From the gauge invariance requirements we obtain[4]

$$\Phi_{\mu\nu} = a\left[(kq)\delta_{\mu\nu} + q^2 k_\mu k_\nu/(kq) - k_\mu q_\nu - k_\nu q_\mu\right] + b\left[q^2\delta_{\mu\nu} - q_\mu q_\nu\right], \tag{8}$$

where a and b are scalar functions.

It can be readily verified that as $q^2 \to 0$ the quantity a goes into σ_{ph}, the cross section of the photoprocess $q + k \to \Sigma\beta_i$. Practically nothing can be said beforehand concerning the quantity β unless the tensor $\Phi_{\mu\nu}$ is calculated directly. We introduce the positive quantity $\Delta^2 = -q^2$ and make the following change of variables in (7)

$$\frac{d^3 p_2}{E_2} = \frac{\pi d\Delta^2 dw^2}{2M|\mathbf{k}|},$$

where w is the energy of the aggregate of the particles $\Sigma\beta_i$ in their center-of-mass system. Substituting (8) in (7) we get

$$\sigma = \frac{Z^2\alpha}{4\pi v}\frac{d\Delta^2 dw^2}{\Delta^4}F^2(\Delta)\left\{a(w^2,\Delta^2)\left[\frac{2\Delta^2}{w^2+\Delta^2-m_i^2} + \frac{\Delta^2}{ME} - \frac{w^2+\Delta^2-m_i^2}{2E^2}\right] + b(w^2,\Delta^2)\frac{\Delta^2}{E^2}\left[1+\frac{\Delta^2}{4M^2}\right]\right\}, \tag{9}$$

where M is the mass of the nucleus, E the energy of the incoming particle in the system where the nucleus is at rest, and v and m_i are the velocity and mass of the particle respectively.

Formula (9) is an exact expression for the cross section, but it is rather cumbersome to use for calculations. We shall show below that when the charged leptons in processes (1) and (2) are muons we can make several simplifications in (9). In this case the form of the expression for the cross section σ is found to be the same as in the formulas obtained by the Weizsäcker-Williams method. The conditions that lead to such a result are:

1) $a(w^2, \Delta^2)$ must differ from $a(w^2, 0)$ in the entire domain of variation of w^2;

2) the function $b(w^2, \Delta^2)$ must not increase more strongly than $a(w^2, \Delta^2)$ with increasing w^2;

3) all the terms in the curly brackets of (9) must be small compared with $a(w^2, \Delta^2)\cdot 2\Delta^2/(w^2 + \Delta^2 - m_i^2)$.

The first condition is satisfied if $\Delta^2_{\max} \lesssim m^2$, where m is the mass of the charged lepton. The second condition is satisfied in our case, and this can be verified without calculating the tensor $\Phi_{\mu\nu}$. In order to explain how the third condition is satisfied in our case, let us examine the ranges of the variables Δ^2 and w^2.

The range of variation of these variables, allowed by the conservation laws, is shown in Fig. 3. However, only part of this domain is essential to the process (shaded). The point is that the presence in (9) of the electromagnetic form factor of the nucleus $F(\Delta)$, which decreases sharply with increasing Δ, leads to an effective cut-off of the possible momentum transfer. For our relatively rough estimates we can approximate the form factor by

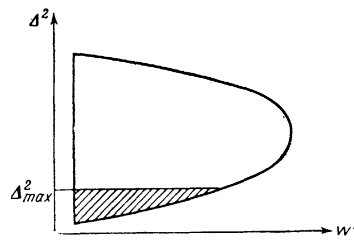

FIG. 3

*I. Ya. Pomeranchuk and I. M. Shumashkevich have shown that additional small terms appear in formula (7) in the case of nuclei with spin ½.

$$F(\Delta) = \int \rho \exp(i\Delta \mathbf{r})\, d^3r \Big/ \int \rho\, d^3r,$$

where ρ is the distribution of the charge in the nucleus by means of the function

$$F(\Delta) = \begin{cases} 1, & \Delta \leqslant 1/R, \\ 0, & \Delta > 1/R \end{cases} \qquad (10)$$

where R is the radius of the nucleus. Thus, we should take for the maximum value of the momentum transfer

$$\Delta_{max}^2 = \frac{1}{R^2} = \mu^2 A^{-2/3}, \qquad (11)$$

where μ is the mass of the pion and A the atomic weight of the nucleus.

We note that the condition 2) is therefore satisfied in the case of muons.

At high incident-particle energy the lower limit of the integration region is determined from the condition[5]

$$\Delta_{min}^2 \approx M^2 (w^2 - m_l^2)^2 / [(W^2 + M^2 - m_l^2)^2 - 4W^2M^2], \qquad (12)$$

where M is the mass of the nucleus and $W = \sqrt{(M+E)^2 - \mathbf{k}^2}$ is the energy in the c.m.s. of the incoming particle and of the original nucleus.

If we now integrate with respect to Δ^2 in formula (9) and neglect in the resultant expression terms of order $\mu/EA^{1/3}$ and $\mu/MA^{1/3}$, we obtain for the cross section

$$\sigma = \frac{Z^2 \alpha}{\pi} \int \sigma_{ph}(w^2) \frac{dw^2}{w^2} \ln \frac{2\mu E A^{-1/3}}{w^2} \eta, \qquad (13)$$

which in fact is the Weizsäcker-Williams formula.

We wish to emphasize, however, that the situation in this case is the complete opposite of the electrodynamic case, for which the Weizsäcker-Williams method was developed. The point that is in electrodynamics $\sigma_{ph}(w^2)$ decreases with increasing w^2, and consequently small w^2 and small Δ^2 are essential to the process. In our case the opposite holds true: $\sigma_{ph}(w^2)$ increases with increasing w^2, and consequently large momentum transfers are significant. Only the presence of the electromagnetic form factor of the nucleus leads to our result, and this furthermore is true only for muons. For electrons the substitution for $\sigma_{ph.el}(w^2)$ instead of $a(w^2, \Delta^2)$ would be too crude an approximation.

We note finally that the formula for the process (1) does not reflect the fact that this is a threshold process. The reason is that the calculations were made for energies $E \gg m$, and we have neglected the lepton mass in the phase volume of the final state.

3. CROSS SECTIONS OF PROCESS (1) AND (2)

In the case when the energy of the incoming particle is much greater than the masses of the light particles participating in the process, the factor η under the logarithm sign in formula (13) tends to unity. Then, substituting into (13) expression (5) for σ_{ph} (taking only the principal term in the formula for σ_{ph}), we obtain for the cross section of process (1)

$$\sigma = \frac{2Z^2\alpha^2G^2\mu A^{-1/3}}{9\pi^3} E \left(\ln \frac{2\mu E A^{-1/3}}{m^2} - 2 \right). \qquad (14)$$

We wish to make the following remark concerning this formula. If we integrate over the entire region of Fig. 3 without imposing any limitations on the maximum momentum transfer, then the cross section will contain terms proportional to $E^2 \ln(E/m)$ and E^2. Such an energy dependence of the cross section, calculated without account of the form factor of the nucleus and consequently incorrect, was obtained, in particular, by Badalyan and Chou Kuang-chao[6] for the process $\nu + A \to \nu + A + e^+ + e^-$.

It follows from the foregoing that the form factor of the nucleus exerts an appreciable influence on the energy dependence of the cross section.

The cross section of process (2) is obtained by substituting (6) into (13):

$$\sigma = \frac{Z^2\alpha^2G^2\mu A^{-1/3}}{18\pi^3} E \left(\ln \frac{2\mu E A^{-1/3}}{m^2} - 2 \right). \qquad (15)$$

The process (2) was considered for nonrelativistic electrons, without account of the form factor, by Gandel'man and Pinaev.[7]

The numerical values of the cross section of the process (1) with an initial-neutrino energy E = 10 Bev are $\sigma \approx 6.6 \times 10^{-44}$ cm^2 and $\sigma \approx 6.0 \times 10^{-41}$ cm^2 for hydrogen and lead, respectively.

The authors are grateful to L. B. Okun' for suggesting the problem and for discussions, and thank I. Ya. Pomeranchuk for a discussion and for interest in the work.

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163

ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2 f^2}{4\pi^3}\omega^2 (\ln\frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2 f^2}{9\pi^3}\omega^2 \left(\ln\frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2f^2/4\pi^3)\,\omega^2 \geqslant \ldots$	$(e^2f^2/9\pi^3)\,\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-[\frac{1}{4}(3\cos^2\theta - 1)\ldots$
14	3	Charakhchyan et al.	533	Table II, col. 6 line 1		1.9	0.9
14	3	Malakhov	550	The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.			
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2 \sum_{s=1}^{\infty} K_2(bs)\right]^2$	$\ldots\left[b^2 \sum_{s=1}^{\infty}(-1)^{s+1} K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	l	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	1 < d/2 < 2	1 < d/r < 2
14	5	Shapiro	1154	l	Table	2306	23.6