ELECTROSTATIC INSTABILITY OF AN INTENSE ELECTRON BEAM IN A PLASMA

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It is demonstrated that the current density in a quasi-neutral fast electrom beam propagating in a plasma has an upper limit above which the beam becomes unstable against formation of a virtual cathode. The mechanism of this instability is discussed.

THE experimental data reported earlier by Pierce [1] and recently by Volosov [2] show that if the current density j in a quasi-neutral electron beam propagating in vacuum exceeds a certain limit j_b, then small negative potential perturbations build up progressively with time in the beam. This results in the production of a virtual cathode, i.e., to the "blocking" of the beam by its own space charge. The limiting current j_b exceeds the maximum beam current without ions j_m by a factor of only 5 or 6, [1,2] i.e., cancellation of the space charge hardly contributes to an appreciable increase in the maximum beam current.

The purpose of the present investigation was to determine the stability of a quasi-neutral electron beam in a concentrated plasma, or, what is the same, the maximum stable current of an electron beam in a plasma.

1. EXPERIMENTAL PROCEDURE

The stability of the electron beam in the plasma was investigated with the apparatus shown schematically in Figs. 1 and 2. An intense beam of fast electrons and a concentrated highly-ionized "cold" plasma were produced in lithium vapor in a strong longitudinal magnetic field by a gas discharge with an incandescent tungsten cathode with direct current. After leaving the discharge chamber (15 cm long with diaphragms 1.2 cm in diameter) the fast electrons of the beam and the plasma particles propagated in a vacuum volume 12.5 cm in diameter and struck an anode which was usually at a distance L = 100 cm from the cathode. In this volume the pressure p of the residual gas was (3—6) × 10^{-6} mm Hg, the lithium-vapor pressure in the discharge chamber was on the order of 10^{-4} mm Hg, the discharge voltage V_d which determined the beam electron energy could be adjusted from several to 400 volts, the beam current I_d was 0.5—5 amp, and the magnetic field H could be varied

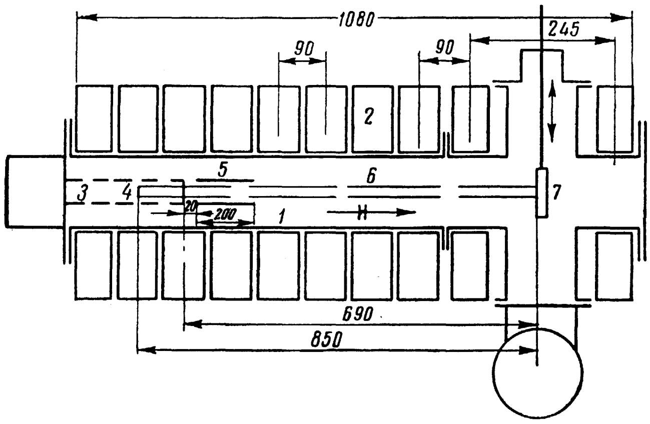

FIG. 1. Experimental apparatus: 1 – vacuum, 2 – solenoid, 3 – plasma source, 4 – cathode of source, 5 – filter, 6 – beam, 7 – anode. The dimensions are in millimeters.

from 400 to 2000 oe. The beam diameter was approximately 1 cm.

The plasma concentration in the investigated volume was regulated either by changing the lithium vapor pressure and the discharge chamber (but not below the discharge ignition threshold) or by using a special filter. The filter consisted of two water-cooled copper plates cut to fit the beam; these plates surrounded the beam over a length of 20 cm and could be brought together or moved apart during the experiment without interfering with the beam. Since the plates "froze out" the vapor and some of the lithium ions emerging from the discharge chamber, the plasma concentration decreased along the filter.

The beam and plasma electron concentrations, n_{e_1} and n_{e_2}, were measured by a probe method, using the volt-ampere characteristic of a bulky collector situated behind a small hole (2 mm in diameter) drilled in the center of the anode. The ion branch of the probe characteristic (Figs. 3 and 4) was determined by the Bohm formula, [3] according to which

$$j_+ = 0.4 n_+ \sqrt{2T_e / m_+}, \qquad (1)$$

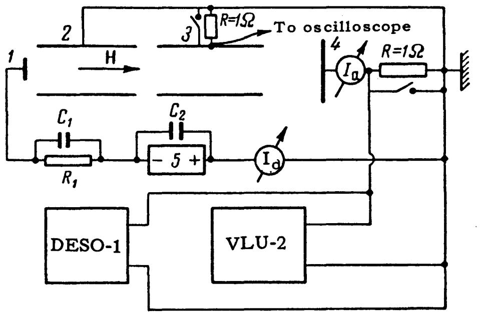

FIG. 2. Electric circuit of apparatus: 1 – cathode, 2 – discharge chamber, 3 – filter, 4 – anode, $C_1 = C_2 = 100\ \mu f$, $R_1 = 40$ ohm, 5 – discharge rectifier.

where j_+ is the ion current density in the probe (collector), n_+ the concentration of the ions in the plasma, m_+ their mass, and T_e the temperature of the plasma electrons. In our case

$$n_+ = n_{e1} + n_{e2}, \tag{2}$$

$$n_{e1} = j_{e1} / v_{e1}, \qquad n_{e2} = 4 j_{e2} / v_{e2}, \tag{3}$$

and j_{e_1} and j_{e_2} are the current densities of the beam and of random plasma electrons. The value of j_{e_1} was assumed equal to the collector current density at zero collector potential (j_0).

We neglect here the plasma-electron current in the probe, i.e., we assume that in the expression

$$j_0 \approx j_{e1} + \frac{n_{e2} v_{e2}}{4} \exp\left(-\frac{e\varphi_a}{T_e}\right), \tag{4}$$

where φ_a is the potential of the plasma relative to the probe (anode)*; the second term in the right half can be neglected compared with the first. The validity of this assumption will be justified later on. Equations (1)–(4) were used to determine the relative plasma concentration ($\alpha = n_{e_2}/n_{e_1}$):

$$\alpha + 1 \approx 2.5 \sqrt{\frac{m_+}{m_e} \frac{eU}{T_e}} \frac{j_+}{j_{e1}} \approx \frac{1.6 \cdot 10^3 \sqrt{U}}{j_{e1} / j_+}, \tag{5}$$

where eU is the energy of the beam electrons (hundreds of ev) and $T_e = 3$ ev (the latter was obtained from probe measurements). We assumed U to equal the discharge voltage V_d.

2. EXPERIMENTAL DATA

In the first series of measurements we plotted the dependence of the time-averaged electron current in the anode, I_{av}, on the plasma concentration. In these measurements I_d (the time-averaged total beam electron current) served as the parameter for the family of functions $I_{av}(n_+)$.

*In the discharge considered here the plasma has a positive potential on the order of several times T_e/e relative to the anode.

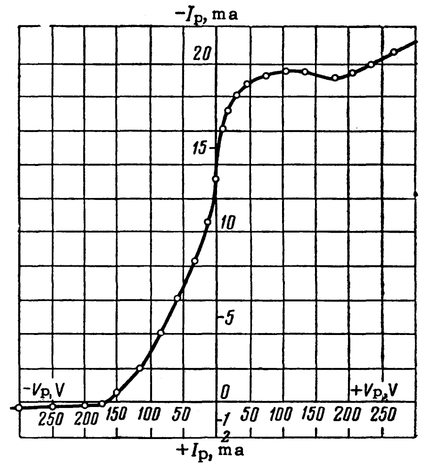

FIG. 3. Probe characteristic: $I_d = 1.2$ amp; $V_d = 120$ v, $I_{av} = 0.96$ amp; H = 1800 oe; L = 85 cm; p = 5×10^{-6} mm Hg.

We measured simultaneously the amplitude A of the oscillations in the beam passing through the plasma. A typical result (Fig. 5) shows that there are two qualitatively different modes of electron beam propagation in the plasma.

1. If the relative concentration of the plasma exceeds a certain "critical" value ($\alpha > \alpha_c$), the beam is stable in the plasma. All the electrons that leave the cathode pass freely through the plasma and reach the anode. The anode electron current I_a is practically independent of the time (i.e., it is equal to I_{av}) and exceeds somewhat the cathode electron current I_d, owing to the plasma-electron current.

2. When $\alpha < \alpha_c$, the beam is unstable in the plasma, as shown by the fact that the electron current in the anode oscillates intensely from practically zero to a maximum value somewhat greater than I_d.

The current oscillations in the unstable beam (Fig. 6) are made up of a relaxation component, with period $T \approx 10^{-4} - 10^{-3}$ sec, and a high-

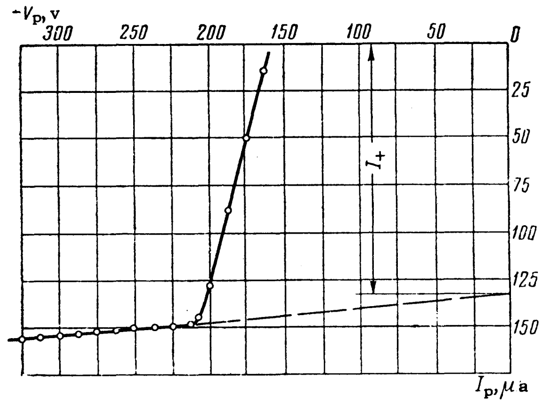

FIG. 4. Ion branch of probe characteristic under the conditions of Fig. 3.

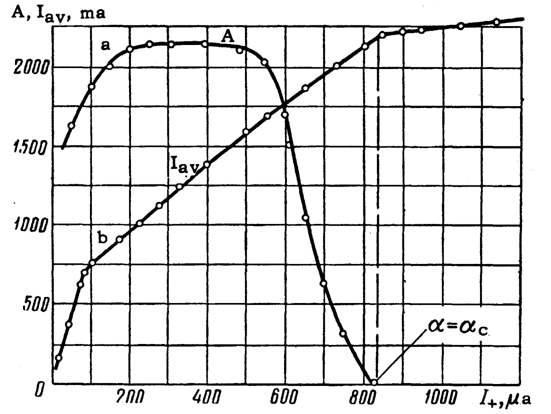

FIG. 5. Dependence of I_{av} and A on the concentration of the plasma. I_+ – ion current in probe collector; I_d = 2 amp; V_d, H, L, and p are the same as in Fig. 3.

frequency component with period $\tau \approx (1-3) \times 10^{-7}$ sec. The high-frequency oscillations (Fig. 7) occupy a portion T_1 of the period of the relaxation oscillations, and their amplitude is somewhat greater than I_d. This means that when $\alpha < \alpha_c$ (we refer here to time-averaged quantities) the beam passes freely through the plasma only during a time interval $T_2 = T - T_1$, while during the time interval T_1 it alternately passes through the plasma (with period τ) or is cut off. Consequently the average anode current I_{av} comprises only part of the total electron current, the remainder of this current (as shown by measurements and by Fig. 8) going to the discharge chamber (or to the filter, if used). With increasing α, the value of T_2/T increases, causing a corresponding increase in I_{av} (Fig. 5); when $\alpha = \alpha_c$ we have $T_2/T = 1$ and I_{av} practically reaches saturation (at a value somewhat greater than I_d).

3. MECHANISM OF INSTABILITY OF THE ELECTRON BEAM IN THE PLASMA

It must be emphasized that these current oscillations in the electron beam are not connected with

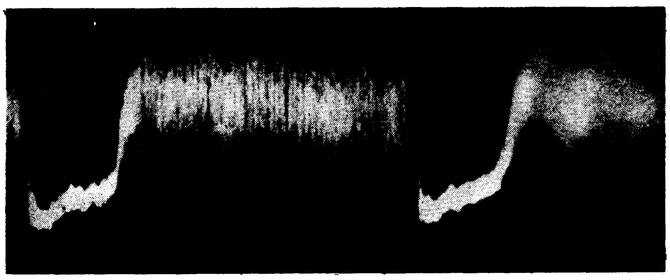

FIG. 6. Dependence of the anode current on the time. Upward deflection corresponds to an increase in the electron current. T = 500 μsec; τ = 0.15 μsec; peak-to-peak oscillation A = 2 amp; I_d = 1.8 amp; V_d = 180 v; I_{av} = 1 amp; H = 1800 oe; p = 4 × 10^{-6} mm Hg; L = 100 cm.

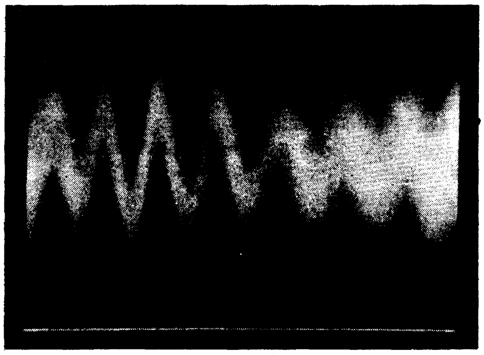

FIG. 7. Sweep of high-frequency oscillations of Fig. 6.

instability in the discharge arc. Such an instability arises, as is well known,[3] when the ratio of the cathode ion current to the cathode electron current does not satisfy the Langmuir relation

$$j_+ / j_{e1} \geqslant \sqrt{m_e / m_+}. \qquad (6)$$

In our experiments such an instability was produced artificially by decreasing V_d to 10 or 15 volts, and manifested itself in the fact that the discharge was alternately ignited and extinguished, with a period on the order of 200–500 μsec (Fig. 9); it is particularly important here that the oscillations of the anode and discharge-chamber current were in step with the oscillations of the total discharge current (no filter was used in this experiment). Unlike the oscillations due to such an instability of the discharge arc itself, the investigated oscillations in the anode and discharge-chamber currents (Fig. 8) are out of phase and are consequently of a different nature (there was likewise no filter in this experiment).

We note that all these phenomena looked qualitatively alike, regardless of whether a filter was used or not, and regardless of the length of the

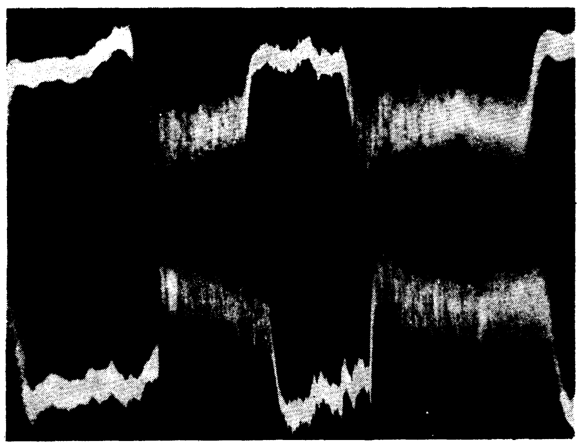

FIG. 8. Oscillograms of anode current (below) and discharge-chamber current (above) in a low-voltage discharge. I_d = 1 amp; V_d = 15 v; T = 300 μsec.

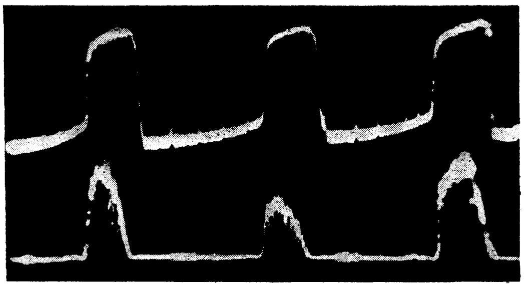

FIG. 9. Oscillations of anode current (top) and discharge-chamber current (bottom). Deflection upward corresponds to an increase in the electron current; T = 300 μsec; τ = 0.1 μsec; A = 2.1 amp; I_d = 2 amp; V_d = 200 v; I_{av} = 1 amp; H = 1800 oe; L = 100 cm; p = 3×10^{-6} mm Hg.

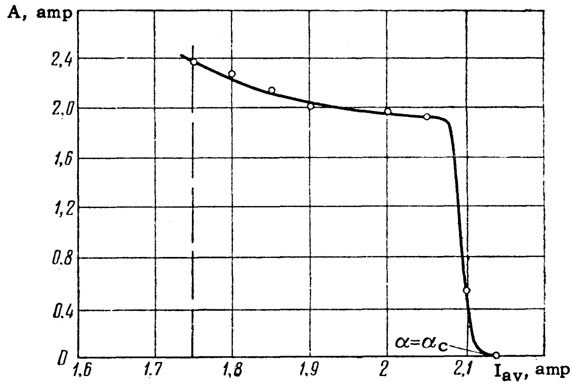

FIG. 10. Dependence of A on the concentration of the plasma at l = 2 cm, I_d = 2 amp, V_d = 120 v, H = 1800 oe.

plasma beam, particularly when the anode was at a distance l = 2 cm from the discharge chamber. The latter is illustrated in Fig. 10, where the measure of α is the quantity $\beta = I_{av}/I_d$ (I_{av} = 1.75 amp corresponds to the threshold of discharge ignition).

We thus arrive, on the basis of the data presented above, at the conclusion that the beam instability observed in the plasma when $\alpha < \alpha_c$ is connected with the alternate appearance and disappearance (with a period $\tau \approx 10^{-7}$ sec) of a virtual cathode in the beam. In these oscillations the vanishing of anode current does not signify extinction of the discharge, but that the electrons reflected from the virtual cathode "drop out" into the discharge chamber and the filter (Fig. 8). As to the coordinates of the virtual cathode, a special experiment, with a probe moving along the beam (on the edge of the beam) has shown that in a long beam the virtual cathode is produced between the discharge chamber and the anode (for example in the filter, if the filter plates are brought sufficiently close together).

Let us explain qualitatively why the oscillations have the appearance shown in Figs. 6 and 8. If the inequality $\alpha < \alpha_c$ obtains at some instant of time, the beam can pass through the plasma only during the time interval τ_0 when the negative potential perturbations do not have time to produce a virtual cathode in the beam. This time interval ($\sim 10^{-7}$ sec), as follows from Figs. 6 and 8, is of the same order as the transit time (τ_{e_1}) of a beam electron in the system. After formation of the virtual cathode, the "beam plus plasma" system, deprived of the potential barrier which confines the plasma electrons, is likewise not stationary. The duration τ_{co} of the cut-off state of the beam will be equal to the time in which the plasma electrons remove from the system a corresponding excess of negative charge, after which the beam is again resumed, etc. This can explain why the virtual cathode pulsates in time (Figs. 6—8).

During these oscillations the plasma electrons apparently "heat up" and begin to ionize the lithium vapor more effectively. If at the same time the plasma concentration increases to the critical value α_c, the oscillations will stop and the plasma electrons will begin to "cool down." Then the plasma concentration, after reaching a certain limit, will again decrease and when $\alpha < \alpha_c$ the oscillations resume and eventually increase the concentration of the plasma to a level $\alpha > \alpha_c$, etc. In our opinion this is precisely why the high-frequency oscillations are modulated by the relaxation oscillations, and why the latter have "platforms" corresponding to the stable state of the beam (the part T_2 of the oscillation period T in Figs. 6 and 8). It is easy to see that the less the time average of α differs from α_c, the greater should be the ratio T_2/T and the average current through the plasma (I_{av}). As already noted, this is indeed observed in the experiments (Fig. 5). If $\alpha \ll \alpha_c$, then the "platforms" of Figs. 6 and 8 should disappear, in spite of the additional ionization of the lithium by the plasma electrons. This phenomenon also takes place when $\beta < 0.2—0.3$.

4. LIMITING STABLE CURRENT OF ELECTRON BEAM IN A PLASMA

Thus, as the electron beam passes through the plasma, there is an upper limit to the current density, beyond which the beam becomes unstable with respect to formation of a virtual cathode. The limiting stable (or critical) beam current density j_c has been shown by the measurements to be proportional to the plasma concentration and to be determined by the relation

$$j_c \approx 0.7\, j_{e2} \approx n_{e2}\, v_{e2} / 6. \quad (7)$$

Thus, for example, the values of I_c are 1.05, 2, and 4 amp for n_{e_2} = 2.4, 5, and 1.5×10^{12} cm^{-3}, etc. (These data pertain to V_d = 120 v). Here

$$\alpha_c \approx 6v_{e1} / v_{e2} \approx 30 - 40. \quad (8)$$

From relations (8), (6), and (1) it follows that α_c is approximately double the relative plasma concentration corresponding to the discharge ignition threshold. Consequently a discharge with a stable arc can be unstable with respect to formation of a virtual cathode (as follows from Fig. 10).

It is easy to see that the presence of a concentrated plasma ($\alpha \approx 30-40$) increases by many times the limiting current and the stable electron beam. In fact, in the absence of plasma ($\alpha = 0$) the limiting current in our case (V_d = 120 v, beam diameter 1 cm, diameter of vacuum volume 12 cm) would be [1,2]

$$j_b \approx 30 \text{ ma cm}^{-2} \approx 10^{-2}\, j_c. \quad (9)$$

In other words, the plasma exerts an appreciable stabilizing action on an electron beam propagating through it. This action is obviously connected with the fact that when a perturbation of potential occurs in the beam, a sufficient number of plasma electrons leave the system under the influence of the resultant electric field, and the development of the perturbation is slowed down, ceasing completely when $\alpha > \alpha_c$.

We can make the following suggestions concerning the mechanism of this additional departure of the plasma electrons. This beam instability, as already noted, develops in a time on the order of the transit time τ_{e_1} of the beam electrons. Within this time the plasma electrons do not have a chance to "drop out" of the beam along the magnetic field, since their velocities are much lower than the velocities of the beam electrons. Thus, there is apparently no time for the longitudinal drift of plasma electrons to come into play, and the principal factor stabilizing the system is the transverse (perpendicular to **H**) drift of the plasma electrons.

The following data favor this conclusion. First, when $\alpha < \alpha_c$ the flow of beam electrons transverse to the discharge chamber is approximately equal to the total electron current from the cathode (Fig. 8) i.e., it amounts to two or three amperes, which is easily verified to be several orders of magnitude greater than the transverse flow of these electrons due to collision with ions and atoms. Second, measurement with a probe located 40 mm away from the beam axis have shown that when $\alpha < \alpha_c$ the plasma electron and ion currents in the probe increase by several times, in spite of the considerable decrease (by several times) of the current in the beam passing through the plasma (the half-width of the beam also increases appreciably). In other words, when $\alpha < \alpha_c$ a very effective ("anomalous") transverse diffusion of electrons is produced in the plasma beam.

Third, experiments have shown that the beam stability depends strongly on the intensity of the magnetic field. It turns out that when α is neither too small nor too large the beam is stable only within a definite range of H, for example from 800 to 1400 oe (this range is greatly extended on both sides as α increases, and its upper limit goes beyond 2000 oe). Fourth, as already noted, in a long beam the virtual cathode is formed between the discharge chamber and the anode. This means that different parts of the beam (having different longitudinal coordinates) are not equally stable against the formation of the virtual cathode. The greatest stability is possessed by the discharge-chamber region. This can be attributed to the greater effectiveness of the transverse motion of the plasma electrons in this region, owing to the increased concentration of the plasma and to the proximity of the walls.

To find a general quantitative criterion of the above instability of a beam in a plasma, and to explain its origin, additional research must be carried out. The same holds for the explanation of the role of the boundary conditions. In the present investigation the "beam plus plasma" system was open and was bounded by electrodes, one of which was the source of the electron beam and the other the receiver. The question whether a similar instability can arise in a closed system, say in a torus, still remains unanswered.

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[1] J. R. Pierce, Theory and Design of Electron Beams, (Russian translation), Soviet Radio Press, 1956, p. 165. [Van Nostrand, 1949].

[2] V. I. Volosov, J. Tech. Phys. (U.S.S.R.), in press.

[3] D. Bohm, The Characteristics of Electrical Discharges in Magnetic Fields, N.Y. 1949.

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ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2 f^2}{4\pi^3}\omega^2 (\ln \frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2 f^2}{9\pi^3}\omega^2 \left(\ln \frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2f^2/4\pi^3)\,\omega^2 \geqslant \ldots$	$(e^2f^2/9\pi^3)\,\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-[\frac{1}{4}(3\cos^2\theta - 1)\ldots$
14	3	Charakhchyan et al.	533	Table II, col. 6 line 1		1.9	0.9
14	3	Malakhov	550	The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.			
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2 \sum_{s=1}^{\infty} K_2(bs)\right]^2$	$\ldots\left[b^2 \sum_{s=1}^{\infty}(-1)^{s+1} K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	l	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	1 < d/2 < 2	1 < d/r < 2
14	5	Shapiro	1154	l	Table	2306	23.6