CALCULATION OF THE FRACTION OF HIGH-ENERGY ELECTRONS AND PHOTONS NEAR THE AXIS OF EXTENSIVE AIR SHOWERS

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The fraction of high-energy electrons and photons in extensive air showers located at distances of up to 3 m from the shower axis is calculated assuming that π^0 mesons are produced continuously throughout the atmosphere. The angles of emission of the π^0 mesons are taken into account. It is shown that the theoretical calculations can be made consistent with the experimental results if, besides taking the π^0 meson emission angles into account, we assume that the energy spectrum of the photons produced is much softer than the spectrum of the nuclear-active component in EAS of energy $\geq 10^{11} - 10^{12}$ ev.

IT has been shown [1-3] that, in the core of extensive air showers (EAS) at distances of 0 — 3 m from the shower axis, the energy spectra of the electron-photon component are considerably softer than should be expected from calculations based on the electromagnetic cascade theory. The fraction of high-energy ($E \geq 10^9$ ev) electrons and photons at 0 — 3 m from the shower axis has been determined experimentally and theoretically:

$$\Delta = N(\geqslant 10^9)/N(>0).$$

The experimental value Δ for showers with a total number of particles $N = 10^4 - 5 \times 10^4$ equals $(16 \pm 3)\%$. For a cascade parameter $s = 1.2$, the theoretical calculation of Δ (see [4]) at different energies E_0 of the primary photon gives

E_0, eV:	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
Δ, %:	15	33	35	39	44

The variation of Δ with the initial photon energy E_0 and the depth of production of the cascade shower t is illustrated in Table I (where t is expressed in radiation lengths, E_0 in ev, and Δ in percent).

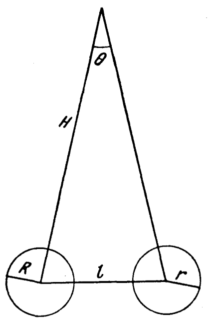
It can be seen from the table that in order to make the theoretical calculations based on the purely electromagnetic theory of EAS development consistent with the experimental value of Δ it is necessary to satisfy one of two conditions: either the electron-photon component of EAS is produced mainly by protons with energy $\leq 10^{10}$ ev, or the energy spectrum of secondary photons with an energy 10^8 ev or higher should be characterized by a large value of the cascade parameter s (> 1.4). It is very difficult to reconcile either of these assumptions with a number of experimental facts.

A comparison of the data on the energy flux carried by electrons and photons near the axis of EAS [5,6] and on the mean energy per electron at various distances from the shower axis [7] with the calculations based on the electromagnetic cascade theory has also shown that the core region in EAS is poor in high-energy electrons.

In the present paper, we attempt to explain this deficiency of electrons by taking into account the

Table I

E_0 \ t		3	5	7	9	11	13	15	17	19	21	23
10^{10}	Δ	32	24	15	9							
	s	0,75	1,0	1,27	1,4							
10^{11}	Δ		46	40	34	29	21					
	s		0,75	1,10	1,12	1,3	1,4					
10^{12}	Δ			56	50	44	36	30	25			
	s			0,8	0,95	1,1	1,2	1,3	1,4			
10^{13}	Δ				63	58	50	44	36	30	27	
	s				0,8	0,9	1,05	1,15	1,24	1,3	1,4	
10^{14}	Δ					70	62	54	46	40	34	29
	s					0,85	0,95	1,03	1,11	1,22	1,26	1,34



angles of emission of π^0 mesons. We have carried out the calculation of the fraction of high-energy electrons and photons in the range of $0-3$ m from the shower axis assuming a continuous production of π^0 mesons in the atmosphere, and taking their angles of emission into account. The lateral distribution of the nuclear-active particles in the showers was neglected; it was assumed that the π^0 mesons are produced within the shower core.* The transverse momentum obtained by the π^0 mesons was assumed to be constant, independent of the energy, and equal to $p_\perp = 4 \times 10^8$ ev/c. The angle θ at which π^0 mesons are inclined to the EAS axis is equal to $\theta = p_\perp c/E_0$.

It has been assumed that the γ rays produced in the decay of π^0 mesons conserve their direction of emission. If the γ ray is produced at an atmospheric depth t, then, at the observation level t_0, the axis of the cascade shower produced by the photon deviates from the center of the EAS by a distance $l \approx H\theta = Hp_\perp c/E_0$ (where H is in meters).

The number of electrons and photons $N(t_0, R, E)$ at depth t_0 in a circle with radius R around the EAS axis in a shower with total energy E′ is equal to

$$N(t_0, R, E) = \int_0^{t_0}\int_0^{E'} \Phi(E_0, t)\, F(E_0, E, R, t_0 - t)\, dE_0\, dt.$$

where $F(E_0, E, R, t_0 - t)$ is a function giving the number of electrons and photons with energy $\geq E$ arriving at the depth t_0 within a circle with radius R originating from one proton with energy E_0 produced at the depth t and deviating from the shower axis by the angle θ, and $\Phi(E_0, t)$ is the number of photons with energy E_0 produced at the depth t as a result of π^0 meson decay.

For the function $F(E_0, E, R, t_0 - t)$ we have

$$F = 2\int_{l-R}^{l+R} r\rho \arccos\frac{r^2 + l^2 - R^2}{2lr}\, dr, \quad l \geqslant R,$$

$$F = \int_0^{R-l} \pi r\rho\, dr + 2\int_{R-l}^{R+l} r\rho \arccos\frac{r^2 + l^2 - R^2}{2lr}\, dr, \quad l < R,$$

where $\rho(r, E_0, E, t_0 - t)$ is the flux density of particles with energy $\geq E$ at a distance r from the cascade shower axis. From the cascade theory, we have

$$\rho = r_1^{-2} f(r, E_0, E, t_0 - t)\, N / 2\pi,$$

$$r_1 = E_s / E, \quad E_s = 21 \text{ Mev},$$

where $f(r, E_0, E, t_0 - t)$ is the lateral distribution function of electrons and photons with energy $\geq E$ at a depth t_0 produced by a photon with energy E_0 at a depth t,[4] and $N(E_0, E, t_0 - t)$ is the total number of such electrons and photons.[8]

Similarly, the number of electrons with energy greater than 0 in the circle of radius R was determined. For this, it was necessary to use the results of the cascade theory taking the ionization losses into account.

1. We have assumed that the primary photons in the atmosphere are produced according to the function $e^{-\mu t}$, and have determined how the fraction Δ of high-energy electrons and photons varies as compared with all electrons within a 3 m radius as a function of the initial photon energy E_0 and of the factor μ (Table II). The calculation was carried out numerically.

Furthermore, we have determined what the photon spectrum $\Phi(E_0, t)$ should be so that the theoretical value of Δ will agree with the experimental one for $1/\mu = 160$ g/cm^2. The spectrum was assumed to be represented by a power law $\Phi(E_0, t) = AE^{-\gamma}$, and the following possibilities were considered:

1) If $\gamma = 2.0$* in the $10^9 - 2 \times 10^{12}$ ev energy range, then $\Delta = 19\%$.

2) If $\gamma = 1.5$ in the $10^9 - 10^{11}$ ev energy range and $\gamma = 2.5$ in the $2 \times 10^{11} - 2 \times 10^{12}$ ev energy range, then $\Delta = 20\%$.

3) If we choose $\gamma = 1.3$ and consider the photon spectrum up to 2×10^{12} ev, then $\Delta = 43\%$.

It is impossible to determine the exact analytical shape of the spectrum from these calculations. It is, however, clear that the flatter the spectrum for low energies, the steeper it should become at higher energies.

*The correction for the lateral distribution of nuclear-active particles in the shower was introduced only for small altitudes (< 1 t), where it plays a larger role than the deflection of photons.

*Earlier,[2,3] the value $\gamma = 1.7$ was obtained for the electron and photon spectra within a 3 m radius about the EAS axis in the $2 \times 10^9 - 10^{10}$ ev energy range. We can assume that the spectrum of photons produced by the electron-photon shower component is not less steep.

Table II. Calculated values of Δ, %

$1/\mu$, g/cm²	E_0, ev				
	$7\cdot10^{10}$	$2\cdot10^{11}$	$5.5\cdot10^{11}$	$1.5\cdot10^{12}$	$4\cdot10^{12}$
180	5	47	59	64	70
160	4	40	53	60	66
120	3.5	31	38	45	
70	1.7	9	17	25	

2. We have used the theoretical spectra of photons produced as a result of the π^0 meson decay at various depths in the atmosphere for an initial nuclear energy $E_1' = 10^{14}$ ev and $E_2' = 10^{16}$ ev calculated by Guzhavin and Zatsepin[9] using the Landau model[10] of the nuclear collision. If the spectrum $\Phi(E_0, t)$ is assumed to be of the power law form $\Phi = AE^{-\gamma}$, then, as shown in [9], the value γ varies with the depth of the atmosphere. The mean value of γ (for $E_1' = 10^{14}$ ev) is then $\bar{\gamma} \approx 1.8$ in the $5.5 \times 10^{11} - 1.1 \times 10^{13}$ ev energy range, while $\bar{\gamma} \approx 0.9$ in the $1.35 \times 10^9 - 10^{10}$ ev energy range.

If we assume that the nuclear-active component produces photons with a probability decreasing as $e^{-\mu t}$, then, according to [9], the value for $1/\mu$ varies with photon energy from 70 g/cm² for $E_0 = 4 \times 10^{12}$ ev to 190 g/cm² for $E_0 = 4 \times 10^9$ ev.

Using this spectrum $\Phi(E_0, t)$, we have determined the values of Δ for E_1' and E_2' within a 3 m radius about the EAS axis, both taking into account and neglecting the angles of emission of π^0 mesons (Table III). As can be seen from the table, when the angles of emission of π^0 mesons are taken into account, the fraction of high-energy electrons and photons within a 3 m radius about the shower axis decreases by a factor of 1.5. However, the theoretical and experimental values of Δ are still markedly different. In order to obtain an agreement with the experiment, we have assumed that the photon spectrum is softer than that calculated by Guzhavin and Zatsepin.[9] In fact, we have simply cut off the spectrum $\Phi(E_0, t)$ at the energy $E_0 = 1.5 \times 10^{12}$ ev at all depths of the atmosphere, obtaining for Δ a value of 18%.*

From the above discussion it can be seen that the calculated value of Δ may agree with the experimental one without making any assumptions concerning the steepness of the π^0 production spectrum in the low-energy range ($10^8 - 10^{10}$ ev). Taking the emission angles of π^0 mesons into account decreases the difference between the theoretical and experimental values of Δ, and enables us to assume a less steep spectrum in this energy range. However, for energies $> 10^{11}$ ev, we still have to assume that the spectrum of the produced photon may be considerably steeper than the spectrum of the nuclear-active component in the showers.* The difference between the spectra of π^0 mesons and the spectra of the nuclear-active particles produced with mesons has been observed independently by the Bristol group.[11]

*In the calculation, we have not taken the distribution of $p_\perp$ into account, but have assumed its value to be constant. In order to estimate the influence of $p_\perp$, we have calculated Δ for $E' = 10^{14}$ ev for $p_\perp = 2 \times 10^8$ and 8×10^8 ev/c, and have obtained Δ = 28% and 26% respectively. After cutting off the spectrum at the energy 1.5×10^{12} ev, we have obtained Δ = 20% and 16% instead of 18% as given in Table III. Hence, it can be seen that taking the exact distribution of $p_\perp$ into account cannot introduce any significant changes.

Table III

	Experiment	Calculations based on the model of[9]					
		Neglecting the π^0 meson angles of emission	Taking into account the π^0 meson angles of emission		Taking into account the angle of emission of π^0 mesons and cutting the spectrum off at 1.5×10^{12} ev	
E', ev*	$(1-5)\cdot10^{14}$	10^{14}	10^{14}	10^{16}	10^{14}	10^{16}
Δ, %	16±3	45	27	39	18	24

*For $E' = 10^{16}$ ev, Δ is somewhat larger, since at this energy the π^0 meson spectrum is flatter than for $E' = 10^{14}$ ev.

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