THERMAL CONDUCTIVITY OF PURE SUPERCONDUCTORS AND ABSORPTION OF SOUND IN SUPERCONDUCTORS

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The electron thermal conductivity produced in very pure superconductors by scattering of electron excitations on phonons is calculated on the basis of the microscopic theory of superconductivity. The absorption of sound in superconductors is also discussed.

We consider in this paper the electron thermal conductivity of superconductors, due to the scattering of electrons on phonons. This interaction plays a major role in the investigation of the thermal conductivity of very pure semiconductors. In addition, it makes a certain contribution also when the impurity concentration is not too high.[1] The final formula for the corresponding coefficient of thermal conductivity was given earlier.[2] We present here more detailed calculations and a comparison with experiment. We consider also the absorption of ultrasound in superconductors.

1. THERMAL CONDUCTIVITY OF PURE SUPERCONDUCTORS

The electron distribution function satisfies the kinetic equation

$$\frac{\partial f}{\partial x}\frac{\partial \varepsilon}{\partial p_x} - \frac{\partial f}{\partial p_x}\frac{\partial \varepsilon}{\partial x} = \left(\frac{\partial f}{\partial t}\right)_{\text{st}}, \qquad (1.1)$$

where f is the electron distribution function, which we seek, as usual, in the form

$$f = f_0 + \frac{\partial f_0}{\partial \varepsilon}\varphi(\varepsilon, \Omega)\frac{\partial T}{\partial x}.$$

In the electron-phonon interaction Hamiltonian

$$H = \sum_{\mathbf{k},\mathbf{k}',s} V_{\mathbf{k}\mathbf{k}'} a^{+}_{k's} a_{ks} b_{\mathbf{q}} + \text{c.c.}$$

we change over to new Fermi amplitudes that describe the electron excitations of the superconductor by making a canonical transformation, following Bogolyubov[3]:

$$\alpha_{k0} = u_k a_{k,\,1/2} - v_k a^{+}_{-k,\,-1/2},$$
$$\alpha_{k1} = u_k a_{-k,\,-1/2} + v_k a^{+}_{k,\,1/2},$$
$$u_k^2 = \tfrac{1}{2}(1+\xi/\varepsilon), \quad v_k^2 = \tfrac{1}{2}(1-\xi/\varepsilon)$$

(ξ — energy of ordinary electron reckoned from the Fermi surface, ϵ — energy of electron excitation, $\epsilon = \sqrt{\xi^2 + \Delta^2(T)}$, $\Delta(T)$ — size of gap in the energy spectrum).

We now rewrite the kinetic equation in terms of the new amplitudes (the left half is transformed in accordance with [4]):

$$f_0^2 e^{\varepsilon/T}\frac{\xi}{\varepsilon}\frac{\varepsilon}{T}\frac{\partial T}{\partial x} = \int |V|^2 N_0\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right) \times [\varphi(\varepsilon', \Omega') - \varphi(\varepsilon, \Omega)]\, e^{\varepsilon'/T} f_0(\varepsilon) f_0(\varepsilon') \times \delta(\varepsilon' - \varepsilon - \hbar\omega)\, d\mathbf{q} + \int |V|^2 N_0\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)[\varphi(\varepsilon', \Omega') - \varphi(\varepsilon, \Omega)]\, e^{\varepsilon/T} f_0(\varepsilon) f_0(\varepsilon')\, \delta(\varepsilon - \varepsilon' - \hbar\omega)\, d\mathbf{q} + \int |V|^2 N_0\left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)[\varphi(\varepsilon', \Omega') - \varphi(\varepsilon, \Omega)] \times e^{\hbar\omega/T} f_0(\varepsilon) f_0(\varepsilon')\, \delta(\varepsilon' + \varepsilon - \hbar\omega)\, d\mathbf{q},$$

$$N_0 = (e^{\hbar\omega/T} - 1)^{-1}. \qquad (1.2)$$

We recognize further that the phonon wave vector **q** is small compared with $|\mathbf{p}|$. We therefore expand $\varphi(\Omega')$ in powers of $\mathbf{q}' = \mathbf{p} - \mathbf{p}^*$ ($\mathbf{p}^*$ is a vector directed in the **p** direction of length $|\mathbf{p}'|$). This method of investigating the kinetic equation was first developed by Landau and Pomeranchuk.[5] Integrating then over the angle ϑ (the polar axis is chosen in the direction of the vector **p**), we obtain*

$$f_0^2 e^{\varepsilon/T}\frac{\varepsilon}{T}\frac{p_x}{m} = \int N_0 |V|^2 \frac{\varepsilon\varepsilon'}{|\xi||\xi'|}\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right) \times [\varphi(\varepsilon+\hbar\omega, \Omega) - \varphi(\varepsilon, \Omega)]\frac{e^{(\varepsilon+\hbar\omega)/T}}{(e^{\varepsilon/T}+1)(e^{(\varepsilon+\hbar\omega)/T}+1)} \times \frac{m}{pq} q^2\, dq\, d\varphi + \int N_0 |V|^2 \frac{\varepsilon\varepsilon'}{|\xi||\xi'|}\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right) \times [\varphi(\varepsilon-\hbar\omega, \Omega) - \varphi(\varepsilon, \Omega)]\frac{e^{\varepsilon/T}}{(e^{\varepsilon/T}+1)(e^{(\varepsilon-\hbar\omega)/T}+1)} \times \frac{m}{pq} q^2\, dq\, d\varphi + \int N_0 |V|^2 \frac{\varepsilon\varepsilon'}{|\xi||\xi'|} \times \left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)[\varphi(\hbar\omega-\varepsilon, \Omega) - \varphi(\varepsilon, \Omega)]$$

*The temperature is in energy units throughout.

$$\times \frac{e^{\hbar\omega/T}}{(e^{\varepsilon/T}+1)(e^{(\hbar\omega-\varepsilon)/T}+1)} \frac{m}{pq} q^2\,dq\,d\varphi$$

$$+\int N_0 |V|^2 \frac{\varepsilon\varepsilon'}{|\xi||\xi'|}\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)$$

$$\times \frac{e^{(\varepsilon+\hbar\omega)/T}}{(e^{\varepsilon/T}+1)(e^{(\varepsilon+\hbar\omega)/T}+1)} \frac{m}{pq} \frac{\partial^2\varphi}{\partial\Omega^2} q_i q_j q^2\,dq\,d\varphi$$

$$+\int N_0 |V|^2 \frac{\varepsilon\varepsilon'}{|\xi||\xi'|}\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right) \frac{e^{\varepsilon/T}}{(e^{\varepsilon/T}+1)(e^{(\varepsilon-\hbar\omega)/T}+1)}$$

$$\times \frac{m}{pq} \frac{\partial^2\varphi}{\partial\Omega^2} q_i q_j q^2\,dq\,d\varphi + \int N_0 |V|^2 \frac{\varepsilon\varepsilon'}{|\xi||\xi'|}\left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)$$

$$\times \frac{e^{\hbar\omega/T}}{(e^{\varepsilon/T}+1)(e^{(\varepsilon-\hbar\omega)/T}+1)} \frac{m}{pq} \frac{\partial^2\varphi}{\partial\Omega^2} q_i q_j q^2\,dq\,d\varphi. \qquad (1.2')$$

We make the approximate substitution

$$\frac{\varepsilon\varepsilon'}{|\xi||\xi'|}\left(1\pm\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\approx 2,$$

as was done in the calculation of the phonon thermal conductivity, [6] and integrate both parts of (1.2′) with respect to ξ. In the right half we make at the same time the expansion [5] $p_x/p = (p_x/p)_{p=p_0} + a_x\xi$. We then seek a solution in the form $\varphi(\epsilon, \Omega) = \varphi_1(\Omega) + \varphi_2(\epsilon, \Omega)$, with $\varphi_1 \gg \varphi_2$. Using $d\xi = \epsilon\, d\epsilon/\xi$ to change over to integration with respect to ϵ we obtain

$$\int_b^\infty f_0^2 e^z a_x z\sqrt{z^2-b^2}\,dz = |V'|^2 T^2 \int_0^\infty \frac{x^2\,dx}{e^x-1}\int [\varphi_2(z+x,\Omega)$$

$$-\varphi_2(z,\Omega)]\frac{e^{z+x}\,dz\,d\varphi}{(e^z+1)(e^{z+x}+1)}$$

$$+|V'|^2T^2\int_0^\infty \frac{x^2\,dx}{e^x-1}\int_{b+x}^\infty [\varphi_2(z-x,\Omega)-\varphi_2(z,\Omega)]$$

$$\times\frac{e^z\,dz\,d\varphi}{(e^z+1)(e^{z-x}+1)} + |V'|^2T^2\int_{2b}^\infty \frac{x^2\,dx}{e^x-1}\int_b^{x-b} [\varphi_2(x-z,\Omega)$$

$$-\varphi_2(z,\Omega)]\frac{e^x\,dz\,d\varphi}{(e^z+1)(e^{x-z}+1)}$$

$$+|V'|^2T^4\int_0^\infty \frac{x^4\,dx}{e^x-1}\int_b^\infty \frac{e^{z+x}\,dz\,d\varphi}{(e^z+1)(e^{z-x}+1)}\frac{\partial^2\varphi_1}{\partial\Omega^2}$$

$$+|V'|^2T^4\int_0^\infty \frac{x^4\,dx}{e^x-1}\int_{b+x}^\infty \frac{e^z\,dz\,d\varphi}{(e^z+1)(e^{z-x}+1)}\frac{\partial^2\varphi_1}{\partial\Omega^2}$$

$$+\frac{1}{2}|V'|^2T^4\int_{2b}^\infty \frac{x^4\,dx}{e^x-1}\int_b^{x-b} \frac{e^x\,dz\,d\varphi}{(e^z+1)(e^{x-z}+1)}\frac{\partial^2\varphi_1}{\partial\Omega^2}.$$

We use the notation $\epsilon/T = z$; $\hbar\omega/T = x$; $\Delta/T = b$; $|V'|^2 = |V^2|/q$.

The first integrals, which contain $\varphi_2(\epsilon, \Omega)$, make a zero contribution (to verify this we make the substitutions $z \to z + x$ in the second integrand and $z \to x - z$ in the third). We then find

$$\varphi = \frac{a(\Omega)}{T^4\Phi(T)}\int_b^\infty f_0^2 e^z z\sqrt{z^2-b^2}\,dz\frac{\partial T}{\partial x};$$

$$\Phi(T) = \int_0^\infty \frac{4x^4}{e^x-1}\int_b^\infty \frac{dz}{(e^z+1)(e^{-z-x}+1)}$$

$$+\int_{2b}^\infty \frac{x^4\,dx}{e^x-1)}\int_b^{x-b} \frac{dz}{(e^z+1)(e^{-z}+e^{-x})} \qquad (1.3)$$

where $a(\Omega)$ depends on the angles that determine the direction of motion of the electron.

We calculate next the heat flux from the formula $Q = \int \epsilon v_x f\, dp$. Taking (1.3) into account, we obtain

$$Q = \frac{\text{const}}{\Phi(T)T^2}\overline{a(\Omega)}\left[\int_b^\infty f_0^2 e^z z\sqrt{z^2-b^2}\,dz\right]^2\frac{\partial T}{\partial x}. \qquad (1.4)$$

Calculation of the integrals in (1.4) leads ultimately to

$$\varkappa = -Q\Big/\frac{\partial T}{dx} = \frac{\text{const}}{\Phi(T)T^2}\left[b^2\sum_{s=1}^\infty K_2(bs)\right]^2, \qquad (1.5)$$

where $K_2(bs)$ is a Bessel function of imaginary argument, and

$$\Phi(T) = 96\zeta(4)\ln(1+e^{-b}) + \sum_{s=1}^\infty s^{-5}e^{-2bs}(80b^4s^4$$

$$+160b^3s^3+240b^2s^2+240bs+120)$$

$$-\ln(e^b+1)\sum_{s=1}^\infty s^{-4}e^{-2bs}(64b^3s^3+96b^2s^2+96bs+48). \qquad (1.6)$$

In the temperature region close to T_C, where the contribution of the electron-phonon interaction to the heat flux is most significant, we obtain

$$\varkappa = \varkappa_n\frac{36}{\pi^4}\frac{\Phi(T_c)}{\Phi(T)}\left[2\sum_{s=1}^\infty\frac{(-1)^{s+1}}{s^2}e^{-bs}\right.$$

$$\left.+2b\ln(1+e^{-b})-\frac{b^2}{2(e^b+1)}\right]^2, \qquad (1.7)$$

where κ_n —thermal conductivity of the normal metal. The results obtained agree quite well with the experimental data. [7,8]

Expression (1.5) for κ differs from that obtained by Bardeen, Rickayzen, and Tewordt. [9] These authors used a direct variational method and trial functions for f that differed appreciably from the true function (1.3).

2. ABSORPTION OF ULTRASOUND IN SUPERCONDUCTORS

Let us examine the absorption of sound in superconductors when the frequency satisfies the condition $\omega \gg 1/\tau$ (ω —frequency of sound, τ —relaxation time for electron excitations; see below).

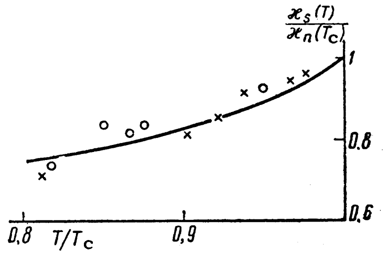

FIG. 1. Experimental points correspond to × – tin,[7] ○ – indium,[8] solid curve – theoretical.

Since the period of the sound wave is in this case less than the relaxation time, we can disregard the relaxation processes completely and consider only the absorption of sound quanta (the number of which is $N \gg 1$) by the electron excitations.

Writing for the probability of absorption of a sound quantum

$$W_{\rm I} = \sum_k |V|^2 [(u_k u_{k'} - v_k v_{k'})^2 N f (1 - f') \delta(\varepsilon' - \varepsilon - \hbar\omega) + (u_k v_{k'} + u_{k'} v_k)^2 N (1 - f)(1 - f') \delta(\varepsilon' + \varepsilon - \hbar\omega)]$$

(f – number of electron excitations with energy ϵ, N – number of phonons of frequency ω) and for the probability of the reverse process

$$W_{\rm II} \sim \sum_k |V|^2 [(u_k u_{k'} - v_k v_{k'})^2 (N + 1) \times (1 - f) f' \delta(\varepsilon' - \varepsilon - \hbar\omega) + (u_k v_{k'} + u_{k'} v_k)^2 (N + 1) f f' \delta(\varepsilon' + \varepsilon - \hbar\omega)]$$

and substituting the expressions for u_k and v_k, we obtain for the absorption coefficient, which is proportional to the difference $W_{\rm I} - W_{\rm II}$, the expression

$$\gamma \sim \int |V|^2 \left\{ \left(1 + \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)(f - f') \delta(\varepsilon' - \varepsilon - \hbar\omega) + \left(1 - \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)(1 - f - f') \delta(\varepsilon' + \varepsilon - \hbar\omega) \right\} \times \frac{p^2\, dp \sin\vartheta\, d\vartheta\, d\varphi}{4\pi^2\hbar^4}. \qquad (2.1)$$

From the momentum conservation law we obtain for the angle ϑ between **p** and **q**, in the case of the scattering of electrons by phonons ($\mathbf{p}' = \mathbf{p} + \mathbf{q}$)

$$\cos\vartheta = (2m\xi' - 2m\xi + q^2)/2pq.$$

In the creation of excitation pairs ($\mathbf{p} + \mathbf{p}' = \mathbf{q}$) we have

$$\cos\vartheta = (-2m\xi' + 2m\xi + q^2)/2pq.$$

Taking these relations into account, we obtain upon integration over the angles (the polar axis is chosen in the direction of the vector **q**) and after changing to integration with respect to ϵ

$$\gamma = \text{const}\cdot \left[\int_\Delta^\infty \left(1 - \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)(f - f') \frac{\varepsilon\varepsilon'}{|\xi||\xi'|} d\varepsilon + D\left(\frac{\hbar\omega}{T}\right) \int_\Delta^{\hbar\omega - \Delta} \frac{1}{2}\left(1 - \frac{\xi\xi' - \Delta^2}{\varepsilon\varepsilon'}\right)(1 - f - f') \frac{\varepsilon\varepsilon'}{|\xi||\xi'|} d\varepsilon \right];$$

$$D\left(\frac{\hbar\omega}{T}\right) = \begin{cases} 1, & \hbar\omega \geqslant 2\Delta \\ 0, & \hbar\omega < 2\Delta. \end{cases} \qquad (2.2)$$

The function $D(\hbar\omega/T)$ has been introduced because creation of excitation pairs is possible only when $\hbar\omega \geq 2\Delta$.

We put approximately

$$1 \pm (\xi\xi' - \Delta^2)/\varepsilon\varepsilon' \approx 2; \qquad (2.3)$$

and consider here frequencies that satisfy the condition $\hbar\omega < T$. We then obtain

$$\gamma = \text{const}\cdot T \left[\int_b^\infty (f - f')\, dz + D(x) \int_b^{x-b} (1 - f - f')\, dz \right];$$

$$f = (e^z + 1)^{-1}, \quad b = \Delta/T, \quad z = \varepsilon/T, \quad x = \hbar\omega/T.$$

Simple integration results in a general formula for the ratio of the ultrasound absorption coefficients in the normal and superconducting states[1]

$$\gamma_s/\gamma_n = \{x - \ln[(e^{b+x} + 1)(e^b + 1)^{-1}] + D(x)[2b - x + 2\ln[(e^{x-b} + 1)(e^b - 1)^{-1}]]\} / \ln[(e^x + 1)/2]. \qquad (2.4)$$

When $x \ll 1$ we have $\gamma_s/\gamma_n = 2/(e^b + 1)$, which agrees with the result obtained by Bardeen, Cooper, and Schrieffer[10] and experimentally confirmed by Morse and Bohm.[11]

3. ABSORPTION OF LONG-WAVE SOUND

We now investigate the absorption of sound in superconductors for the case when $\omega \ll 1/\tau$; for electron excitations we have according to [11] $\tau \sim 10^{-7} - 10^{-6}$ sec. We consider first the absorption of sound by electron excitations, which makes the main contribution to this absorption. We consider the sound field as a factor that deforms the lattice. The irreversibility of the deformation process indeed leads to the absorption of sound energy. The problem thus reduces to a solution of the corresponding kinetic equation and subsequent calculation of the dissipation function, which determines the absorption of the sound wave.

We write the kinetic equation

$$-\left(\frac{\partial f}{\partial t}\right)_s = \sum_q |V|^2 (u_k u_{k'} - v_k v_{k'})^2 \{[f'(1 - f)(N + 1) - f(1 - f')N] \delta(\varepsilon' - \varepsilon - \hbar\omega) + [f'(1 - f)N - f(1 - f')(N + 1)] \delta(\varepsilon - \varepsilon' - \hbar\omega)\} + |V|^2 (u_k v_{k'} + u_{k'} v_k)^2 [N(1 - f)(1 - f') - (N + 1) f f'] \delta(\varepsilon' + \varepsilon - \hbar\omega); \qquad (3.1)$$

where N is the number of phonons of frequency ω.

When the sound field is turned on, the electron is in a lattice with a somewhat modified constant, so that its momentum becomes dependent on the deformation tensor.[12] Therefore

$$\left(\frac{\partial f}{\partial t}\right)_s = \frac{\partial f}{\partial \varepsilon}\frac{\xi}{\varepsilon}\varepsilon_{ik}(\mathbf{k})\,u_{ik}$$

(u_{ik} — deformation tensor; $\epsilon_{ik}(\mathbf{k})$ — tensor dependent on the direction of k). We assume that $l \gg \lambda$ (l — mean free path, λ — wavelength of sound). We can then neglect the electric deformation fields.[13]

We seek the distribution function, as usual, in the form $f = f_0 + g(\epsilon, \Omega)$, where $f_0 = (e^{\epsilon/T} + 1)^{-1}$ and $g(\epsilon, \Omega)$ depends on the energy of the electron excitation and on the angles that determine its direction of motion. We expand $g(\epsilon, \Omega)$ in Legendre polynomials and confine ourselves as usual to the first term of the expansion, i.e., we assume $g(\epsilon, \Omega) = g(\epsilon)p_x/p$ (p — electron momentum, p_x — its projection on the direction of the sound), $g(\epsilon)$ being sought in the form

$$g(\varepsilon) = \frac{\partial f_0}{\partial \varepsilon}\varphi(\varepsilon)\frac{\xi}{|\xi|}.$$

Then (3.1) is rewritten as

$$f_0^2 e^{\varepsilon/T}\frac{|\xi|}{\varepsilon}\varepsilon_{ik}u_{ik} = \int N_0|V|^2\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)$$
$$\times\left[\varphi(\varepsilon')\frac{p'_x}{p'} - \varphi(\varepsilon)\frac{p_x}{p}\right]f_0 f'_0 e^{\varepsilon'/T}\delta(\varepsilon'-\varepsilon-\hbar\omega)\,d\mathbf{q}$$
$$+\int N_0|V|^2\left(1+\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\left[\varphi(\varepsilon')\frac{p'_x}{p'}\right.$$
$$\left.-\varphi(\varepsilon)\frac{p_x}{p}\right]f_0 f'_0 e^{\varepsilon/T}\delta(\varepsilon-\varepsilon'-\hbar\omega)\,d\mathbf{q}$$
$$+\int N_0|V|^2\left(1-\frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\left[\varphi(\varepsilon')\frac{p'_x}{p'}\right.$$
$$\left.-\varphi(\varepsilon)\frac{p_x}{p}\right]f_0 f'_0 e^{\hbar\omega/T}\delta(\varepsilon+\varepsilon'-\hbar\omega)\,d\mathbf{q}. \quad (3.2)$$

For the momentum projection averaged over the azimuth φ we have $q_x = 2\pi(p_x/p)q\cos\vartheta$ (ϑ — angle between **p** and **q**; the polar axis is chosen in the direction of the vector p), with $\cos\vartheta \approx -q/2p$. We integrate further over the angle ϑ and put approximately

$$\left(1 \pm \frac{\xi\xi'-\Delta^2}{\varepsilon\varepsilon'}\right)\frac{\varepsilon\varepsilon'}{|\xi||\xi'|} \approx 2.$$

We seek $\varphi(\epsilon)$ in the form $\varphi(\epsilon) = \varphi_0 + \varphi_1(\epsilon)$, with $\varphi_0 \gg \varphi_1$ (φ_0 = const). Introducing further the variables $z = \epsilon/T$, $b = \Delta/T$, $x = \hbar\omega/T$ and integrating with respect to z and with respect to the angles in the momentum space of the electrons, we obtain

$$\int_b^\infty \frac{e^z}{(e^z+1)^2}\varepsilon_{ik}\dot{u}_{ik}\,dz\,d\Omega = |V'^2|\,T^5\int_0^\infty\frac{2x^4\,dx}{e^x-1}\int_b^\infty\frac{dz\,d\Omega\varphi_0}{(e^z+1)(e^{-z-x}+1)}$$
$$+|V'|^2\frac{T^5}{2}\int_{2b}^\infty\frac{x^4\,dx}{e^x-1}\int_b^\infty\frac{e^z\,dz\,d\Omega}{(e^z+1)(e^{x-z}+1)}\varphi_0,$$

$$|V'|^2 = |V|^2/q.$$

We see therefore that the sought function φ has the form

$$\varphi = \frac{\text{const}}{T^5}\frac{1}{(e^b+1)\Phi(T)}. \quad (3.3)$$

For $\Phi(T)$ see (1.6).

The impurities do not play any role in this process. They cause elastic scattering of the electrons, and their effect can be accounted for by adding a term $(f-f_0)/\tau$ to (3.1). This term, however, drops out in the integration over the angles in the momentum space of the electrons, a natural fact, for in this case elastic scattering cannot lead to establishment of equilibrium.

We now find the dissipation function, which determines the absorption of the sound energy. The entropy of a gas of electron excitations is

$$S = \sum_{\mathbf{k}}\{(f_k-1)\ln(1-f_k) - f_k\ln f_k\},$$

which leads to the following expression for the dissipation function:

$$W = T\dot{S} = -T\sum_k \dot{f}_k\frac{g}{(1-f_0)f_0}. \quad (3.4)$$

Substituting f_k from (3.1) and then integrating with respect to ϵ and the angles in the phonon momentum space, we obtain, with account of approximation (2.3),

$$W = \int_0^\infty x^2\,dx\int_b^\infty dz\varphi(z)\,[\varphi(z+x,\Omega') - \varphi(z,\Omega)]$$
$$\times\frac{e^{z+x}}{(e^x-1)(e^z+1)(e^{z+x}+1)} + \int_0^\infty x^2\,dx\int_{b+x}^\infty dz\,\varphi(z)$$
$$\times[\varphi(z-x,\Omega') - \varphi(z,\Omega)]\frac{e^z}{(e^x-1)(e^z+1)(e^{z-x}+1)}$$
$$-\frac{1}{2}\int_{2b}^\infty x^2\,dx\int_b^{x-b}\varphi(z)\,[\varphi(x-z,\Omega')$$
$$-\varphi(z,\Omega)]\frac{e^x\,dz}{(e^z+1)(e^{x-z}+1)(e^x-1)}.$$

We therefore find

$$W = \frac{\text{const}}{T^5}\frac{1}{(e^b+1)^2\Phi(T)}. \quad (3.5)$$

This formula was first derived in [2]; it is seen from (3.5) that W is a universal temperature function. Consequently, the coefficient of absorption of long-wave sound by electron excitations, which

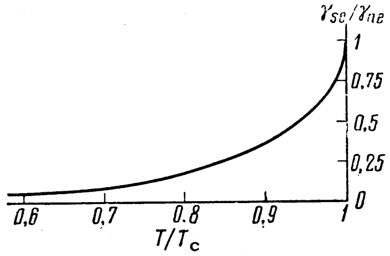

FIG. 2

is proportional to W, is given by

$$\gamma_{se} = \gamma_{ne}\frac{4\Phi(T_c)}{(e^b+1)^2\Phi(T)}, \qquad (3.6)$$

where $\gamma_{ne} = \mathrm{const}/T^5$ is the coefficient of absorption of sound in the normal metal[13]; $\Phi(T)$ is given by (1.6) (see Fig. 2).

We determine similarly the absorption of sound energy by phonons. The sound field changes the phonon frequency to $\omega' = \omega(1 + \alpha_{i\nu}u_{i\nu})$; here $\alpha_{i\nu}$ is a tensor dependent on the direction of the phonon wave vector and $u_{i\nu}$ the deformation tensor. The kinetic equation for the distribution function of phonons interacting with electrons in a superconductor in a sound field has the form

$$-N_0^2 e^{\hbar\omega/T}\frac{\hbar\omega}{T}\alpha_{i\nu}\dot{u}_{i\nu} = \int |V|^2\{2(u_k u_{k'} - v_k v_{k'})^2$$
$$\times [f'(1-f)(N+1) - Nf(1-f')]\,\delta(\varepsilon'-\varepsilon-\hbar\omega)$$
$$+ (u_k v_{k'} + u_{k'}v_k)^2[(N+1)ff'$$
$$- N(1-f)(1-f')]\,\delta(\varepsilon'+\varepsilon-\hbar\omega)\}\, p^2 dp\, d\cos\vartheta\, d\varphi. \qquad (3.7)$$

The perturbation of the phonon distribution function $R = N - N_0$ can be determined from this equation by a method similar to that used in [6], and has the form

$$R = -N_0^2 e^x r(x)/T;$$

$$\frac{1}{r} = \frac{\mathrm{const}}{x}\left\{\int_b^\infty \frac{e^z\,dz}{(e^z+1)(e^{z+x}+1)} + D(x)\int_b^{x-b}\frac{dz}{(e^z+1)(e^{x-z}+1)}\right\}$$

$$\left(D(x) = \begin{cases}0, & x < 2b\\ 1, & x \geqslant 2b\end{cases}\right).$$

For the dissipation function of the phonon gas we have

$$W_{\mathrm{ph}} = -T\sum_q R\dot{N}/N_0(N_0+1).$$

Evaluation of this function yields (see Fig. 3)

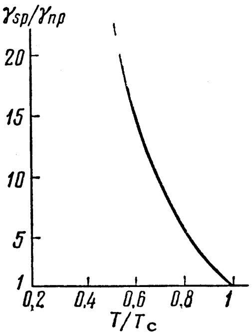

FIG. 3

$$\gamma_{sp}/\gamma_{np} = F(T)/F(T_c);$$

$$F(T) = -8(b^4 + b^3)(e^b-1)^{-1} - 6\zeta(3)(e^b+1)$$
$$-3(e^b+1)\sum_{s=1}^\infty s^{-3}e^{-2bs}(4b^2s^2 + 4bs + 2)$$
$$+6\zeta(4)(e^b-1) - (e^b-1)\sum_{s=1}^\infty s^{-4}e^{-2bs}(8b^3s^3 + 12b^2s^2$$
$$+ 12bs + 6) + 32b^3(e^{2b}-1)^{-1}$$
$$- a^4\sum_{s=1}^\infty\{se^{-2bs}\,\mathrm{Ei}[-s(2b-a)]\} + 6\sum_{s=1}^\infty s^{-3}e^{-2bs};$$

$$a \approx 2b - 0.16;$$

$$\zeta(s) = \sum_{n=1}^\infty n^{-s};\quad \zeta(3) = 1.202;\quad \zeta(4) = 1.082. \qquad (3.8)$$

Here γ_{sp} and γ_{np} are the coefficients of absorption of sound by phonons in the superconducting and normal states. γ_{sp} is also a universal function of the temperature. The total absorption coefficient is $\gamma_s = \gamma_{se} + \gamma_{sp}$.

The absorption of sound by phonons increases with decreasing temperature because the phonon mean free path is increased by the deduction in the number of electron excitations. This absorption mechanism is significant at temperatures not too close to T_c.

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ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2 f^2}{4\pi^3}\omega^2 (\ln \frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2 f^2}{9\pi^3}\omega^2 \left(\ln \frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2 f^2/4\pi^3)\,\omega^2 \geqslant \ldots$	$(e^2 f^2/9\pi^3)\,\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-\left[\frac{1}{4}(3\cos^2\theta - 1)\ldots\right.$
14	3	Charakhchyan et al.	533	Table II, col. 6 line 1		1.9	0.9
14	3	Malakhov	550	The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.			
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2 \sum_{s=1}^{\infty} K_2(bs)\right]^2$	$\ldots\left[b^2 \sum_{s=1}^{\infty} (-1)^{s+1} K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	l	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	1 < d/2 < 2	1 < d/r < 2
14	5	Shapiro	1154	l	Table	2306	23.6