THEORY OF PLASMA DIFFUSION IN A MAGNETIC FIELD

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Submitted to JETP editor April 1, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1151-1167 (October, 1961)

The transverse diffusion coefficient for particles in a fully ionized plasma in a magnetic field is calculated by a graphical method. The Born parameter $e^2/\hbar v_T$ is assumed small. The magnetic field is considered to be strong, the electron Larmor frequency being considerably larger than the collision frequency. Account is taken of the role of electrons and ions in screening the electron-ion interaction, of the deformation of the interacting-particle Debye clouds, and of the inelastic nature of electron-ion scattering. The numerical factor in the Coulomb logarithm is estimated in the case when the electron Larmor radius is much greater than the Debye radius. Corrections to the diffusion coefficient due to the effect of the magnetic field on electron-ion collisions are estimated. The case when the Debye radius is smaller than the Larmor radius of the electron but larger than that of the ion and when the magnetic field exerts a significant influence on the collision act is also considered.

1. A characteristic feature of the kinetic properties of a system of particles interacting in accordance with Coulomb's law is the divergence of the Coulomb scattering cross section at small momentum transfers. In a neutral system, the interaction between charged particles, which leads to screening, cuts off the interaction potential at distances on the order of the Debye radius $1/\kappa = (4\pi ne^2/kT)^{-1/2}$ (n — plasma electron concentration, e — electron charge, T — temperature). The cross section for the scattering on a screened potential is always finite, and the principal role is played by collisions with small momentum transfers, on the order of $\hbar\kappa$. Starting from these considerations, Landau[1] derived a kinetic equation for a system of charged particles.

These qualitative considerations, however, still leave a few points not completely clear. Thus, the Debye clouds of the particles colliding in the plasma can become noticeably deformed, and it would be interesting to ascertain the influence of this effect on the kinetic phenomena. Further, the screening of the interaction in an electron-ion plasma should be not only by the electrons but also by the ions, and it would be necessary to ascertain the extent of the ion contribution. Finally, we wish to know whether low inelasticity of the collisions between the electrons and ions of the plasma is essential; such inelasticity would cause, for example, the interaction between the electrons and ions to proceed via exchange of collective excitations of the plasmon type.

These questions are answered to some extent by Konstantinov and Perel',[2] whose proposed diagram technique[3] yielded a kinetic equation for the plasma electrons in a weak electric field. The Born parameter $e^2/\hbar v_T$ was assumed small ($v_T = \sqrt{2kT/m}$, where m is the electron mass).

In the present paper we investigate, also in the Born approximation, the transverse diffusion of particles of a completely ionized plasma in a magnetic field H. We consider the case $\Omega\tau \gg 1$ ($\Omega = eH/mc$ is the Larmor frequency and τ is the electron relaxation time). In this case both the screening by the ions (see [2]) and the deformation of their Debye clouds proved to be significant. However, exchange processes between the electron and the ion via plasmon-like oscillations, although theoretically possible, always make a negligibly small contribution.

When $\kappa R \gg 1$ ($R = v_T/\Omega$), the coefficient of transverse diffusion is proportional to $\ln (q_T/\kappa)$,[4,5] where $q_T = mv_T/\hbar$. An account of the foregoing factors enables us to find the coefficient under the sign of this logarithm. The corrections for the diffusion due to the effect of the magnetic field on the electric scattering are small when $\kappa R \gg 1$. We also investigated the case $\kappa R \ll 1$, when the effect of the magnetic field on the electron scattering cannot be neglected, and the solution cannot be obtained by iterating the equation of [2], the left half of which contains the Lorentz force. In this case the Coulomb logarithm is a function of H.

2. Let the magnetic field **H** be directed along the z axis and let the electron and ion concentration gradients and the electric field **E** be directed along the x axis. Then we have for the electron and ion current densities (the ions are assumed to be singly charged) in the linear approximation ($E = -\nabla\varphi$):

$$i_x^e = -D_{xx}^{ee}\nabla_x n_e - D_{xx}^{ei}\nabla_x n_i + \sigma_{xx}^e \nabla_x \varphi/e, \quad (1)$$

$$i_x^i = -D_{xx}^{ie}\nabla_x n_e - D_{xx}^{ii}\nabla_x n_i - \sigma_{xx}^i \nabla_x \varphi/e. \quad (2)$$

The first term in (1) is the diffusion electron current, while the second is the electron current proportional to the gradient of the ion concentration, brought about by the action of the non-equilibrium ion distribution on the equilibrium electron distribution, i.e., the "dragging" of the electrons by the ions. The expression for σ_{xx}^e has the following form [2,6]

$$\sigma_{xx}^e = \sigma_{xx}^{ee} - \sigma_{xx}^{ei}, \quad (3)$$

$$\sigma_{xx}^{ee} = \frac{e^2\beta}{V_0}\,\mathrm{Re}\int_{-\infty}^{0} e^{st}\,dt \sum_{\lambda\lambda'}\sum_{\mu\mu'}\Big\langle e^{-\beta\mathcal{H}_0'} T_C \exp\Big[\int U(z)\,\frac{dz}{i\hbar}\Big]$$
$$\times (a_\mu^+ a_{\mu'})_0 (a_\lambda^+ a_{\lambda'})_t \Big\rangle v_{\lambda\lambda'}^x v_{\mu\mu'}^x; \quad (4)$$

$$\sigma_{xx}^{ei} = \frac{e^2\beta}{V_0}\,\mathrm{Re}\int_{-\infty}^{0} e^{st}\,dt \sum_{\lambda\lambda'}\sum_{\mu\mu'}\Big\langle e^{-\beta\mathcal{H}_0'} T_C \exp\Big[\int U(z)\,\frac{dz}{i\hbar}\Big]$$
$$\times (a_\mu^+ a_{\mu'})_0 (A_\lambda^+ A_{\lambda'})_t \rangle V_{\lambda\lambda'}^x v_{\mu\mu'}^x. \quad (5)$$

Here $\beta = 1/kT$, V_0 is the normalization volume, s is the adiabatic parameter ($s > 0$, $s \to 0$),

$$\langle \ldots \rangle = \mathrm{Sp}\,(\ldots)/\,\mathrm{Sp}\exp(-\beta\mathcal{H}_0'),$$
$$\mathcal{H}' = \mathcal{H}_0 - \zeta_e N_e - \zeta_i N_i + U = \mathcal{H}_0' + U, \quad (6)$$

$\mathcal{H}_0$ is the Hamiltonian of the free electrons and ions, N_e (N_i) is the operator of the number of electrons (ions), ζ_e (ζ_i) is the chemical potential of the electrons (ions), and T_C is the symbol of ordering along the contour C, shown in the figures as a dash-dot line. Furthermore,

$$U(z) = \exp(\mathcal{H}_0 z/i\hbar)\, U \exp(-\mathcal{H}_0 z/i\hbar);$$

$U = U_{ee} + U_{ei} + U_{ii}$ is the interaction operator, with

$$U_{ee} = \frac{1}{2}\sum_{\gamma\gamma'}\sum_{\delta\delta'}\sum_{\mathbf{q}} u_{\mathbf{q}} \langle\gamma|e^{i\mathbf{qr}}|\gamma'\rangle\langle\delta|e^{-i\mathbf{qr}}|\delta'\rangle a_\gamma^+ a_{\gamma'} a_\delta^+ a_{\delta'}, \quad (7)$$

$$U_{ei} = -\sum_{\gamma\gamma'}\sum_{\delta\delta'}\sum_{\mathbf{q}} u_{\mathbf{q}} \langle\gamma|e^{i\mathbf{qr}}|\gamma'\rangle\langle\delta|e^{-i\mathbf{qr}}|\delta'\rangle a_\gamma^+ a_{\gamma'} A_\delta^+ A_{\delta'}, \quad (8)$$

$$U_{ii} = \frac{1}{2}\sum_{\gamma\gamma'}\sum_{\delta\delta'}\sum_{\mathbf{q}} u_{\mathbf{q}} \langle\gamma|e^{i\mathbf{qr}}|\gamma'\rangle\langle\delta|e^{-i\mathbf{qr}}|\delta'\rangle A_\gamma^+ A_{\gamma} A_\delta^+ A_{\delta'}. \quad (9)$$

Here a_γ^+ and a_γ (A_γ^+ and A_γ) are the operators of creation and annihilation of the electron (ion) in the state γ; when the vector-potential has a gauge $\mathbf{A} = (0, Hx, 0)$ this state is determined by the quantum numbers n, p_z (momentum projection on the z axis), and X (coordinate of the center of the Landau oscillator); ϵ_γ (E_γ) is the energy of the electron (ion) in this state

$$u_{\mathbf{q}} = \int d^3r\, u(\mathbf{r})\, e^{i\mathbf{qr}}/V_0 = 4\pi e^2/V_0 q^2;$$

$v_{\lambda\lambda'}^x$ ($V_{\lambda\lambda'}^x$) are the matrix elements of the operator of the x component of the electron (ion) velocity.

From the meaning of the formula (3) [2] it is clear that σ_{xx}^{ee} determines the electric current due to the action of the external electric field on the electrons only, while σ_{xx}^{ei} is the electron current due to action of the electric field on the ions only, which then drag the "equilibrium" electrons. The possibility of representing σ_{xx}^e in the form of a difference of these two quantities stems from the fact that the current is calculated in an approximation which is linear in the field. But such a representation enables us to relate σ and D with the aid of the Einstein equations

$$D_{xx}^{ee} = \sigma_{xx}^{ee}/ne^2\beta, \qquad D_{xx}^{ei} = \sigma_{xx}^{ei}/ne^2\beta \quad (10)$$

(analogously for D_{xx}^{ie} and D_{xx}^{ii}).

On the other hand, it is clear from general consideration that $\sigma_{xx}^e = \sigma_{xx}^i = 0$. This can be verified by writing out the equations of motion for the system of interacting electrons and ions and going over to a new system of coordinates with the aid of the transformation $\mathbf{r} = \mathbf{r}' + ct[\mathbf{E}\times\mathbf{H}]/H^2$ (see [4,5]). But then

$$\sigma_{xx}^{ee} = \sigma_{xx}^{ei}, \qquad \sigma_{xx}^{ii} = \sigma_{xx}^{ie}. \quad (11)$$

Finally, it is easy to show that $\sigma_{xx}^{ei} = \sigma_{xx}^{ie}$. Of the equal quantities given in (11), we deemed it most expedient to calculate σ_{xx}^{ee}.

The general formula (4) can be expanded in powers of the small parameter $1/\Omega\tau$. We note that the first term to be calculated in this expansion contains the electron-electron interaction only to the extent that it determines the screening of the electron-ion potential, while the direct electron-electron scattering produces a zero contribution.

3. To calculate σ_{xx}^{ee} we apply to Eq. (4) the Konstantinov and Perel' diagram technique. [3,2] We calculate the term of lowest (namely second) order in the particle concentration n in σ_{xx}^{ee}. For this purpose it is necessary to take into account graphs with not more than two irregular lines, and the factor $1 - n_\gamma$ corresponding to the regular lines must be replaced by unity.

If the electron-ion potential were to fall off

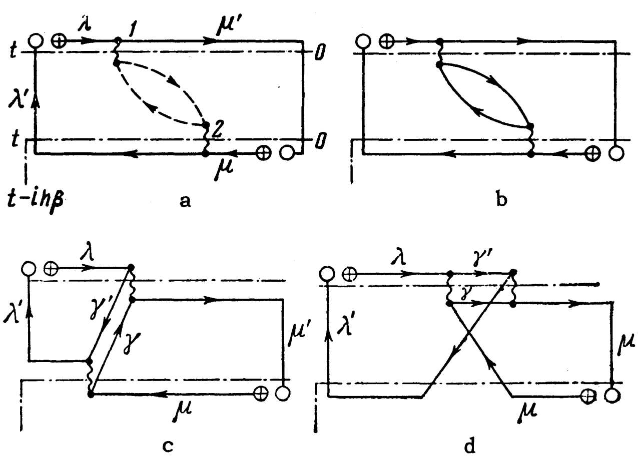

FIG. 1

with distance more rapidly than the Coulomb potential, then the conductivity would be determined in the Born approximation by the sum of the graph 1a and the other graphs of the like type with vertices 1 and 2 occupying all possible positions on the contour C. In order to eliminate the Coulomb divergence at small q, it is necessary to add here the sum of other graphs of higher order in the concentration, but containing on the other hand higher powers of 1/q. These renormalizing graphs are chains which can end only in ion loops and have in the middle either ion or electron loops, since both particles participate in the screening of the electron-ion interaction. It can be shown that if we sum over all the electron (or ion) indices on which such a loop depends, then the incoming and outgoing momenta are equal, and therefore such chains actually produce an accumulation of divergences in 1/q.

As noted earlier, direct electron-electron interaction does not contribute to σ_{xx}^{ee}. But we cannot draw such a conclusion beforehand when this interaction is renormalized via the ions. Consequently we must consider along with the chain-like graphs of type 1a also graphs of types 1b, c, and d.

To calculate the sums of these graphs we employ the procedure proposed in [2]. We consider the expression

$$l_{\mathbf{q}}(z_1, z_2) = \langle e^{-\beta\mathscr{H}'} T_C \, [(B_{\mathbf{q}})_{z_1} (B_{-\mathbf{q}})_{z_2}] \rangle, \tag{12}$$

which we call the plasmon line. Here

$$B_{\mathbf{q}} = \sum_{\gamma\gamma'} \langle \gamma | e^{i\mathbf{qr}} | \gamma' \rangle (a_\gamma^+ a_{\gamma'} - A_\gamma^+ A_{\gamma'}). \tag{13}$$

The plasmon line is the sum of all the graphs attached by means of the two interaction lines to the point z_1 and z_2, and consequently is the renormalized interaction line.

If the plasmon line is regular (z_2 ahead of z_1 on the contour), then

$$l_{\mathbf{q}}(z_1, z_2) = l_{\mathbf{q}}(z_1 - z_2) = \int_{-\infty}^{\infty} d\omega \Phi_{\mathbf{q}}(\omega) e^{i\omega(z_1 - z_2)}. \tag{14a}$$

On the other hand if the plasmon line is irregular, then

$$l_{\mathbf{q}}(z_1, z_2) = \bar{l}_{\mathbf{q}}(z_1 - z_2) = \int_{-\infty}^{\infty} d\omega \Phi_{\mathbf{q}}(\omega) e^{\hbar\omega\beta} e^{i\omega(z_1 - z_2)}. \tag{14b}$$

Here

$$\Phi_{\mathbf{q}}(\omega) = Z^{-1} \sum_{kk'} e^{-\beta\mathscr{H}'_k} |\langle k | B_{\mathbf{q}} | k' \rangle|^2 \delta(\omega_{kk'} - \omega)$$
$$= e^{-\hbar\omega\beta} \Phi_{\mathbf{q}}(-\omega), \tag{15}$$

k and k′ label the states of the entire interacting electron-ion system; $Z = \mathrm{Sp}\, e^{-\beta\mathscr{H}'}$; $\mathscr{H}'_k = \langle k | \mathscr{H}' | k \rangle$.

We put furthermore

$$l_{\mathbf{q}}(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} L_{\mathbf{q}}(\eta) e^{\eta t} d\eta, \quad \bar{l}_{\mathbf{q}}(x) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \bar{L}_{\mathbf{q}}(\eta) e^{\eta t} d\eta, \tag{16}$$

$$L_{\mathbf{q}}(\eta) = \int_0^\infty e^{-\eta t} l_{\mathbf{q}}(t)\, dt = \int_{-\infty}^{\infty} d\omega \Phi_{\mathbf{q}}(\omega) \frac{1}{\eta - i\omega}, \tag{17a}$$

$$\bar{L}_{\mathbf{q}}(\eta) = \int_0^\infty e^{-\eta t} \bar{l}_{\mathbf{q}}(t)\, dt = \int_{-\infty}^{\infty} d\omega \Phi_{\mathbf{q}}(\omega) e^{\hbar\omega\beta} \frac{1}{\eta - i\omega}; \tag{17b}$$

$L_{\mathbf{q}}(\eta)$ and $\bar{L}_{\mathbf{q}}(\eta)$ are functions analytic in the right half of the complex η plane.

In the calculations it was found convenient to leave the integrals with respect to λ, but carry out the integration with respect to t. If we consider each graph as a result of such an integration, then the rules for setting up the analytic expressions in correspondence with the diagrams are as follows: 1) to each regular fermion line corresponds a factor $1 - n_\gamma$, and to each irregular line—a factor n_γ; 2) to each vertex on the upper (lower) time axis corresponds a factor $u_{\mathbf{q}}/i\hbar$ $(-u_{\mathbf{q}}/i\hbar)$; 3) to each point on the "temperature" axis corresponds a factor $(-u_{\mathbf{q}})$ and one integration with respect to dλ; 4) to each regular plasmon line corresponds a factor $L_{\mathbf{q}}(\eta)$; 5) to each interval between points on the horizontal portion of the contour corresponds a factor $[s + i(\omega_i - \omega_e)]^{-1}$ where $\hbar\omega_i$ is the total energy of the lines entering from the left into this interval and $\hbar\omega_e$ the total energy of the lines leaving to the right; the plasmon line is assigned a direction from left to right and an "energy" $i\hbar\eta$; 6) the graph is multiplied by $(-1)^{P_i + P_e}$, where $P_{i,e}$ is the number of intersections of the fermion lines of one type with each other; 7) in the final expression the summation is over all the electrons and ion indices and over all **q**, and the integration is over λ and η. The integration over η can be carried out by closing the contour in the right half-plane.

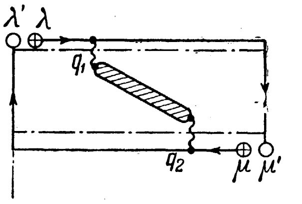

FIG. 2

4. Our problem is to express the sought sum of graphs in terms of known functions $\Phi_{\mathbf{q}}(\omega)$ and $K_{\mathbf{q}}(\omega)$ (see below).

The sum of the graphs 1a and b assumes after renormalization the form of a graph with a single plasmon line (Fig. 2). There are six graphs of this type with different placements of the vertices on the contour C (see Fig. 3). Graphs with points on the "temperature" axis only are pure imaginary and therefore not considered here. For the case when one end of the plasmon line is on the "temperature" axis, the integral with respect to λ is calculated in Appendix 1.

The sum of graphs of the type of Fig. 2, with the points placed as in Figs. 3a, b, and c, is

$$-\frac{2}{\hbar^2}\sum_{\mathbf{q}} u_{\mathbf{q}}^2 \operatorname{Re}\sum_{\lambda\mu} n_\lambda L_{\mathbf{q}}(s+i\omega_{\mu\lambda})\langle\lambda\,|\,(x-X_\lambda)\,e^{i\mathbf{qr}}\,|\,\mu\rangle \times \langle\mu\,|\,(x-X_\mu)\,e^{-i\mathbf{qr}}\,|\,\lambda\rangle.$$

We made use here of the identity

$$v_{\lambda\lambda'}/(s+i\omega_{\lambda\lambda'}) = \langle\lambda\,|\,x-X_\lambda\,|\,\lambda'\rangle \qquad (s\to 0). \tag{18}$$

The sum of the graphs of the type of Fig. 2, with points situated as in Figs. 3d, e, and f, is

$$\frac{2}{\hbar^2}\sum_{\mathbf{q}} u_{\mathbf{q}}^2 \operatorname{Re}\sum_{\lambda\mu} n_\lambda L_{\mathbf{q}}(s+i\omega_{\mu\lambda})\langle\lambda\,|\,(x-X_\mu)\,e^{i\mathbf{qr}}\,|\,\mu\rangle \times \langle\mu\,|\,(x-X_\mu)\,e^{-i\mathbf{qr}}\,|\,\lambda\rangle.$$

Consequently the sum of all the graphs of the type shown in Fig. 2 is

$$\frac{2}{\hbar^2}\sum_{\mathbf{q}} u_{\mathbf{q}}^2 \operatorname{Re}\sum_{\lambda\mu}(X_\lambda - X_\mu)\,n_\lambda L_{\mathbf{q}}(s+i\omega_{\mu\lambda})\langle\lambda\,|\,e^{i\mathbf{qr}}\,|\,\mu\rangle \times \langle\mu\,|\,(x-X_\mu)\,e^{-i\mathbf{qr}}\,|\,\lambda\rangle. \tag{19}$$

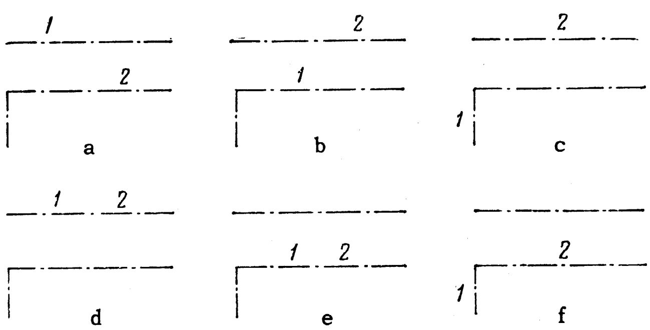

FIG. 3

We interchange the dummy indices λ and μ in (19) and take half the sum of the resultant expressions. We also take account of the fact that

$$(X_\lambda - X_\mu)^2 = (c/eH)^2\,(p_{y\lambda}-p_{y\mu})^2 = (c\hbar/eH)^2\,q_y^2, \tag{20}$$

and that the difference

$$L_{\mathbf{q}}(s-i\omega) - e^{\hbar\omega\beta}L_{\mathbf{q}}(s+i\omega)$$

is, on the basis of (17) pure imaginary when $s \to 0$. Then we obtain ultimately

$$\frac{\pi}{\hbar^2}\left(\frac{c\hbar}{eH}\right)^2 \sum_{\mathbf{q}} q_y^2 u_{\mathbf{q}}^2 \sum_{\lambda\mu} n_\lambda\,|\langle\lambda\,|\,e^{i\mathbf{qr}}\,|\,\mu\rangle|^2\,\Phi_{\mathbf{q}}(\omega_{\mu\lambda}). \tag{21}$$

We consider now non-renormalized electron-electron interaction graphs such as Figs. 1c and d. Their sum is

$$-\pi\left(\frac{c}{eH}\right)^2 \sum_{\mathbf{q}} q_y^2 u_{\mathbf{q}}^2 \sum_{\lambda\mu}\sum_{\gamma\gamma'} n_\lambda n_\gamma \delta(\omega_{\mu\lambda} + \omega_{\gamma'\gamma})\,|\langle\lambda\,|\,e^{i\mathbf{qr}}\,|\,\gamma'\rangle|^2\,|\langle\mu\,|\,e^{i\mathbf{qr}}\,|\,\gamma\rangle|^2. \tag{22}$$

There can be four types of electron-electron scattering graphs with a single plasmon line, each type including 14 graphs. Figure 4 shows the principal graphs of the first type. The remainder are obtained by transferring the non-renormalized vertex from the upper segment of the time axis to the lower one (or vice versa). Figure 5 shows the characteristic graphs of the three remaining types. For the cases when both ends of the plasmon line (or one end and one simple vertex) are on the temperature axis, the integrals with respect to λ are calculated by a method similar to that developed in Appendix 1.

The sum of graphs with one plasmon line is

$$\left(\frac{2\pi}{\hbar}\right)\left(\frac{c}{eH}\right)^2 \sum_{\mathbf{q}} q_y^2 u_{\mathbf{q}}^3 \sum_{\lambda\mu}\sum_{\gamma\gamma'} |\langle\gamma\,|\,e^{i\mathbf{qr}}\,|\,\lambda\rangle|^2 \times |\langle\gamma'\,|\,e^{i\mathbf{qr}}\,|\,\mu\rangle|^2\,n_\lambda n_{\gamma'}\delta(\omega_{\lambda\mu}+\omega_{\gamma'\gamma})\operatorname{Re}K_{\mathbf{q}}(\omega_{\gamma'\beta}-is). \tag{23}$$

We have introduced here the notation

$$\overline{L}_{\mathbf{q}}(s+i\omega) - L_{\mathbf{q}}(s+i\omega) = iK_{\mathbf{q}}(\omega - is), \tag{24}$$

the meaning of which will become clear below (see Sec. 5).

There are four types of graphs with two plasmon lines (see Fig. 6). Each corresponds to 30 graphs. Their sum is

$$-\frac{\pi}{\hbar^2}\left(\frac{c}{eH}\right)^2 \sum_{\mathbf{q}} q_y^2 u_{\mathbf{q}}^4 \sum_{\lambda\mu}\sum_{\gamma\gamma'} |\langle\gamma\,|\,e^{i\mathbf{qr}}\,|\,\lambda\rangle|^2\,|\langle\gamma'\,|\,e^{i\mathbf{qr}}\,|\,\mu\rangle|^2 \times n_\lambda n_{\gamma'}\delta(\omega_{\mu\lambda}+\omega_{\gamma\gamma'})\,|K_{\mathbf{q}}(\omega_{\gamma'\mu}-is)|^2. \tag{25}$$

The total contribution to the conductivity from the renormalized electron-electron interaction is the sum of the expressions (22), (23), and (25):

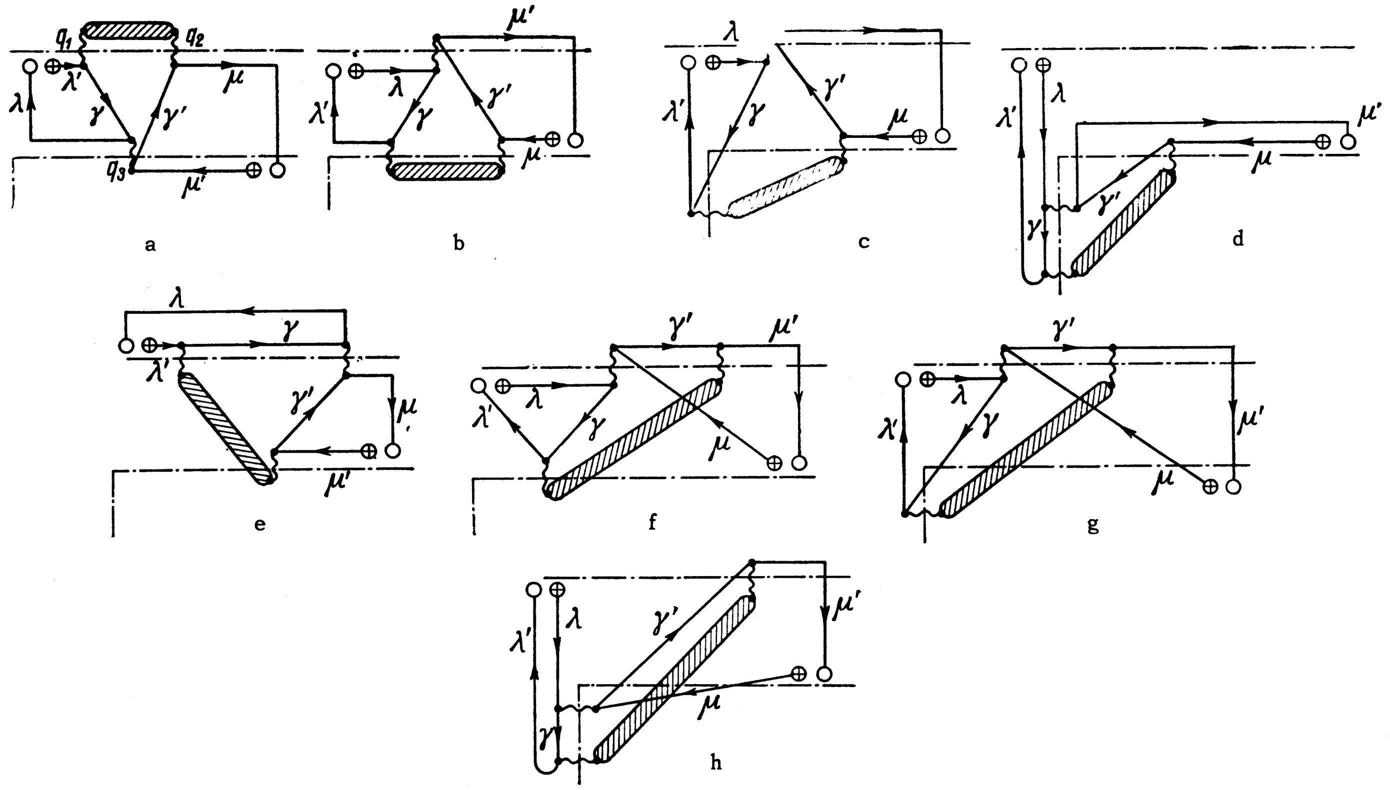

FIG. 4

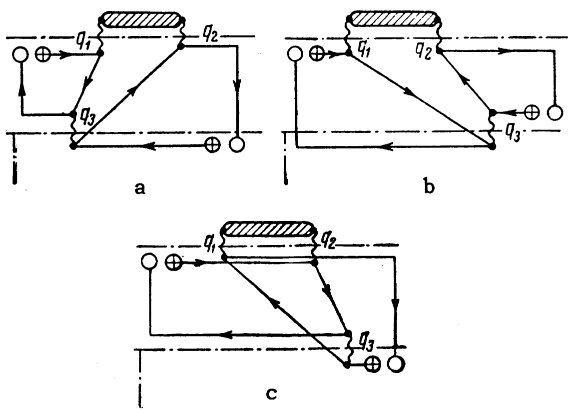

FIG. 5

$$-\pi\left(\frac{c}{eH}\right)^2 \sum_{\mathbf{q}} q_y^2 u_{\mathbf{q}}^2 \sum_{\lambda\mu} \sum_{\gamma\gamma'} n_{\gamma'} n_\lambda \delta(\omega_{\lambda\mu} + \omega_{\gamma'\gamma})$$
$$\times |\langle\lambda|e^{i\mathbf{q}\mathbf{r}}|\gamma\rangle|^2 |\langle\mu|e^{i\mathbf{q}\mathbf{r}}|\gamma'\rangle|^2 |1 - (u_{\mathbf{q}}/\hbar) K_{\mathbf{q}}(\omega_{\gamma'\mu} - is)|^2. \quad (26)$$

5. Generalizing the methods of [2,7] to the case $H \neq 0$, we obtain for $e^2 n^{1/3}/kT \ll 1$ the following expression

$$K_{\mathbf{q}}(\omega) = \beta\hbar n V_0 [P_{\mathbf{q}}^e(\omega) + P_{\mathbf{q}}^i(\omega)] \{1 + (\varkappa^2/q^2) [P_{\mathbf{q}}^e(\omega) + P_{\mathbf{q}}^i(\omega)]\}^{-1}. \quad (27)$$

The function $P_{\mathbf{q}}^e(\omega)$ has been calculated in [8,9] and is equal to

$$P_{\mathbf{q}}^e(\omega) = \frac{2}{\hbar\beta}\int_0^\infty dt\, e^{-i\omega t} \sin\left(\frac{\hbar q_z^2}{2m} t + \frac{\hbar q_\perp^2}{2m\Omega}\sin\Omega t\right)$$
$$\times \exp\left(-\frac{q_z^2}{2m\beta}t^2 - \frac{\hbar q_\perp^2}{m\Omega}\operatorname{cth}\frac{\hbar\Omega\beta}{2}\sin^2\frac{\Omega t}{2}\right), \quad (28)^*$$

where $q_\perp^2 = q_x^2 + q_y^2$ and $P_{\mathbf{q}}^i(\omega)$ is obtained by replacing the electron mass m in (28) by the ion mass M.

In analogy with [2] we obtain relations (24) and

$$L_{\mathbf{q}}(s + i\omega) + \bar{L}_{\mathbf{q}}(s + i\omega) = 2\pi\Phi_{\mathbf{q}}(\omega)$$
$$= \operatorname{Im} K_{\mathbf{q}}(\omega - is)/(1 - e^{\hbar\omega\beta}). \quad (29)$$

The first of these was already used.

We note that in the absence of interaction

$$\Phi_{\mathbf{q}}^0(\omega) = nV_0 e^{-\hbar\omega\beta/2} [F_{\mathbf{q}}^e(\omega) + F_{\mathbf{q}}^i(\omega)], \quad (30)$$

where

$$F_{\mathbf{q}}^e(\omega) = \hbar\beta \frac{\operatorname{Im} P_{\mathbf{q}}^e(\omega)}{2\pi \operatorname{sh}(\hbar\omega\beta/2)}$$
$$= \frac{1}{nV_0} e^{\hbar\omega\beta/2} \sum_{\lambda\lambda'} |\langle\lambda|e^{i\mathbf{q}\mathbf{r}}|\lambda'\rangle|^2 n_\lambda \delta(\omega - \omega_{\lambda\lambda'}) \quad (31)$$

and similarly for the ion function.

We represent the δ function in (26) in the form

$$\int_{-\infty}^{\infty} d\omega\, \delta(\omega + \omega_{\lambda\gamma})\, \delta(-\omega + \omega_{\gamma'\mu}) \quad (32)$$

and transform this expression using (31). We introduce into (21) an additional integration with re-

*cth = coth.

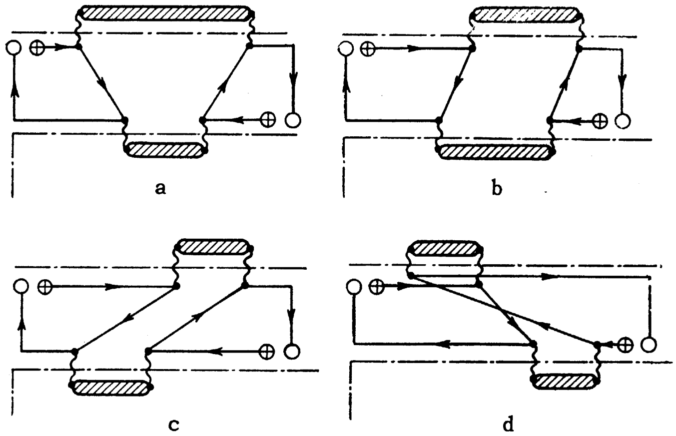

FIG. 6

spect to $d\omega$, with a factor $\delta(\omega - \omega_{\mu\lambda})$, and again use (31). Adding the two resultant expressions we obtain ultimately

$$D^{ee}_{xx} = \frac{n}{8\pi^2}\left(\frac{c}{eH}\right)^2 \int d^3q\, q_y^2 \left(\frac{4\pi e^2}{q^2}\right)^2 \int_{-\infty}^{\infty} d\omega \frac{F^i_{\mathbf{q}}(\omega) F^e_{\mathbf{q}}(\omega)}{A^2(\omega, \mathbf{q}) + B^2(\omega, \mathbf{q})}$$

$$\equiv [ne^4/\pi\sqrt{2}\, m^{3/2}(kT)^{1/2}\,\Omega^2]\,\mathcal{Y}, \qquad (33)$$

$$A(\omega, \mathbf{q}) = 1 + \varkappa^2 q^{-2}\,[\mathrm{Re}\, P^e_{\mathbf{q}}(\omega) + \mathrm{Re}\, P^i_{\mathbf{q}}(\omega)], \qquad (34)$$

$$B(\omega, \mathbf{q}) = \varkappa^2 q^{-2}\,[\mathrm{Im}\, P^e_{\mathbf{q}}(\omega) + \mathrm{Im}\, P^i_{\mathbf{q}}(\omega)]. \qquad (35)$$

The products of the electron functions in the numerator of the sum of (21) and (26) cancel each other, i.e., the electron-electron scattering, even when renormalized with the ions, makes no contribution to σ^{ee}_{xx} or D^{ee}_{xx}.

The final expression is an integral over all the momentum transfers $\hbar\mathbf{q}$ and the energy transfer $\hbar\omega$ due to the product of the electron and ion functions, divided by the renormalizing denominator. If the interaction potential were to decrease with distance more rapidly than the Coulomb potential, simple considerations following Titeica[10] would yield for D^{ee}_{xx} the formula (33), but without the renormalizing denominator. In the case of Coulomb interaction, this denominator takes into account both the screening of the electron-ion potential and non-static effects connected with the inelasticity of the electron-ion collisions. In particular, the quantity A can vanish for certain connections between ω and $\mathbf{q}$. If B is also small, then this connection between ω and $\mathbf{q}$ yields the dispersion of the collective excitations in the plasma. An integrand with a sharp maximum in (33) corresponds in this case to a process in which the electron and ion exchange one of these excitations with each other.

Let us make one fundamental remark. The functions $F^e_{\mathbf{q}}(\omega)$ and $F^i_{\mathbf{q}}(\omega)$ tend to infinity as

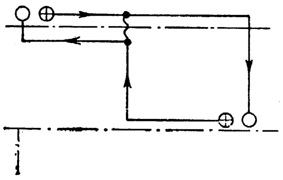

FIG. 7

$q_z \to 0$ and $\omega \to 0$. By virtue of this fact, Eq. (33) diverges logarithmically at small q_z when integrated with respect to ω. This divergence is connected with the specific properties of scattering in a magnetic field, discussed, for example, by Adams and Holstein[11] and by Skobov.[12] It is eliminated in fact by taking some cutoff mechanism into account. This gives rise in the expression for D^{ee}_{xx} to a term containing $\ln q_c R$, where q_c is the characteristic cutoff momentum (which can be estimated[11,12]). In many interesting cases, however, the coefficient of this logarithm is so small that the entire term can be discarded compared with the others. Then the final expression does not contain q_c at all. This is precisely the case, for example, when $\kappa R \gg (m/M)^{1/2}$.

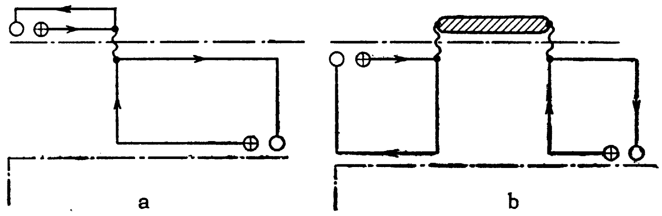

FIG. 8

6. Let us ascertain on what basis we can discard other graphs with four (or less) points.

The contribution from graphs such as shown in Fig. 7, being pure imaginary, is zero. The graphs consisting of two parts connected by a single interaction line or by a single plasmon line (Figs. 8a and b) are proportional to $u_{\mathbf{q}}|_{q=0}$ and are consequently equal to zero. The graphs on Fig. 9 represent corrections to the electron-ion scattering in an approximation higher than that of Born, and are therefore disregarded. The graphs on Fig. 10 are corrections to σ^{ee}_{xx} of next higher order in

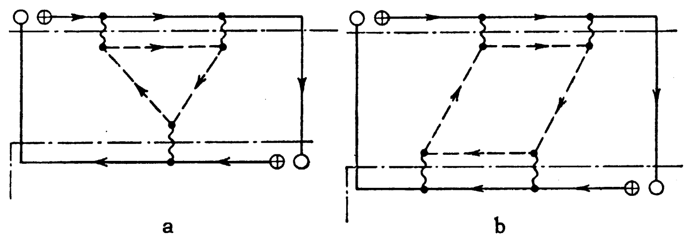

FIG. 9

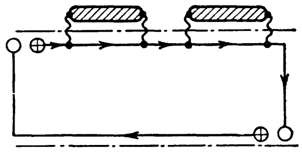

FIG. 10

$1/\Omega\tau$, and should therefore yield zero when summed. Graphs with overlapping or superimposed plasmon lines were estimated in [2] for the case $H = 0$. They are in any case small if $e^2/\hbar v_T \ll 1$.

7. Let us start with an investigation of the case $\kappa R \gg 1$. Expression (28) is no longer dependent on H and is noticeably simplified when $q_z \gg 1/R$. In fact, the terms significant in the integral of (28) are those with $t^2 \lesssim m\beta/q_z^2$. But for these values of t we have $\Omega t \lesssim (m\beta)^{1/2}\Omega/q_z = 1/q_z R \ll 1$, and we must therefore expand the sinusoidal terms in (28) in powers of the small parameter Ωt, retaining the first term. Then for $\hbar\Omega/kT \ll 1$ (this is the case of interest to us) we obtain

$$P_{\mathbf{q}}^e(\omega) = \frac{2}{\hbar\beta}\int_0^\infty dt e^{-i\omega t}\sin\left(\frac{\hbar q^2 t}{2m}\right)\exp\left(-\frac{q^2t^2}{2m\beta}\right). \tag{36}$$

Hence [7]

$$\operatorname{Re} P_{\mathbf{q}}^e(\omega) = \frac{q_T}{q}\left[W\left(\frac{\omega}{qv_T}+\frac{q}{2q_T}\right) - W\left(\frac{\omega}{qv_T}-\frac{q}{2q_T}\right)\right], \tag{37}$$

$$\operatorname{Im} P_{\mathbf{q}}^e(\omega) = -\sqrt{\pi}\,\frac{q_T}{q}\,\mathrm{sh}\,\frac{\hbar\omega\beta}{2}\exp\left[-\left(\frac{\omega}{qv_T}\right)^2 - \left(\frac{q}{2q_T}\right)^2\right], \tag{38*}$$

and $P_{\mathbf{q}}^i(\omega)$ is obtained from $P_{\mathbf{q}}^e(\omega)$ by replacing $\mathbf{q}$ with $\mathbf{q}/\gamma$, where $\gamma^2 = M/m$. Here

$$W(x) = e^{-x^2}\int_0^x e^{y^2}dy = \begin{cases} 1/2x & \text{for } x \gg 1, \\ x & \text{for } x \ll 1. \end{cases} \tag{39}$$

Obviously, the values $q \gtrsim \kappa$ play a role in (33). But if $\kappa R \gg 1$ and q is significant, we can use (37) and (38) for $P_{\mathbf{q}}^e(\omega)$, and consequently also for $P_{\mathbf{q}}^i(\omega)$. The entire dependence of D_{xx} on the magnetic field is determined by the factor $1/\Omega^2$ in (33). Therefore, to calculate D_{xx} in this case it is natural to replace $P_{\mathbf{q}}^e(\omega)$ and $P_{\mathbf{q}}^i(\omega)$ by their limiting values (37) and (38) with $H = 0$ over the entire region of q, and then verify that the contribution from $q_z \lesssim 1/R$ is small where such a substitution cannot be made.

Let us substitute (37) and (38) in (33) and take (32) into account. It is easiest to integrate over the angles, and the matter reduces to calculating the expression

$$\mathcal{J}(\xi) = \frac{8\pi}{3}\int_0^\infty \frac{dz}{z}\exp\left[-\frac{z^2(1+\gamma^{-2})}{4}\right]\int_{-\infty}^\infty \exp[-x^2(1+\gamma^{-2})]\frac{dx}{A^2+B^2}, \tag{40}$$

where the dimensionless variables are $z = q/q_T$, $x = \hbar\omega\beta\gamma/2z$;

$$4\xi^2 = \varkappa^2/q_T^2 \ll 1. \tag{41}$$

If $A \neq 0$, then the exponential term in the numerator of the integrand of (40) makes $x \lesssim 1$, $z \lesssim 1$ the significant region of the variables. In this region A and B are given by the following expressions (we neglect quantities of order $1/\gamma$):

$$A = 1 + 8(\xi/z)^2 a(x), \qquad B = 8(\xi/z)^2 b(x),$$
$$a(x) = 1 - x\,W(x), \qquad b(x) = (x\sqrt{\pi}/2)\,e^{-x^2}. \tag{42}$$

We show later that only this region plays any role, since the contribution from the region where $A = 0$, i.e., from the collective effects, is negligibly small. Then the expression for D_{xx} can be represented for $\xi \ll 1$ in the form

$$D_{xx} = \frac{2}{3}\left(\frac{2\pi m}{kT}\right)^{1/2}\left(\frac{c}{eH}\right)^2 ne^4\left\{\ln\frac{1}{2eC_1\xi^2} - \frac{1}{\sqrt{\pi}}\int_0^\infty e^{-x^2}\ln[a^2(x) + b^2(x)]\,dx - \frac{2}{\sqrt{\pi}}\int_0^\infty\left[\frac{a(x)}{b(x)}\operatorname{arctg}\frac{b(x)}{a(x)} - 1\right]e^{-x^2}dx\right\}, \tag{43*}$$

where $C = \ln C_1 = 0.577$ is Euler's constant.

The quantity $x = \hbar\omega\beta\gamma q_T/2q$ serves as a measure of the screening role of the ions. When $x \ll 1$ the screening can be regarded as static. If we replace $a(x)$ and $b(x)$ in (43) by $a(0)$ and $b(0)$, then the second and third terms vanish, corresponding to inclusion of the static effects only. To the contrary, when $x \gg 1$, the ions practically do not participate in the screening. If we replace $a(x)$ and $b(x)$ in (43) by $a(\infty)$ and $b(\infty)$, then the expression in the curly brackets becomes equal to $\ln(1/eC_1\xi^2)$, i.e., the factor under the logarithm sign is twice as large as in the first case, corresponding to complete neglect of the ion screening.

Actually, however, the sum of the second and third terms in the curly brackets of (43) is $\ln 1.34 = 0.296$ and is different from the two limiting values because of the non-static ion screening. Finally

$$D_{xx} = \frac{2}{3}\left(\frac{2\pi m}{kT}\right)^{1/2}\left(\frac{c}{eH}\right)^2 ne^4\ln(0.55q_T^2/\varkappa^2). \tag{44}$$

This result differs from the expressions obtained earlier with logarithmic accuracy (see [5], where references can be found to earlier papers) in that the factor under the logarithm sign is exact.

We note that the same methods can be used to calculate the transverse conductivity σ_{xx} of a semiconductor with quadratic carrier dispersion law, if $e^2/\epsilon\hbar v_T \ll 1$ (ϵ is the dielectric constant of the semiconductor). The ions must be regarded here as stationary, and the scattering of the electrons by the ions must be assumed elastic. When

*sh = sinh.

*arctg = $\tan^{-1}$.

$\kappa R \gg 1$ we obtain for σ_{xx} the following formula, with an exact coefficient under the logarithm sign (compare with the result of Brooks [13]):

$$\sigma_{xx} = \frac{ne^2}{m^*\Omega^2\tau}, \qquad \frac{1}{\tau} = \frac{2}{3}\left(\frac{2\pi}{m^*}\right)^{1/2} \frac{n_i}{(kT)^{3/2}} \left(\frac{e^2}{\varepsilon}\right)^2 \ln \frac{1}{eC_1\xi^2}$$

(n_i is the concentration of the ions and m^* the effective mass).

8. We show now that the contribution to (33) from regions where the collective effects are significant, i.e., where $A(\omega, q) = 0$, is negligibly small. First we determine when the second or third term of (34) can actually be negative, which is the necessary condition for $A(\omega, q)$ to vanish. The second term can be negative when $\omega/qv_T \gg 1$. The third term is small in this case, and $A(\omega, q)$ is in the first approximation independent of q and has a value $A = 1 - 4\pi ne^2/m\omega^2 = 1 - \omega_p^2/\omega^2$. When $\omega = \omega_p$ we have $A = 0$ (plasma "pole").

But if $\omega/qv_T \gg 1$, then $\operatorname{Im} P_q^e(\omega)$ is exponentially small [essentially like $\exp(-\omega^2/q^2v_T^2)$] and $\operatorname{Im} P_q^i(\omega)$ is small [like $\exp(-\gamma^2\omega^2/q^2v_T^2)$]. Inasmuch as when $A = 0$ the denominator of (33) is proportional to the square of the sum of these functions, i.e., in fact to the square of the greater of the two [$\operatorname{Im} P_q^e(\omega)$], and the numerator is proportional to their product, the contribution to (33) from the plasma poles is exponentially small, essentially like $e^{-\gamma^2}$.

If $\gamma\omega/qv_T \gg 1 \gg \omega/qv_T$, the second term in (34) is positive and the third is negative, so that with account of the inequality $q/\gamma q_T \ll 1$ the function A assumes the form $A = 1 + 2(\kappa^2/q^2) \times [1 - xW(x)]$. Since $xW(x) < 1$, A does not vanish in this region. This is demonstrated by the known fact [14] that sound cannot propagate in a fully ionized Boltzmann gas.

9. We proceed to investigate the region $q_z \lesssim 1/R$. We separate in this region two intervals of the variation of q_z. In the first region, where

$$1/R \gtrsim q_z \gtrsim 1/R_i \tag{45}$$

($R_i = \gamma R$ is the ionic Larmor radius) we must use for $P_q^e(\omega)$ the exact expression (28) and for $P_q^i(\omega)$ the approximate formulas (37) and (38). In the second interval, where

$$1/R_i \gtrsim q_z \gtrsim q_c, \tag{46*}$$

we must use the exact expressions for both quantities. In (46) q_c is the characteristic value of the wave vector introduced above, which cuts off the divergence at small q_z.

*cth = coth.

For what follows we must use the series expansion obtained in [9] for the integral in (28):

$$F_q^e(\omega) = \frac{1}{\sqrt{\pi}\,|q_z|\,v_T} \exp\left\{-\frac{u^2}{2}\alpha \operatorname{cth}\alpha\right\} \sum_{n=-\infty}^{\infty} I_n\left(\frac{\alpha u^2}{2\operatorname{sh}\alpha}\right) \times \exp\left[-\left(\frac{n\Omega - \omega}{q_z v_T}\right)^2 - \left(\frac{q_z}{2q_T}\right)^2\right]. \tag{47}$$

By the methods of [9] we obtain a similar expansion for $\operatorname{Re} P_q^e(\omega)$:

$$\operatorname{Re} P_q^e(\omega) = \frac{q_T}{|q_z|} \exp\left\{-\frac{u^2}{2}\alpha\operatorname{cth}\alpha\right\} \sum_{n=-\infty}^{\infty} e^{\alpha n} I_n\left(\frac{\alpha u^2}{2\operatorname{sh}\alpha}\right) \times [W(z_n^+) + W(z_n^-)]. \tag{48}$$

Here

$$u = q_\perp R, \qquad \alpha = \frac{\hbar\Omega}{2kT}, \qquad z_n^{\pm} = \frac{\hbar q_z^2/2m \pm \omega + n\Omega}{|q_z|\,v_T}. \tag{49}$$

It is further essential to verify that the contribution to (33) from the region of values of q and ω where $A = 0$, and from its vicinity, is negligibly small. Since the plasma and sound zeros of $A(\omega, q)$ are different when $q_z R < 1$, the corresponding analysis is much more complicated than when $q_z R \gg 1$. Its principal features are outlined in Appendix 2. But once this is proved, it is easy to show that we can replace the sum in (33), with sufficient degree of accuracy, by

$$1 + \varkappa^2/q^2. \tag{50}$$

We can thus get rid of the series in the denominator of (33). By the same token, the role of the Coulomb interaction reduces in the first approximation to pure screening. The non-static nature of the screening, when $\kappa R \gg 1$ can affect only the numerical factor in the small corrections, is neglected in such estimates.

The next stage consists of estimating the integral of and expression with the denominator of (33) and with a numerator containing the double series. When $\alpha \ll 1$ and $q \ll q_T$, the sum of the series (47) is represented by the following integral (see [9] for derivation), where $w = |q_z|R$:

$$F_q^i(\omega) = \frac{\gamma}{\pi w\Omega} \int_{-\infty}^{\infty} \exp\left\{-\lambda^2 + \frac{2i\gamma\omega}{\Omega w}\lambda - \frac{\gamma^2 u^2}{2}\left[1 - \cos\left(\frac{2\lambda}{\gamma w}\right)\right]\right\} d\lambda. \tag{51}$$

The denominator of (50) is independent of ω, and the following integral enters into (33):

$$\int_{-\infty}^{\infty} d\omega F_q^e(\omega)\, F_q^i(\omega) = \frac{\gamma^2 R}{2\pi v_T} e^{-u^2/2} \sum_{n=-\infty}^{\infty} I_n\left(\frac{u^2}{2}\right) \int_{-\infty}^{\infty} d\mu \times \exp\left[in\gamma^2\mu - \frac{\gamma^4 w^2\mu^2}{4} - \frac{\gamma^2 u}{2}(1 - \cos\mu)\right]. \tag{52}$$

When $\gamma^4 w^2 \ll 1$, the difference $1-\cos\mu$ can be expanded in powers of μ and the first term retained; then

$$\int_{-\infty}^{\infty} d\omega F_q^e(\omega) F_q^i(\omega) = \frac{1}{\sqrt{\pi}\Omega} e^{-u^2/2} \frac{1}{\sqrt{u^2/\gamma^2+w^2}} \sum_{n=-\infty}^{\infty} I_n\left(\frac{u^2}{2}\right) \times \exp\left(-\frac{n^2}{u^2/\gamma^2+w^2}\right). \quad (53)$$

Thus, we actually obtain instead of intervals (45) and (46) the two following regions of variation of q_z:

$$1 \gtrsim q_z R \gtrsim 1/\gamma^2 \quad \text{and} \quad 1/\gamma^2 \gtrsim q_z R \gtrsim q_c R. \quad (54)$$

The contribution to $\mathcal{J}$ from the first of these regions is, on the basis of (53)

$$\Delta_1\mathcal{J} \approx 4\pi^{3/2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} du \int_{1/\gamma^2}^{1} \frac{dw}{\sqrt{w^2+u^2/\gamma^2}} \frac{u^3 I_n(u^2/2)\, e^{-u^2/2}}{(u^2+w^2+\varkappa^2R^2)^2} \times \exp\left(-\frac{n^2}{w^2+u^2/\gamma^2}\right). \quad (55)$$

The second region may not exist at all if $q_c R \gtrsim 1/\gamma^2$ [the lower limit of the integral with respect to w in (55) is then not $1/\gamma^2$ but $q_c R$]. If this region does exist, we have in it

$$\Delta_2\mathcal{J} \approx 4\pi^{3/2} \sum_{n,N} \int_{q_c R}^{1/\gamma^2} \frac{dw}{w} \int_0^{\infty} du \frac{u^3 I_n(u^2/2)\, e^{-u^2/2}}{(u^2+w^2+\varkappa^2R^2)^2} \times I_N(\gamma^2u^2/2)\, e^{-\gamma^2u^2/2} \exp\left[-(N\gamma^{-2}-n)^2/w^2\right]. \quad (56)$$

Only "resonant" terms, for which $N \approx \gamma^2 n$, can play a role in the series (56) (the others are exponentially small), and consequently the double series is replaced by a single one, in which the zeroth term is the principal one. When $\kappa R \gtrsim 1$ its order of magnitude is

$$-(1/\gamma\varkappa^2R^2)\ln(q_c R\gamma^2).$$

We note that when $1 \gg \kappa R \gg 1/\gamma$, the value of this term is

$$-(1/\gamma\varkappa R)\ln(q_c R\gamma^2).$$

In the series (55) the sum of all but the zeroth term is of the same order as the smaller of the two quantities $1/\gamma$ or $1/\kappa R$. In the calculation of the zeroth term we can replace the lower limit in the integral with respect to w by zero (the error due to such a replacement is small if $\gamma\kappa R \gg 1$). When $\kappa R \gg 1$ this term is of order $(\kappa R)^{-4}\ln\gamma$. We note that when $1/\gamma \ll \kappa R \ll 1$, its value, accurate to the numbers under the logarithm sign is

$$2\pi^{3/2}\ln\left[1/(\varkappa R)^2\right]\ln\gamma. \quad (57)$$

To obtain the final estimates of the corrections $\Delta\mathcal{J}$ to the value $\mathcal{J}$ from the region $q_z R < 1$, we must subtract from the sum of the correct expressions (55) and (56) the contribution from the integral containing for $P_q^{e,i}(\omega)$ expressions (37) and (38) which do not hold in this region. When $\kappa R \gg 1$ this contribution is

$$\pi^{5/2}/2\varkappa R, \quad (58)$$

and when $\kappa R \ll 1$

$$\frac{4\pi^{3/2}}{3}\ln\frac{1}{(\varkappa R)^2}. \quad (59)$$

Thus, the correction to $\mathcal{J}$, due to the dependence of the collision operator on the magnetic field, is $\pi^{5/2}/2\kappa R$ when $\kappa R \gg \gamma$ and is of order $1/\gamma$ when $\gamma \gg \kappa R \gg 1$. When $1 \gg \kappa R \gg 1/\gamma$, if $1/\gamma^2 \gtrsim q_c R$, we have accurate to factors under the logarithm sign

$$\mathcal{J} = \frac{8\pi^{3/2}}{3}\ln(q_T R) + 4\pi^{3/2}\ln\left(\frac{1}{\varkappa R}\right)\ln\gamma. \quad (60)$$

The dependence of this expression on the magnetic field reflects the effect of the magnetic field on collisions with small momentum transfers. Obviously the result (60) could not be obtained from a kinetic equation of the type given in [2], with a collision operator independent of the magnetic field. From the nature of the derivation it might appear that this result has limited applicability, determined by the condition $e^2/\hbar v_z \ll 1$ for all the significant values of the z component of the electron velocity v_z. This result, however differs from the formula obtained by Belyaev [15] by classical analysis only in that the first term in (60) contains q_T rather than kT/e^2. This suggests that the smallness of the Born parameter manifests itself only in collisions with large momentum transfers, and the contribution from collisions with small momentum transfers is independent of whether the Born approximation is valid for their description or not. If this is so, our result (60) is valid under the less stringent condition $e^2/\hbar v_T \ll 1$.

The authors are grateful to V. I. Perel' for a discussion of the work and for many valuable remarks.

APPENDIX 1

Let us calculate the Laplace transform of a plasmon line with one end on the "temperature" axis. Let $z_1 = t_1$, $z_2 = t - i\hbar\lambda$, and let $\hbar a$ be the difference of the electron energies entering and leaving the point z_2. Then the sought quantity is

$$\dot{\overline{m}}_a(t_1 - t) = -\int_0^{\beta} d\lambda \exp(\hbar a\lambda)\, \overline{l}_q[t_1 - (t - i\hbar\lambda)].$$

Calculating the integral with respect to λ with the aid of (16) and (17b) we get

$$\overline{m}_a(t_1 - t) = -\frac{1}{2\pi i\hbar}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{\eta(t_1-t)}\,d\eta \int_{-\infty}^{\infty}\frac{\Phi_{\mathbf{q}}(\omega)}{\eta - i\omega}\,\frac{e^{\hbar a\beta} - e^{\hbar\omega\beta}}{a-\omega}.$$

Expanding the denominator in partial fractions and using (17), we obtain finally

$$\overline{M}_a(\eta) = \frac{1}{i\hbar(\eta - ia)}\{[e^{\hbar a\beta}L(\eta) - \overline{L}(\eta)] - [e^{\hbar a\beta}L(s+ia) - \overline{L}(s+ia)]\}.$$

APPENDIX 2

Let us verify that the contribution to (33) from the region where $A(\omega, \mathbf{q}) = 0$ and from its vicinity is small. We consider the interval (45), where expressions such as (37) and (38) hold for $P^i_{\mathbf{q}}(\omega)$. If the series (48) contains a large number of significant terms, it can be summed in the case of interest to us ($\alpha \ll 1$, $\hbar\omega\beta \ll 1$, $q_\perp \ll q_T$). As a result of rather cumbersome transformations we obtain for $q_z R \ll 1$

$$\operatorname{Re} P^e_{\mathbf{q}}(\omega) = 1 - \frac{\omega}{\Omega}\operatorname{ctg}(\pi\delta)\int_0^{\pi}\exp\left[\frac{u^2}{2}(\cos\psi - 1)\right] \times \cos\left(\frac{\omega}{\Omega}\psi\right)d\psi - \frac{\omega}{\Omega}\int_0^{\pi}\exp\left[\frac{u^2}{2}(\cos\psi - 1)\right] \times \sin\left(\frac{\omega}{\Omega}\psi\right)d\psi + \frac{n_0}{\delta}e^{-u^2/2}I_{n_0}\left(\frac{u^2}{2}\right) + \frac{q_T}{|q_z|}e^{-u^2/2}\left\{I_0\left(\frac{u^2}{2}\right)[W(z_0^+) + W(z_0^-)] + I_{n_0}\left(\frac{u^2}{2}\right)[e^{\alpha n_0}W(z^-_{n_0}) + e^{-\alpha n_0}W(z^+_{-n_0})]\right\}. \quad \text{(A.1)*}$$

Here $\delta = \omega/\Omega - n_0$, where n_0 is the integer closest to ω/Ω. (When $q_z R \sim 1$, Eq. (A.1) yields an estimate of the order of magnitude.)

When $\delta \gg q_z R$ this yields the known expression

$$\operatorname{Re} P^e_{\mathbf{q}}(\omega) = -\frac{q_\perp^2 v_T^2}{\omega^2 - \Omega^2} - \frac{q_z^2 v_T^2}{\omega^2}. \quad \text{(A.2)}$$

It is negative, and therefore $A(\omega, \mathbf{q})$ can vanish. However, in this case $\operatorname{Im} P^e_{\mathbf{q}}(\omega)$ and $\operatorname{Im} P^i_{\mathbf{q}}(\omega)$ are exponentially small. Therefore, by virtue of considerations similar to those advanced in Sec. 8, the contribution from this region and from its vicinity turns out to be small.

An investigation shows that when $u^2 \gg 1$, Eq. (A.1) can be negative only near "gyroresonances," when $\delta \ll q_z R$. $A(\omega, \mathbf{q})$ can then vanish only if $n_0/q_z R \gtrsim u \gtrsim n_0$. When $w \ll 1$ and $u \gtrsim n_0$, only the n_0-th term plays any role in the series for $\operatorname{Im} P^e_{\mathbf{q}}(\omega)$ (we shall denote it B_{n_0}) and all others are exponentially small. For B_{n_0} we have ($s \equiv \delta/w$)

$$B_{n_0}(s) = \frac{\sqrt{\pi}}{2}(\varkappa R)^2\left\{\frac{\gamma}{\sqrt{u^2+w^2}}\exp\left[-\frac{\gamma^2 n_0^2}{u^2+w^2}\right] + \frac{1}{w}I_{n_0}\left(\frac{u^2}{2}\right)e^{-u^2/2-s^2}\right\} \quad (n_0 \neq 0), \quad \text{(A.3)}$$

$$B_0(s) = \frac{\sqrt{\pi}}{2}(\varkappa R)^2 s\left\{\frac{\gamma w}{\sqrt{u^2+w^2}}\exp\left[-\frac{s^2\gamma^2 w^2}{u^2+w^2}\right] + I_0\left(\frac{u^2}{2}\right)e^{-u^2/2-s^2}\right\}.$$

The entire region of integration with respect to ω can be subdivided into the intervals

$$(n_0 - 1/2)\,\Omega \leqslant \omega \leqslant (n_0 + 1/2)\,\Omega, \quad \text{(A.4)}$$

i.e., $-w/2 \le s \le w/2$, and with the accuracy employed here the contribution to $\mathcal{J}$ in region (45) can be represented in the form

$$\Delta_1\mathcal{J} \approx 8\pi\gamma \sum_{n=-\infty}^{\infty}{}' \int_{1/\gamma}^{1} dw \times \int_0^{\infty}\frac{du\,u^3 I_n(u^2/2)\exp[-u^2/2 - \gamma^2 n^2/(u^2+w^2)]}{(u^2+w^2)^{5/2}}\int_0^{1/2w}\frac{e^{-s^2}ds}{A_n^2 + B_n^2} + 8\pi\gamma\int_{1/\gamma}^{1}dw\int_0^{\infty}du\,\frac{u^3 I_0(u^2/2)\,e^{-u^2/2}}{(u^2+w^2)^{5/2}} \times \int_0^{1/2w}ds\,\frac{\exp[-s^2 - s^2\gamma^2 w^2/(u^2+w^2)]}{A_0^2 + B_0^2}ds. \quad \text{(A.5)}$$

Here

$$A_{n_0} \approx 1 + \frac{\varkappa^2 R^2}{u^2+w^2}\left\{1 + I_{n_0}\left(\frac{u^2}{2}\right)e^{-u^2/2}\frac{q_T}{|q_z|}[e^{\alpha n_0}W(z^-_{n_0}) + e^{-\alpha n_0}W(z^+_{-n_0})] + \left[1 - \frac{n_0\gamma}{\sqrt{u^2+w^2}}W\left(\frac{n_0\gamma}{2\sqrt{u^2+w^2}}\right)\right]\right\},$$

and A_0 can be readily obtained from (A.1) by expansion in powers of ω/Ω.

When $s \lesssim 1$, the width of the interval Δs, defined by the condition

$$A_n(s_n + \Delta s) = B_n(s_n), \qquad A_n(s_n) = 0, \quad \text{(A.6)}$$

is proportional to $B_n(s_n)$ and is consequently exponentially small [we recall that when $w \lesssim 1$, $A(s)$ can have zeros only if $u \lesssim \gamma n$]. Thus, the contribution from the plasma "poles" is proportional in this region to

$$\exp\left[-s_0^2 - \frac{\gamma^2 n^2}{u^2+w^2}\right]\Big/\left[\exp(-s_0^2) + \exp\left(-\frac{\gamma^2 n^2}{u^2+w^2}\right)\right] \quad \text{(A.7)}$$

and is consequently exponentially small.

If $s_0 \lesssim 1$, then B is essentially independent of s and A is linear in s, so that it is easy to integrate with respect to s. We can thus readily verify that the contribution from the "poles" is negligibly small in this region. In similar fashion we can verify the smallness of the contribution from the poles in region (46).

*ctg = cot.

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Translated by J. G. Adashko
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