INTERFERENCE BETWEEN DIRECT AND RESONANCE CAPTURE OF SLOW NEUTRONS

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The effective cross section for resonance capture of slow neutrons is known to be symmetric if a compound nucleus is formed. It is shown that when compound nucleus formation and direct capture occur simultaneously, the effective cross section is not symmetric.

1. INTRODUCTION

EXPERIMENTAL and theoretical investigations of direct reactions have been at the center of interest for nuclear physics for many years. Until recently we were able to consider and describe the capture of slow neutrons only by means of statistical theory, based on the Bohr hypothesis of a "compound nucleus". But in studying the anomalies which appear in the γ spectrum from (n, γ) reactions, Lane and Lynn [1] came to the conclusion that a direct mechanism may also play an important role in the process of radiative capture, at least for certain definite groups of nuclei.

According to the statistical theory, the spectral distribution of the primary γ rays appearing in an (n, γ) reaction is proportional to the quantity

$$E_\gamma^3 \rho (E_k - E_\gamma),$$

where E_γ is the energy of the γ ray, E_k is the binding energy of the neutron and $\rho(E)$ is the average level density of the nucleus.

In the case of the capture of slow neutrons, this function has a maximum in the region of 2 — 3 Mev and falls off monotonically toward higher energies. But experiment shows that there are also intensity maxima at higher energies in the γ spectra, at least in the region of nuclei with mass numbers between 70 and 208. [2] As we approach the mass numbers 70 and 208, the peaks which are seen at high energies approach a value corresponding to the binding energy.

Since the 2p and 3p neutron shells close near the mass numbers 70 and 208 respectively, the following explanation of this phenomenon seems very likely. In the field of the nuclear forces, the neutrons are subject to potential scattering; from the free s state they go directly, without compound nucleus formation, into a bound p state, emitting electric dipole radiation. As we approach the mass numbers 70 and 208, the p states correspond to lower and lower excitation, so that the energy of the emitted γ rays increases. Apparently the probability of direct capture is higher the closer the p state is to a pure single particle state.

On the basis of the dispersion theory of nuclear reactions, Lane and Lynn showed that such a direct capture is actually possible and compared the theoretical values for the cross section with the available experimental data; [3] despite the fact that only an order of magnitude comparison was possible, because of the complexity of the phenomena, it seems to be entirely probable that for certain nuclei the process of direct neutron capture actually exists.

From the analysis of Lane and Lynn we see that, in those cases where the final state is an approximate single particle state, the channel region, i.e., the region of configuration space outside the range of nuclear forces, makes a sizable contribution to the matrix element determining the cross section, as had already been pointed out by Thomas. [4] According to them, if the final state is an approximate single particle state, the probability of a transition which leads to this state is increased, and thus it may be possible to explain the anomalies observed in the γ spectra.

From these remarks it follows that it would be extremely useful to carry out a direct experiment to settle unequivocably the question of the existence and role of the direct capture process. The interference that occurs between the direct and resonance capture processes makes it possible, in principle, to set up such a direct experiment. Just as for the case of interference between resonance and potential scattering, here too one may expect that the partial effective cross section for neutron capture accompanied by γ radiation of high energy goes through a minimum value in the region below an isolated resonance, and begins to increase rapidly only after one has passed the resonance.

In the following we shall study the phenomena to be expected in the capture process as a result of the interference between direct and resonance capture.

2. EFFECTIVE CROSS SECTION FOR (n, γ) REACTIONS

We shall consider (n, γ) reactions with slow neutrons, where the orbital angular momentum of the neutron is zero, and assume that the total angular momentum of the system consisting of the neutron and the target nucleus takes on one definite value:

$$\mathbf{J} = \mathbf{s} + \mathbf{s}_n,$$

where $\mathbf{s}$ is the spin of the target nucleus and $\mathbf{s}_n$ is the neutron spin. In this case the effective cross section for the (n, γ) reaction is

$$\sigma_{n\gamma} = \frac{\pi}{k^2} \frac{2J+1}{(2s+1)(2s_n+1)} |S_{fc}|^2, \tag{1}$$

where S_{fc} is the corresponding element of the scattering matrix:

$$S_{fc} = \left(\frac{16\pi}{9\hbar}\right)^{1/2} k_\gamma^{3/2} \frac{(\psi_f \mid H^{(1)} \mid \psi_c)}{(2J+1)^{1/2}}. \tag{2}$$

Here $(\psi_f | H^{(1)} | \psi_c)$ is the reduced matrix element of the dipole operator between the free state ψ_c and the bound state ψ_f; k_γ is the wave vector of the emitted γ rays.

To determine the reduced matrix element of the dipole operator we must calculate the contributions from the interior of the nucleus and from the channel region. Therefore the functions for the initial and final states must be given both in the interior of the nucleus and also in the channel region. The initial state wave function in the channel is

$$\psi_c = v^{-1/2}(I_c(kr) - S_{cc}O_c(kr))\,\varphi_c, \qquad r > R, \tag{3}$$

where v is the velocity and k the wave vector of the incident neutron ($v = \hbar k/M$, where M is the reduced mass); φ_c is the so-called channel function, which depends on all the internal coordinates of the nucleus and also contains the spin function of the neutron; $I_c(kr)$ and $O_c(kr)$ describe incoming and outgoing waves, and have the asymptotic form:

$$I_c(kr) \to e^{-ikr}, \qquad O_c(kr) \to e^{ikr};$$

S_{cc} is the diagonal element of the scattering matrix.

In the interior of the nucleus, S_{cc} and the function ψ_c can be determined using the R-matrix theory.[5] If we are studying neutron capture in the region of an isolated resonance, we may assume that the matrix

$$R = \sum_{\lambda'} \frac{[\gamma_{\lambda'}\gamma_{\lambda'}]}{E_{\lambda'} - E}$$

splits into two parts:

$$R = \sum_{\lambda' \neq \lambda} \frac{[\gamma_{\lambda'}\gamma_{\lambda'}]}{E_{\lambda'} - E} + \frac{[\gamma_\lambda\gamma_\lambda]}{E_\lambda - E} = R^\infty + \frac{[\gamma_\lambda\gamma_\lambda]}{E_\lambda - E}, \tag{4}$$

where the matrix R^∞ contains all resonances except for the one near E_λ and is diagonal.

The quantities $\gamma_{\lambda c}$ are the components of the vector for the reduced width of the level, γ_λ, and are defined by the formula

$$\gamma_{\lambda c} = (\hbar^2/2MR)^{1/2} \int X_\lambda \varphi_c^* dF.$$

In this case the diagonal element of the matrix S can be written in the form

$$S_{cc} = e^{-2i\delta_c'}\left[1 + i\frac{\Gamma_{\lambda c}}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}\right], \tag{5}$$

where the relation between the phase shift δ_c' coming from the potential scattering and the phase shift δ_c for an impenetrable sphere is given by

$$e^{-2i\delta_c'} = e^{-2i\delta_c}\left(\frac{1 - R_{cc}^\infty L_c^*}{1 - R_{cc}^\infty L_c}\right).$$

We can then calculate the partial width

$$\Gamma_{\lambda c} = \frac{2P_c\gamma_{\lambda c}^2}{(1 - R_{cc}^\infty S_c)^2 + (R_{cc}^\infty P_c)^2},$$

the total width

$$\Gamma_\lambda = \sum_c \Gamma_{\lambda c}$$

and the level shift

$$\Delta_\lambda = \sum_c \frac{P_c(R_{cc}^\infty P_c) - S_c(1 - R_{cc}^\infty S_c)}{(1 - R_{cc}^\infty S_c)^2 + (R_{cc}^\infty P_c)^2}\gamma_{\lambda c}.$$

S_c and P_c denote the real and imaginary parts of the logarithmic derivative L_c calculated at a distance corresponding to the radius of interaction:

$$L_c = \left[kr\frac{dO_c(kr)}{d(kr)}\frac{1}{O_c(kr)}\right]_{r=R} = S_c + iP_c.$$

In the interior of the nucleus, the function ψ_c has the form

$$\psi_c = -i\hbar^{1/2}e^{-i\delta_c'}\frac{\Gamma_{\lambda c}^{1/2}X_\lambda}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}, \qquad r < R. \tag{6}$$

First we determine the contribution to the scattering matrix element from the interior region of the nucleus:

$$S_{fc}^{B} = -i\left(\frac{16\pi}{9}\right)^{1/2} \frac{k_\gamma^{3/2}\Gamma_{\lambda c}^{1/2} e^{-i\delta_c'}}{(2J+1)^{1/2}} \frac{(\psi_f \mid H^{(1)} \mid X_\lambda)}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}$$

$$= -ie^{-i\delta_c'} \frac{\Gamma_{\lambda c}^{1/2}\Gamma_{\lambda f}^{1/2}}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}. \qquad (7)$$

In this case the integral appearing in the reduced matrix element should be extended over the region $r < R$.

The contribution from the channel region consists of two parts, resonant and nonresonant, as one sees from (5). The determination of the contribution from the resonance channel is done in a way analogous to that for the internal region. In the region of the channels, X_λ corresponds to a continuation of the diverging wave

$$X_\lambda'(r) = (X_\lambda(R)/O_c(kR))\,O_c(kr), \qquad r > R. \qquad (8)$$

Similarly, the final state wave function ψ_f in the channel region has the form

$$\psi_f'(r) = \int \psi_f \varphi_f^* dF \frac{O_f(k_f r)}{O_f(k_f R)} \varphi_f, \qquad r > R, \qquad (9)$$

where φ_f depends on all the coordinates of the total system, which is in a bound final state, except for the radial coordinate of the captured neutron. $O_f(k_f r)$ is the exponentially decreasing radial function for the neutron in the bound final state. Thus the contribution to the scattering matrix element from the resonance is

$$S_{fc}^{K} = -i\left(\frac{16\pi}{9}\right)^{1/2} \frac{k_\gamma^{3/2}\Gamma_{\lambda c}^{1/2} e^{-i\delta_c'}}{(2J+1)^{1/2}} \frac{(\psi_f' \mid H^{(1)} \mid X_\lambda')}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}$$

$$= -ie^{-i\delta_c'} \frac{\Gamma_{\lambda c}^{1/2}\delta\Gamma_{\lambda f}^{1/2}}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}. \qquad (10)$$

The integration is of course extended over the region $r > R$.

We note that if the penetration factor is small ($P_c \ll 1$), the photon width is real. One can see that the contribution from the resonance channel is significant if the final state function extends out strongly into the channel region. This is the case when the final state is a single particle state or contains a large single particle component. From this it follows that the transition probability is increased for a final state of single particle type. This fact would already explain the anomalies which appear in the γ spectra. But there is still another contribution to the scattering matrix element S_{fc}, coming from the channel region. This contribution comes from the potential scattering, and consequently does not have resonance character:

$$S_{fc}^{p} = \left(\frac{16\pi}{9\hbar v}\right)^{1/2} \frac{k_\gamma^{3/2}}{(2J+1)^{1/2}} (\psi_f' \mid H^{(1)} \mid [I_c - e^{-2i\delta_c'} O_c]\,\varphi_c) = D^p. \qquad (11)$$

The effective cross section corresponding to this contribution may be called the cross section for "direct" capture. Thus the total scattering matrix element will be equal to

$$S_{fc} = S_{fc}^{p} + S_{fc}^{K} + S_{fc}^{B}$$

$$= D^p - ie^{-i\delta_c'} \frac{\Gamma_{\lambda c}^{1/2}\Gamma_{\lambda f}^{1/2}}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}$$

$$- ie^{-i\delta_c'} \frac{\Gamma_{\lambda c}^{1/2}\delta\Gamma_{\lambda f}^{1/2}}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2}, \qquad (12)$$

and the effective cross section will be

$$\sigma_{n\gamma} = \frac{\pi}{k^2} \frac{2J+1}{2(2s+1)} \Big\{ |D^p|^2 + \frac{A^2 + 2A\,\mathrm{Re}\,(D^{p*}e^{-i\delta_c'}) - 2xA\,\mathrm{Im}\,(D^{p*}e^{-i\delta_c'})}{x^2+1} \Big\};$$

$$A \equiv 2\Gamma_{\lambda c}^{1/2}(\Gamma_{\lambda f}^{1/2} + \delta\Gamma_{\lambda f}^{1/2})/\Gamma_\lambda. \qquad (13)$$

Here we have introduced the quantity $x \equiv [E - (E_\lambda + \Delta_\lambda)/(\Gamma_\lambda/2)]$, which gives the difference between the energy E and the resonance energy $E_\lambda + \Delta_\lambda$, measured in units of the halfwidth.

Since the phase shift for scattering from an impenetrable sphere is $\delta_c = -kR$ (where R is the nuclear radius), and the potential scattering phase shift δ_c' is of order δ_c, for the case of slow neutrons the quantity $e^{-i\delta_c'}$ is approximately equal to unity. By studying the quantities appearing in formula (11), one can easily show that D^p is almost a pure imaginary quantity, so that

$$\mathrm{Re}\,(D^{p*}e^{-i\delta_c'}) \approx 0, \qquad \mathrm{Im}\,(D^{p*}e^{-i\delta_c'}) \approx -|D^p|.$$

Using this, we get

$$\sigma_{n\gamma}(x) = \sigma_n(0) \left\{ \frac{\sigma_p}{\sigma_r(0)} + \frac{1 + 2\sqrt{\sigma_p/\sigma_r(0)}\,x}{x^2+1} \right\}, \qquad (14)$$

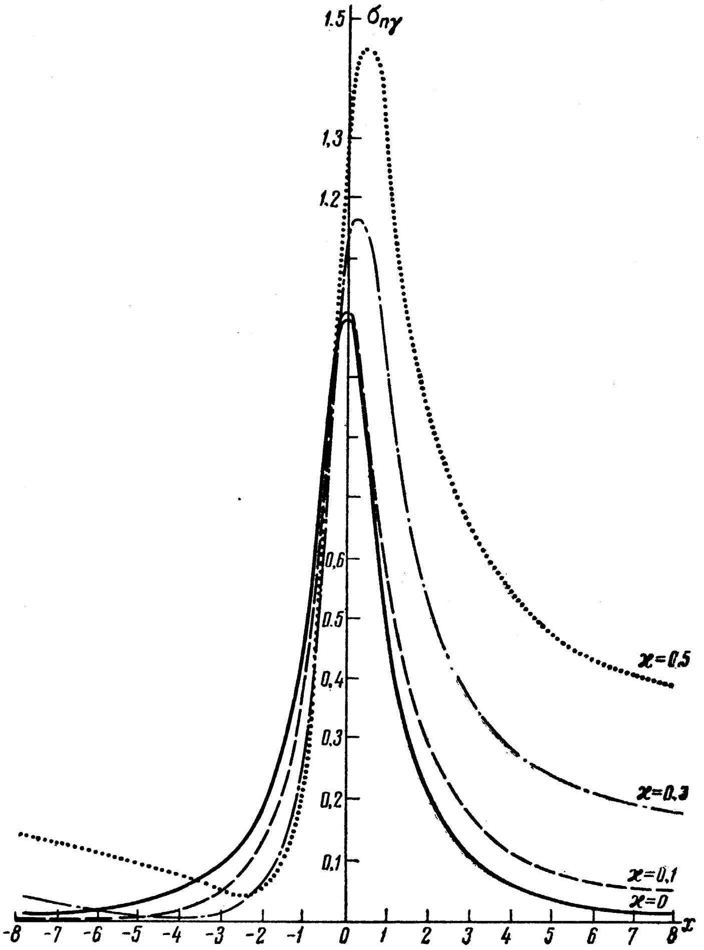
where

$$\sigma_r(0) = \frac{\pi}{k^2} \frac{2J+1}{2(2s+1)} \frac{\Gamma_{\lambda c}(\Gamma_{\lambda f}^{1/2} + \delta\Gamma_{\lambda f}^{1/2})^2}{\Gamma_\lambda^2/4} \qquad (15)$$

is the effective resonance cross section at the resonance energy, and

$$\sigma_p = \frac{16\pi^2}{9\hbar k^2 v} \frac{k_\gamma^3}{2(2s+1)} |(\psi_f' \mid H^{(1)} \mid [I_c - e^{-2i\delta_c'} O_c]\varphi_c)|^2 \qquad (16)$$

is the effective cross section for "potential" or "direct" capture. Thus, by determining experimentally the energy dependence of the cross section, one can obtain information concerning the ratio of the cross sections. In the figure we show the behavior of $\sigma_{n\gamma}(x)$ for various values of $\kappa = (\sigma_p/\sigma_r(0))^{1/2}$. The difference in the energies



Effective cross section for an (n, γ) reaction in the neighborhood of an isolated resonance.

corresponding to the maximum and minimum effective cross sections is equal to

$$E_{max} - E_{min} = \frac{1}{2}\Gamma_\lambda\sqrt{\sigma_r(0)/\sigma_p + 4}. \qquad (17)$$

3. CONCLUSION

From the arguments given we see that if there is actually a mechanism for direct reaction in (n, γ) reactions then one manifestation of its effects is that, in contrast to our present pictures, the cross section curve will not be symmetric. The experimental determination of the asymmetry and of the ratio $\sigma_p/\sigma_r(0)$ seems to be possible only for those nuclei in which there is an unoccupied state which is close to a single particle p state, and in which isolated resonances occur. It is also obvious that one can exhibit the asymmetry only if one measures the partial cross section $\sigma_{n\gamma}(c \rightarrow f)$, i.e., not the effective cross section for the (n, γ) reaction as a whole but only that part which leads to just one final state or several final states with identical character. In practice this means that one selects one or several intense peaks which appear in the high energy region of the γ spectrum, and measures the cross section for those (n, γ) processes in which γ quanta having the selected energy appear.

Formula (14) shows that the detection of the asymmetry is to be expected only for those isolated resonances for which the ratio $\sigma_p/\sigma_r(0)$ is not too small: $\sigma_p/\sigma_r(0) \geq 10^{-2}$.

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