

## A MODEL FOR ANOMALOUS MUON INTERACTION

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Submitted to JETP editor April 27, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 1205-1214 (October, 1961)

A model is considered in which the muon and electron mass difference is caused by the interaction of the muon with a hypothetical neutral vector meson  $\chi$ . Possible manifestations of this interaction are discussed. The interaction of the muon neutrino and the baryons with the  $\chi$  mesons is analyzed.

## INTRODUCTION

IN all experiments performed so far, the interactions of the electron and the muon are identical, whereas the masses of these particles differ by a factor of more than 200. One can imagine two possibilities: 1) either subsequent experiments will continue to give no evidence for a difference between the electron and muon interactions, or 2) through increased accuracy of the experiments and an analysis of the properties of the muon at small distances, some "anomalous" interaction of the muon will be observed, which is absent in the case of the electron. The second possibility seems to us more natural. As will be shown below, the experiments carried out up to now still leave very much room for a possible anomalous interaction of the muon.

For definiteness, we shall consider a model in which the anomalous interaction is given by the interaction of the muon with a hypothetical neutral vector meson  $\chi$ . Such a model has a number of attractive features: it is renormalizable, it gives finite results in a number of cases, and it is  $\gamma_5$  invariant. On the other hand, many of the relations between various physically observable quantities established within the framework of this model should also be qualitatively valid in other models, as, for example, in the case of an anomalous four-fermion interaction.

The anomalous interaction of the muon with a hypothetical intermediate neutral boson has been considered in a number of theoretical papers. Schwinger,<sup>[1]</sup> Cowland,<sup>[2]</sup> and Gatland<sup>[3]</sup> have discussed a model with pseudoscalar and scalar mesons. Jouvét and Goldzahl<sup>[4]</sup> considered also several aspects of the vector model with which we are concerned (see below). A neutral vector meson  $\chi$ , if it exists, can interact not only with

the muon, but also possibly with certain other particles. The larger the number of particles which interact with the  $\chi$  meson, the more numerous and varied will also, of course, be the possibilities for an experimental verification of the model.

1. INTERACTION OF THE  $\chi$  MESON WITH THE MUON

Let us, then, assume that there exists an interaction

$$\sqrt{4\pi}f\psi_\mu\gamma_\alpha\psi_\mu\chi_\alpha,$$

where  $f$  is a dimensionless constant and  $\chi_\alpha$  is the wave function of the  $\chi$  meson field. In the following we shall call this interaction the  $f$  interaction.

The constant  $f$  and the mass of the  $\chi$  meson are parameters of our theory which must be determined from experiment. We shall consider only values of  $f < 1$ . The only "justification" for this restriction is the absence of a theory of strong interactions. For  $f > 1$  we would not be able to employ perturbation theory. The possibility  $f^2 = e^2 = \alpha = 1/137$  appears particularly pleasing to us.

1. The mass of the muon. The interaction of the muon with the  $\chi$  meson might explain the difference between the masses of the muon and the electron. One could think that the bare mass of the muon becomes equal to the mass of the electron if the  $\chi$  interaction is "switched off" ( $m_\mu^0 = m_e$ ). Since for  $f < 1$  the normal polar type mass correction due to the graph of Fig. 1 is small,

$$\delta m_\mu = m_\mu^0 \frac{3f^2}{4\pi} \ln \frac{\Lambda^2}{m_\chi^2},$$

one can assume that the main part of the muon mass  $m_\mu^0$  ( $m_\mu^0 \sim m_\mu$ ) is due to a modification of

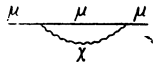


FIG. 1

the  $f$  interaction at small distances ( $r < 1/\Lambda$ ). This "explanation" of the mass difference between  $\mu$  and  $e$  is no better or worse than the widely known explanation of the mass difference between  $e$  and  $\nu$  by the fact that the electron has an electric charge. For in this case the normal electromagnetic mass correction of the electron is also small,

$$\delta m_e = m_e^{0'} \frac{3e^2}{4\pi} \ln \frac{\Lambda^2}{m_e^{0'2}},$$

and the main part of the electron mass is due to a modification of electrodynamics at distances  $r \lesssim 1/\Lambda$ .

**2. Universality of the weak interaction.** The presence of the  $f$  interaction should lead to a lowering of the value of the weak coupling constant for  $\mu$  mesons. For example, the  $\mu$ -decay constant should become smaller than the vector constant for  $\beta$  decay. The ratio of the probabilities for the decays  $\pi \rightarrow \mu + \nu$  and  $\pi \rightarrow e + \nu$ , as well as for the decays  $K \rightarrow \mu + \nu$  and  $K \rightarrow e + \nu$ , should become smaller than predicted by the theory of the universal V-A interaction. This can be easily understood physically: part of the time, when the muon goes over into the state  $\mu + \chi$ , it cannot participate in the weak interactions. Let us consider, for example, the correction due to the  $f$  interaction to the decay of the muon  $\mu \rightarrow e + \bar{\nu} + \nu$ . The effective decay constant  $G_\mu$  will be equal to

$$G_\mu = GZ_2^{1/2}, \quad G = 10^{-5} m_N^{-2},$$

where  $Z_2^{1/2}$  is the renormalization of the  $\psi$  function of the muon owing to the  $f$  interaction.

It is natural to expect that the larger renormalization of the mass of the muon  $\delta m$  gives rise to a large renormalization of the  $\psi$  function  $Z_2^{1/2}$ . Unfortunately, the relation between  $\delta m$  and  $Z_2$  established on the basis of Lehmann's spectral representation<sup>[5]</sup> is rather weak:

$$\delta m_\mu = m_\mu - m_\mu^0 = Z_2 \int_0^\infty [m_\mu \sigma_1(x^2) + x \sigma_2(x^2)] dx^2,$$

$$Z_2 = 1 + \int_0^\infty \sigma_1(x^2) dx^2,$$

where the spectral functions  $\sigma_1$  and  $\sigma_2$  satisfy the conditions

$$\sigma_1(x^2) \geq 0, \quad \sigma_1(x^2) \geq \sigma_2(x^2) \geq -\sigma_1(x^2).$$

In first nonvanishing order of perturbation theory (see Fig. 2)

$$Z_2^{-1} = 1 + (f^2/4\pi) \ln(\Lambda^2/m_\chi^2),$$

which gives

$$(G - G_\mu)/G \approx (f^2/8\pi) \ln(\Lambda^2/m_\chi^2).$$

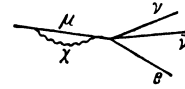


FIG. 2

The analysis of the experimental data indicates that this quantity is not larger than 5%, and may even be equal to zero (taking electromagnetic corrections into account).<sup>[6]</sup> This imposes definite restrictions on the value of  $f$ ,  $m_\chi$ , and  $\Lambda$ . Thus, for  $f = 1/2$ ,  $m_\chi = 10$  Bev,  $\Lambda = 1000$  Bev, the quantity  $(G - G_\mu)/G$  would be larger than 10%. In the following we shall consider two possibilities. In this and the next section, we shall consider a scheme in which only one type of neutrino exists. In such a scheme we expect a violation of the universality of the weak interaction, although we cannot now predict the extent of this violation.

The second possibility, which will be considered in Secs. 3 and 4, is to preserve the universality of the weak interaction even for values of  $f$  close to unity by assuming that there exist two types of neutrinos,<sup>[7,8]</sup> a muonic,  $\nu_\mu$ , and an electronic neutrino,  $\nu_e$ , and that the muonic neutrino, like the muon, interacts with the  $\chi$  meson through the  $f$  interaction.

**3. Anomalous magnetic moment of the muon.** The magnetic moment of the muon with account of the electromagnetic corrections is equal to<sup>[9]</sup>

$$\mu = g_e e / 4m_\mu, \quad g_e/2 = 1 + \alpha/2\pi + 0.75\alpha^2/\pi^2 = 0.001165.$$

The inclusion of the graph of Fig. 3 would lead to an increase in the value of  $g$ :

$$g_{e+f}/2 = g_e/2 + \delta; \quad \delta = \frac{f^2}{\pi} \int_0^1 \frac{x^2(1-x) dx}{x^2 + (m_\chi/m_\mu)^2(1-x)}.$$

For  $m_\mu/m_\chi \ll 1$  we have\*

$$\delta \cong \frac{f^2}{3\pi} \frac{m_\mu^2}{m_\chi^2}.$$

\*This formula is contained in a paper by Jouvét and Goldzahl.<sup>[4]</sup>

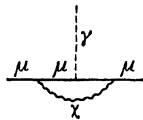


FIG. 3

Recalling that experimentally<sup>[10]</sup>  $(g/2)_{\text{exp}} = 1.001145 \pm 0.000022$ , we assume

$$(f^2/3\pi) (m_\mu^2/m_\chi^2) \lesssim 10^{-5}.$$

From this we obtain the following limit on the mass of the  $\chi$  meson:

$$m_\chi \gtrsim 100m_\mu f \approx 10m_{\text{nucl}} f.$$

4. Four-fermion  $\mu\mu$  interaction. The exchange of  $\chi$  mesons should lead to an effective interaction between muons (see the graph of Fig. 4). This interaction has the form

$$\frac{4\pi f^2}{m_\chi^2 - k^2} (\bar{u}\gamma_\alpha u) (\bar{u}\gamma_\alpha u),$$

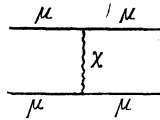


FIG. 4

where  $k$  is the four-momentum transferred by the  $\chi$  meson and the  $u$ 's are the spinors of the muon field. For  $k^2 \ll m_\chi^2$  this interaction has the form of an effective four-fermion interaction

$$F (\bar{u}\gamma_\alpha u) (\bar{u}\gamma_\alpha u).$$

The data on the anomalous magnetic moment of the muon limit the possible values of the constant  $F$ :

$$F = 4\pi f^2/m_\chi^2 \lesssim 12\pi^2 \cdot 10^{-5}/m_\mu^2 = 1/10 m_{\text{nucl}}^2.$$

The presence of such a  $\mu\mu$  interaction should, in particular, lead to the creation of muon pairs by muons in the Coulomb field of the nucleus (one of the four graphs for this process is shown in Fig. 5). The contribution of this graph must be compared with the contribution of the pure electrodynamic graphs of the type of Fig. 6. The contributions of these graphs are comparable if

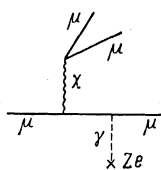


FIG. 5

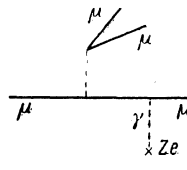


FIG. 6

$$f^2/m_\chi^2 \approx e^2/k^2, \text{ or } \sqrt{k^2} \approx 10 m_\mu \approx 1 \text{ Bev.}$$

Here  $e^2 = \alpha = 1/137$ , and  $k$  is the total four-momentum of the pair.  $\sqrt{k^2}$  is the energy of the created pair  $\mu^+ + \mu^-$  in its center-of-mass system (c.m.s.).

5. Production and decay of the  $\chi$  meson. If the energy of the muons is sufficiently large, they can create  $\chi$  mesons by scattering in the Coulomb field of the nucleus (two graphs of the type of Fig. 7). The cross section for this process  $\sigma_{Z\chi}$  can be computed by the Weizsäcker-Williams method if the cross section  $\sigma_{\gamma\chi}$  of the corresponding photo process  $\gamma + \mu \rightarrow \chi + \mu$  is known. The cross section  $\sigma_{\gamma\chi}$  is easily computed and is equal to

$$\sigma_{\gamma\chi} \approx (2\pi\alpha f^2/\omega^2) \ln(\omega^2/m_\mu^2).$$

Here  $w$  is the total energy of the photon and the muon in their c.m.s. (in deriving this formula we have assumed  $w \geq m_\chi^2 \geq m_\mu^2$ ).

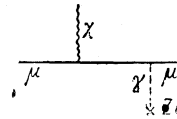


FIG. 7

For the cross section  $\sigma_{Z\chi}$  we have

$$\sigma_{Z\chi} = \int \frac{Z^2\alpha}{\pi} \frac{dq^2}{q^2} \frac{d\omega^2}{\omega^2} \sigma_{\gamma\chi}.$$

Taking account of the fact that the finite dimensions of the nucleus "cut off" the momenta  $\sqrt{q^2} > q_0$  ( $q_0 \sim \pi A^{-1/3}$ , where  $A$  is the number of nucleons in the nucleus), and that the conservation laws prescribe that the minimum value of  $\sqrt{q^2}$  is equal to  $\sqrt{q_{\text{min}}^2} = m_\chi^2/2E$  ( $E$  is the energy of the muon in the laboratory system), we find that for energies  $E \ll E_0 = m_\chi^2/2q_0$  ( $q_0^2 \ll q_{\text{min}}^2$ ) the cross section for production of  $\chi$  mesons is small owing to the "friability" of the nucleus. The main contribution therefore comes from the creation of  $\chi$  mesons, not on the nucleus as a whole, but on the separate nucleons in the nucleus. For a single proton ( $q_0^{(p)} \approx 500 \text{ Mev}/c$ ) with  $E_\mu \gg m_\chi^2/2q_0^{(p)}$

$$\sigma_\chi \approx 10^{-33} - 10^{-34} \text{ cm}^2.$$

The lifetime of the  $\chi$  meson with respect to the decay  $\chi \rightarrow \mu^+ + \mu^-$ , computed in perturbation theory, is equal to

$$\tau = 1/f^2 m_\chi.$$

Hence the  $\chi$  meson should decay within a very short time ( $10^{-23}$  to  $10^{-25}$  sec).

In Secs. 2, 3, and 4 we shall consider the possible interactions of the  $\chi$  meson with other par-

ticles. These interactions would imply the possibility of the decays  $\chi \rightarrow \nu + \bar{\nu}$ ,  $\chi \rightarrow n\pi$ ,  $\chi \rightarrow K + \bar{K}$ , etc., the corresponding decay rates being comparable with the decay rate for  $\chi \rightarrow \mu^+ + \mu^-$ .

#### 6. Interaction of the $\chi$ meson with the photon.

Since the  $\chi$  meson and the photon are vector particles, the transition  $\chi \leftrightarrow \gamma$  is possible (Fig. 8).\*



FIG. 8

The perturbation theoretical amplitude for this transition for small  $k^2$  is equal to

$$M_{\chi\gamma} = k^2 (ef/3\pi) \ln(\lambda^2/m_\mu^2),$$

where  $k$  is the four-momentum of the photon and  $\lambda$  is the cut-off parameter. The occurrence of this transition implies that all charged particles (electron, proton, etc.) interact with the  $\chi$  meson, where the coupling constant is equal to

$$f' = (\alpha f / 3\pi) \ln(\lambda^2 / m_\mu^2).$$

For  $\lambda \sim 1000$  Bev we have  $f' = 0.02f$ .

Thus there occurs, for example, an effective four-fermion interaction between the muon and the electron

$$F' (\bar{u}_\mu \gamma_\alpha u_\mu) (\bar{u}_e \gamma_\alpha u_e), \text{ where } F' \approx 10^{-2} F.$$

We also obtain an interaction between the electron and the proton  $F'' (\bar{u}_e \gamma_\alpha u_e) (\bar{u}_p \gamma_\alpha u_p)$  and between two electrons  $F'' (\bar{u}_e \gamma_\alpha u_e) (\bar{u}_e \gamma_\alpha u_e)$ , where  $F'' \approx 10^{-4} F$ .

## 2. INTERACTION OF THE $\chi$ MESON WITH THE MUON AND WITH BARYONS

Up to this point we have only considered the interaction of the  $\chi$  meson with the muon. Let us now discuss what observable effects would arise if the  $\chi$  meson could also interact with strongly interacting particles. In the spirit of the Sakata model it is natural to consider the interaction of the  $\chi$  meson with the baryon current  $J$ . In choosing a baryon current  $J$  which interacts with the  $\chi$  meson there is some arbitrariness. In the following we shall consider two possible forms of the current:

$$J_\alpha = \bar{\Lambda} \gamma_\alpha \Lambda, \quad J_\alpha = \bar{p} \gamma_\alpha p + \bar{n} \gamma_\alpha n.$$

This choice corresponds to a highly symmetric scheme of elementary particles.

\*For a detailed discussion of this point see<sup>[11]</sup>.

### 1. Symmetric scheme of elementary particles.

If there exists only one type of neutrino, then there is a symmetry between the three leptons ( $\mu^-$ ,  $e^-$ ,  $\nu$ ) and the three baryons in the Sakata model ( $\Lambda$ ,  $n$ ,  $p$ ) which shows up in the Lagrangian of the weak interaction, as noticed by Gamba, Marshak, and Okubo.<sup>[12]</sup> This symmetry can be traced down more deeply\* by assuming that all six particles are degenerate if all interactions are "switched off" and that all interactions are of the form of the electromagnetic interaction and are represented by the interaction of a conserved current with a vector field.

The electromagnetic interaction has the form

$$\sqrt{4\pi e} (\bar{e} \gamma_\alpha e + \bar{\mu} \gamma_\alpha \mu - \bar{p} \gamma_\alpha p) A_\alpha.$$

The strong interaction has the form<sup>[14,15]</sup>

$$\sqrt{4\pi} (\bar{\Lambda} \gamma_\alpha \Lambda + \bar{p} \gamma_\alpha p + \bar{n} \gamma_\alpha n) \rho_\alpha,$$

where  $\rho_\alpha$  is the field of the neutral vector mesons (vectons, in the terminology of<sup>[15]</sup>). The switching-on of a strong charge  $g$  separates the baryons from the leptons and gives rise to the baryon masses. The introduction of the electric charges removes the degeneracy between  $e$  and  $\nu$  and between  $n$  and  $p$ , whereas the degeneracy between  $n$  and  $\Lambda$  and between  $e$  and  $\mu$  still persists. This degeneracy can be removed by the  $f$  interaction. Here we can assume that the  $f$  charge is attached either to the  $\Lambda$  hyperon or to the nucleons.

In the first case the  $f$  interaction will be of the form

$$\sqrt{4\pi f} (\bar{\mu} \gamma_\alpha \mu + \bar{\Lambda} \gamma_\alpha \Lambda) \chi_\alpha,$$

and in the second case, of the form

$$\sqrt{4\pi f} (\bar{\mu} \gamma_\alpha \mu + \bar{n} \gamma_\alpha n + \bar{p} \gamma_\alpha p) \chi_\alpha.$$

In this model the  $f$  interaction is responsible for the difference in the masses not only of  $\mu$  and  $e$ , but also of  $\Lambda$  and  $n$ , and the  $f$  charge is a kind of representative of strangeness. Such a scheme, in which all particles are identical if the interactions are turned off and all interactions are universal (the charges of the various particles are identical, unless they are zero), is extremely symmetric and compact and provides a natural explanation of the isotopic invariance and strangeness.

**2. Anomalous scattering of muons.** The extension of the anomalous interaction to the baryons leads to a difference in the scattering of electrons and muons by nucleons and nuclei. This difference is related to the fact that in the scattering

\*Certain other versions of the symmetric scheme have been discussed by the Nagoya group and by Marshak and Okubo.<sup>[13]</sup>

of muons not only the photon graph (Fig. 9), but also the  $\chi$  meson graph (Fig. 10) gives a contribution, whereas the scattering of electrons is described by the photon graph alone. The vertex part  $NN\gamma$  has the form

$$\Gamma_{\alpha}^{(\gamma)} = \gamma_{\alpha} C_{\gamma}(q^2) + q_{\beta} \sigma_{\alpha\beta} M_{\gamma}(q^2),$$

where  $C_{\gamma}(q^2)$  and  $M_{\gamma}(q^2)$  are the electric and magnetic form factors of the nucleon.

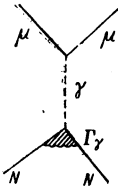


FIG. 9

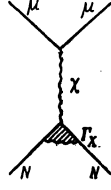


FIG. 10

The vertex part  $NN\chi$  must have an analogous form:

$$\Gamma_{\alpha}^{(\chi)} = \gamma_{\alpha} C_{\chi}(q^2) + q_{\beta} \sigma_{\alpha\beta} M_{\chi}(q^2).$$

The amplitude for  $eN$  scattering, corresponding to the graph of Fig. 9, is equal to

$$e^2 q^{-2} (\bar{u}_e \gamma_{\alpha} u_e) (\bar{u}_N \Gamma_{\alpha}^{(\gamma)} u_N).$$

The amplitude for  $\mu N$  scattering corresponding to the sum of the graphs of Figs. 9 and 10 can be written down in an analogous form:

$$e^2 q^{-2} (\bar{u}_{\mu} \gamma_{\alpha} u_{\mu}) (\bar{u}_N \Gamma_{\alpha}^{(\chi)} u_N),$$

$$\Gamma_{\alpha} = \gamma_{\alpha} C(q^2) + q_{\beta} \sigma_{\alpha\beta} M(q^2).$$

The deviations of the "effective"  $\mu$  meson form factors  $C(q^2)$  and  $M(q^2)$  from the corresponding form factors of the electron are equal to

$$\frac{C(q^2)}{C_{\gamma}(q^2)} = 1 + \frac{f^2}{e^2} \frac{q^2}{q^2 - m_{\chi}^2} \frac{C_{\chi}(q^2)}{C_{\gamma}(q^2)},$$

$$\frac{M(q^2)}{M_{\gamma}(q^2)} = 1 + \frac{f^2}{e^2} \frac{q^2}{q^2 - m_{\chi}^2} \frac{M_{\chi}(q^2)}{M_{\gamma}(q^2)}.$$

These deviations can be of order unity for  $q \sim 1$  Bev. Unfortunately, the cross section is in this case very small on account of the Coulomb form factor. The data<sup>[16]</sup> indicate that for momentum transfers  $q \sim 300$  Mev/c there is no deviation of the observed angular distribution from the pure electrodynamic distribution within an accuracy of 20%.

In the case of the interaction

$$\sqrt{4\pi} f (\bar{p} \gamma_{\alpha} p + \bar{n} \gamma_{\alpha} n) \chi_{\alpha},$$

it is easy to see that

$$\Gamma^{(\chi)}(q^2) = \Gamma^{(\gamma)s}(q^2),$$

where  $\Gamma^{(\gamma)s}$  is the isoscalar component of the form factor  $\Gamma^{(\gamma)}$ .

In the case where the  $\Lambda$  particle interacts with the  $\chi$  meson, the interaction of the  $\chi$  meson with the nucleons can be due only to graphs of the type of Fig. 11. One should expect in this case that

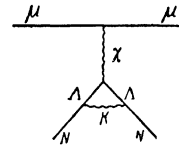


FIG. 11

for nucleons  $C_{\chi} \sim q^2/m_K^2$ ,  $M_{\chi} \sim 1/m_K$ .

**3. Pair production by muons.** The existence of the interaction

$$\sqrt{4\pi} f (\bar{p} \gamma_{\alpha} p + \bar{n} \gamma_{\alpha} n + \bar{\mu} \gamma_{\alpha} \mu) \chi_{\alpha} \quad \text{or} \quad \sqrt{4\pi} f (\bar{\Lambda} \gamma_{\alpha} \Lambda + \bar{\mu} \gamma_{\alpha} \mu) \chi_{\alpha}$$

should lead to the production of pairs  $\mu^+ + \mu^-$  in nuclear collisions. If the energy in the center of mass system exceeds the threshold for creation of a real  $\chi$  meson, such pairs will be produced through creation of a real  $\chi$  meson with subsequent decay  $\chi \rightarrow \mu^+ + \mu^-$ . In practice such a process should look like the emission of a  $\mu^+$ ,  $\mu^-$  pair from a high energy star. For  $f \sim 1$ , the probability for this process relative to multiple production processes will be determined by its statistical weight. If the energy is sufficiently large compared to the threshold for  $\chi$  production, the cross section for production of  $\chi$  can be of the order of one percent of the total strong interaction cross section. For  $f^2 = \alpha$  the probability for  $\chi$  production will be comparable with the probability for emission of hard photons with energies larger than  $m_{\chi}$ . If the energy is insufficient for  $\chi$  production, the contribution of the graph with a virtual  $\chi$  meson relative to the contribution of the corresponding electromagnetic graph will be given by the ratio of  $f^2/m_{\chi}^2$  to  $e^2/k^2$ .

### 3. INTERACTION OF THE $\chi$ MESON WITH THE MUON AND WITH THE MUONIC NEUTRINO

**1. Universality of the weak interaction in the scheme with two neutrinos.** If the  $\mu$  meson is the only lepton with an  $f$  interaction and there exists only one neutrino, then the decay  $\mu \rightarrow e + \gamma$  should occur and the universality of the weak interaction should be strongly violated, as shown above. These difficulties can be avoided by assuming that the neutrinos which appear in the weak current paired with  $\mu$  and  $e$  are different particles.<sup>[7]</sup> Then the weak lepton current is written in the form

$$J_\alpha = (\bar{\mu}\gamma_\alpha(1 + \gamma_5) \nu_\mu) + (e\bar{\gamma}_\alpha(1 + \gamma_5) \nu_e),$$

One further assumes that not only the  $\mu$  meson, but also the neutrino  $\nu_\mu$  has an  $f$  interaction:

$$\sqrt{4\pi f}(\bar{\mu}\gamma_\alpha\mu + \bar{\nu}_\mu\gamma_\alpha\nu_\mu)\chi_\alpha.$$

We note that the  $\gamma_5$  invariance of the  $f$  interaction insures that the anomalous interaction of the  $\nu_\mu$  neutrino will not give it a nonvanishing mass if the bare mass  $m_{\nu_\mu}^0$  is exactly equal to zero.

In the case under consideration the renormalized weak coupling constant is equal to

$$G_\mu = Z_1^{-1}\sqrt{Z_2^{\nu_\mu}Z_2^\nu}G,$$

where  $Z_1$ ,  $Z_2^\mu$ , and  $Z_2^\nu$  are the renormalization constants of the  $\bar{\mu}\nu_\mu$  vertex parts of the weak interactions and of the Green's functions of the muon and the muonic neutrino. The corresponding first order graphs in perturbation theory are shown in Fig. 12. The renormalization will, in general, be different for the vector and axial vector vertices of the  $\bar{\mu}\nu_\mu$  current as a consequence

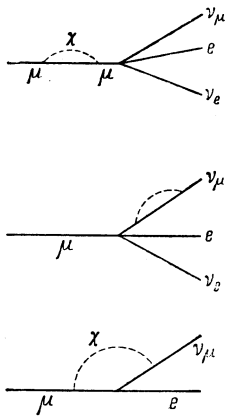


FIG. 12

of the difference between  $Z_1(V)$  and  $Z_1(A)$ . However, it is easily seen that the divergent terms in  $Z_2$  and  $Z_1$  cancel both for  $G_V$  and  $G_A$ . This is connected with the fact that the divergent terms in  $Z_2$  and  $Z_1$ , which are proportional to  $\ln(\Lambda^2/m_\chi^2)$ , occur in the region of virtual momenta  $\Lambda^2 > k^2 > m_\chi^2$ . In this region the mass difference of the muon and neutrino can be neglected and the  $A$  and  $V$  interactions are indistinguishable, so that the divergent terms in  $Z_2$  and  $Z_1$  cancel in the same way as in the case of electrodynamics.

The remaining finite corrections leading to the renormalization of  $G$  and to a removal of the equality of  $G_V^{(\mu)}$  and  $G_A^{(\mu)}$  are of the order

$$f^2(m_\mu^2/m_\chi^2)\ln(m_\chi^2/m_\mu^2)$$

and are small for  $f^2 \leq 1$  ( $m_\mu \ll m_\chi$ ). We note that the existence of an interaction between the  $\chi$  meson and the muon and the muonic neutrino gives rise to a form factor in the  $\bar{\mu}\nu_\mu$  vertex of the  $\mu$  decay in the same way as the strong interaction gives rise to a form factor in the  $\bar{p}n$  vertex in  $\beta$  decay. This may serve as an explanation for the fact that the equality of the  $\beta$  and  $\mu$  decay constants is not destroyed by higher corrections in the weak interaction.

**2. Interaction of the muonic neutrino.** The existence of the interaction  $\sqrt{4\pi f}(\bar{\nu}_\mu\gamma_\alpha\nu_\mu)\chi_\alpha$  would lead to interactions of the muonic neutrino which are considerably stronger than in the case where the only interaction of the neutrino is the usual weak interaction. Sufficiently energetic neutrinos will, in particular, lead to the creation of  $\mu$  meson pairs in the Coulomb field of the nucleus (graph of Fig. 13). The cross section for this process, computed by the Weizsäcker-Williams method, is equal to, for  $E_\nu \gg 2m_\mu^2/q_0$ ,

$$\sigma \sim \frac{e^2 F^2 Z^2 E_\nu q_0}{\pi^3} \ln \frac{q_0 E_\nu}{m_\mu^2},$$

where  $q_0$  is the maximal momentum which can be transferred to the nucleus. For  $F = 1/10 m_N^2$ ,

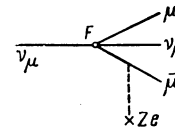


FIG. 13

$Z = 82$ ,  $E_\nu \sim 10$  Bev, this cross section is of the order of  $10^{-32}$  to  $10^{-31}$   $\text{cm}^2$ .

Since the  $\chi$  meson interacts with the electromagnetic currents with a strength given by the constant  $f'$  (see Sec. 1, item 6), the muonic neutrinos will also have a similar interaction with a strength given by the constant  $F'$ . We note that an interaction of this strength is excluded for the electronic neutrinos by the experiment of Reines and Cowen,<sup>[17]</sup> which gives the upper limit  $\sigma < 10^{-42}$   $\text{cm}^2$  for the cross section of the scattering of a  $\beta$  decay neutrino by an electron.

#### 4. INTERACTION OF $\chi$ MESONS WITH MUONS, NEUTRINOS, AND BARYONS

In this section we shall consider the case where the  $\chi$  meson interacts not only with the current  $\bar{\mu}\gamma_\alpha\mu + \bar{\nu}_\mu\gamma_\alpha\nu_\mu$ , but also with the baryon currents

$$\bar{n}\gamma_\alpha n + \bar{p}\gamma_\alpha p \quad \text{or} \quad \bar{\Lambda}\gamma_\alpha \Lambda.$$

In this case the scattering of the neutrino by nucleons will be described by the graph of Fig. 14.

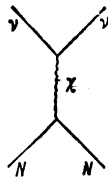


FIG. 14

For an interaction of the first type at small energies, where the nucleon form factors are unimportant, the cross section for the scattering of a neutrino by a nucleon will be equal to

$$\sigma = \frac{F^2}{2\pi} \frac{E^2}{1 + 2E/m_N} \left[ 1 + \frac{4}{3} \frac{(E/m_N)^2}{(1 + 2E/m_N)^2} \right],$$

where  $E$  is the initial energy of the neutrino in the laboratory system and  $m_N$  is the nucleon mass.

Beginning with an energy of order 1 Bev, where the cross section reaches values of order  $10^{-31}$   $\text{cm}^2$  (for  $F = (1/10)m_N^2$ ), the cross section will cease to increase because of the nucleon form factor.

In the scattering of a high energy neutrino by nucleons and nuclei the cross section for inelastic processes with creation of  $\pi$  mesons and strange particles will make up a considerable part of the total cross section. We note also that an interaction of this type can lead to the emission of  $\bar{\nu}, \nu$  pairs by metastable nuclei.

CONCLUSION

On the basis of the preceding analysis of the anomalous interaction of the  $\mu$  meson we can draw the following conclusions:

1. If there exists only one neutrino, one may expect a violation of the universality of the weak interaction.
2. If there exist two neutrinos, one may expect that the muonic neutrino has an anomalous interaction.

The available experimental data do not exclude the possibility of neutrino cross sections of order  $10^{-31}$   $\text{cm}^2$ , so that the corresponding experiments

are not in contradiction with our arguments. One may think that some of our conclusions are not specific to the model considered by us and will therefore also be valid for more general forms of the structure of the anomalous interaction.

The authors thank A. I. Alikhanov, A. A. Ansel'm, V. N. Gribov, Ya. B. Zel'dovich, B. L. Ioffe, V. A. Lyubimov, V. B. Mandel'tsvaig, I. Ya. Pomeranchuk, and B. M. Pontecorvo for numerous discussions of the theoretical questions related to our model and of the possibilities of its experimental verification.

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