# ON THE $\bar{K} + N \to \Lambda(\Sigma) + \gamma$ PROCESS

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Some information on $\Lambda(\Sigma) + \pi \to \Lambda(\Sigma) + \gamma$ processes can be obtained by investigating the $\bar{K} + N \to \Lambda(\Sigma) + \gamma$ reaction. A detailed phenomenological analysis of these processes in the s state is performed. The Kroll-Ruderman theorem for photoproduction of pions on hyperons near threshold is considered.

1. One of the most important problems in elementary-particle physics is the study of interactions between unstable particles, where for lack of an unstable-particle target it becomes necessary to use indirect methods for this purpose.

We have shown earlier [1] that by using the unitarity condition for the S matrix we can establish certain relations between the matrix elements for the processes $\bar{K} + N \to \bar{K} + N$, $\bar{K} + N \to \Lambda(\Sigma) + \pi$ and $\Lambda(\Sigma) + \pi \to \Lambda(\Sigma) + \pi$ for states with arbitrary values of the angular momentum. It is therefore necessary to obtain certain information on the processes $\Lambda(\Sigma) + \pi \to \Lambda(\Sigma) + \pi$ by analyzing the cross sections and polarizations of the baryons in elastic scattering and in reactions involving K mesons and nucleons. Similar conclusions were reached later by other authors. [2,3] In [2] and [3] there is a detailed analysis of elastic scattering and interaction of K mesons with nucleons in the s state. Existing experimental data allow us to establish the phase difference of the s waves in $\pi\Sigma$ scattering with isospin I = 1 and I = 0.

In order to obtain certain information on the electromagnetic and strong interactions of hyperons, we consider in the present article the processes

$$\bar{K} + N \to \Lambda(\Sigma) + \gamma. \tag{1}$$

The S-matrix unitarity conditions cause the matrices for the processes $\pi + \Lambda(\Sigma) \to \Lambda(\Sigma) + \gamma$ to be related with the matrix elements of processes (1).

2. For simplicity we consider the reactions (1) in the s state only. We use the K-matrix method developed in [3]. For our problem it is convenient to use a symmetrical and Hermitian K matrix, expressed in terms of a T matrix with the aid of the relation

$$K = T - i\pi K\rho T = T - i\pi T\rho K, \tag{2}$$

where $\rho$ is the density matrix of the phase volume for the intermediate states with fixed total energy. For two-particle (binary) reactions with a definite angular momentum, the matrix $\rho$ is diagonal. In the relativistic normalization of the wave functions, the diagonal elements of the $\rho$ matrix are

$$\rho_{nn} = M_n k/\pi E, \tag{3}$$

where k is the relative momentum of the particle in the c.m.s., $M_n$ is the mass of the baryons in the intermediate states, and E is the total energy of the system:

$$E = (k^2 + M_n^2)^{1/2} + (k^2 + m^2)^{1/2}. \tag{4}$$

If we introduce the notation

$$K' = \pi\rho^{1/2}K\rho^{1/2}, \qquad T' = \pi\rho^{1/2}T\rho^{1/2}, \tag{5}$$

then Eq. (2) can be rewritten as

$$K' = T' - iK'T' = T' - iT'K'. \tag{6}$$

From (6) we obtain

$$T' = (1 - iK')^{-1} K' = K' (1 - iK')^{-1}. \tag{7}$$

The cross section of reaction (1) expressed in terms of the T′ matrix, in a state with definite angular momentum J and with definite parity, is

$$\sigma (i \to j) = 4\pi k_i^{-2} (J + 1/2) \, |\langle j | T' | i \rangle|^2. \tag{8}$$

Let us consider the submatrices of the introduced K and T matrices, which we denote by

$$\begin{aligned}
\alpha &= \langle \bar{K}N | K | \bar{K}N \rangle, & T_{KK} &= \langle \bar{K}N | T | \bar{K}N \rangle, \\
\beta &= \langle \bar{K}N | K | Y\pi \rangle, & T_{KY} &= \langle \bar{K}N | T | Y\pi \rangle, \\
\beta^+ &= \langle Y\pi | K | \bar{K}N \rangle, & T_{YK} &= \langle Y\pi | T | \bar{K}N \rangle, \\
\gamma &= \langle Y\pi | K | Y\pi \rangle, & T_{YY} &= \langle Y\pi | T | Y\pi \rangle, \\
\xi &= \langle \bar{K}N | K | Y\gamma \rangle, & T_{K\gamma} &= \langle \bar{K}N | T | Y\gamma \rangle, \\
\xi^+ &= \langle Y\gamma | K | \bar{K}N \rangle, & T_{\gamma K} &= \langle Y\gamma | T | \bar{K}N \rangle, \\
\eta &= \langle Y\pi | K | Y\gamma \rangle, & T_{Y\gamma} &= \langle Y\pi | T | Y\gamma \rangle, \\
\eta^+ &= \langle Y\gamma | K | Y\pi \rangle, & T_{\gamma Y} &= \langle Y\gamma | T | Y\pi \rangle, \\
\zeta &= \langle Y\gamma | K | Y\gamma \rangle, & T_{\gamma\gamma} &= \langle Y\gamma | T | Y\gamma \rangle.
\end{aligned} \tag{9}$$

We denote the submatrices of the K′ and T′ matrices by the corresponding primed letters. We neglect the matrix $\zeta$, which is at least one order of magnitude smaller than the other matrices.

If we introduce

$$K_0 = \begin{pmatrix} \alpha & \beta \\ \beta^+ & \gamma \end{pmatrix}, \quad \delta = \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \tag{10}$$

then we can write

$$K = \begin{pmatrix} K_0 & \delta \\ \delta^+ & 0 \end{pmatrix}. \tag{11}$$

From (5), (7), (10), and (11) we readily find that

$$\begin{aligned}
T'_{KK} &= (1 - iX')^{-1} X', \\
T'_{KY} &= (1 - iX')^{-1} \beta' (1 - i\gamma')^{-1} \\
&= (1 - i\alpha')^{-1} \beta' (1 - iZ')^{-1}, \\
T'_{YK} &= (1 - iZ')^{-1} \beta'^T (1 - i\alpha')^{-1} \\
&= (1 - i\gamma')^{-1} \beta'^T (1 - iX')^{-1}, \\
T'_{YY} &= (1 - iZ')^{-1} Z', \ T'_{K\gamma} \\
&= (1 - iX')^{-1} \xi' + i (1 - iX')^{-1} \beta' (1 - i\gamma')^{-1} \eta', \\
T'_{Y\gamma} &= i (1 - iZ')^{-1} \beta'^T (1 - i\alpha')^{-1} \xi' + (1 - iZ')^{-1} \eta', \\
T'_{\gamma K} &= \xi'^T (1 - iX')^{-1} + i\eta'^T (1 - i\gamma')^{-1} \beta'^T (1 - iX')^{-1}, \\
T'_{\gamma Y} &= i\xi'^T (1 - i\alpha')^{-1} \beta' (1 - iZ')^{-1} + \eta'^T (1 - iZ')^{-1},
\end{aligned} \tag{12}$$

where

$$X' = \alpha' + i\beta' (1 - i\gamma')^{-1} \beta'^T,$$
$$Z' = \gamma' + i\beta'^T (1 - i\alpha')^{-1} \beta'. \tag{13}$$

**3.** In our discussion it is sufficient to take into account the electromagnetic interaction in first-order perturbation theory, considering separately the contributions from the iso-scalar and iso-vector parts of the electromagnetic interaction.

We start from the iso-scalar current. In this case the total isospin is $I = 0$ for the $\Lambda + \gamma$ system and $I = 1$ for the $\Sigma + \gamma$ system. We denote by $\xi_\Lambda^0$, $\xi_\Sigma^1$, $\eta_\Lambda^0$, and $\eta_\Sigma^1$ the matrix elements with iso-scalar current for the processes $\bar{K} + N \to \Lambda(\Sigma) + \gamma$ and $\Lambda(\Sigma) + \pi \to \Lambda(\Sigma) + \gamma$, respectively. In the case of the iso-vector current, the total isospin is $I = 1$ for the $\Lambda + \gamma$ system and $I = 0$ or $1$ for the $\Sigma + \gamma$ system. The corresponding matrix elements will be denoted by $\xi_\Lambda^1$, $\xi_\Sigma^1$, $\xi_\Sigma'^1$, $\eta_\Lambda^1$, $\eta_\Sigma^0$, and $\eta_\Sigma'^1$.

Let us consider the channels with isospin $I = 0$. In this case the submatrices $\alpha$, $\beta$, and $\gamma$ are simply numbers. Expressions (13) are then reduced to

$$X = \alpha + i\pi\beta^2 \rho_\Sigma / (1 - i\pi\rho_\Sigma\gamma) = a + ib, \tag{14}$$

where

$$a = \alpha - \pi^2\beta^2\gamma\rho_\Sigma^2 / [1 + \pi^2\rho_\Sigma^2\gamma^2],$$
$$b = \pi\beta^2\rho_\Sigma / [1 + \pi^2\rho_\Sigma^2\gamma^2] > 0. \tag{15}$$

Substituting (14) in (12) we get

$$T'_{KK} = (1 - iX')^{-1} X' = \pi\rho_K (a^0 + ib^0) \Delta_0^{-1},$$
$$T'_{\Sigma K} = \pi^{1/2} \rho_K^{1/2} (b^0)^{1/2} e^{i\lambda_\Sigma} \Delta_0^{-1}, \tag{16}$$

where

$$\tan\lambda_\Sigma = \pi\rho_\Sigma\gamma, \qquad \Delta_0 = 1 - i\pi\rho_K (a^0 + ib^0).$$

Formulas (16) for the processes $\bar{K} + N \to \bar{K} + N$ and $\bar{K} + N \to \Sigma + \pi$ were obtained by many authors.[3]

Let us write

$$T'_{\gamma K} = \xi^T (1 - iX')^{-1} + i\eta^T (1 - i\gamma')^{-1} \beta'^T (1 - iX')^{-1}$$
$$= \begin{pmatrix} T'_{\Lambda\gamma K} \\ T'_{\Sigma\gamma K} \end{pmatrix}, \quad \eta^T = \begin{pmatrix} \eta_{\Lambda\Sigma}^0 \\ \eta_{\Sigma\Sigma}^0 \end{pmatrix}, \quad \xi^T = \begin{pmatrix} \xi_{\Lambda K}^0 \\ \xi_{\Sigma K}^0 \end{pmatrix}. \tag{17}$$

From (14)—(17) we readily find that

$$T'_{\Lambda\gamma K} = \pi\rho_{\gamma\Lambda}^{1/2} \rho_K^{1/2} [\xi_{\Lambda K}^0 + i\eta_{\Lambda\Sigma}^0 \pi^{1/2} \rho_\Sigma^{1/2} (b^0)^{1/2} e^{i\lambda_\Sigma}] \Delta_0^{-1},$$
$$T'_{\Sigma\gamma K} = \pi\rho_{\gamma\Sigma}^{1/2} \rho_K^{1/2} [\xi_{\Sigma K}^0 + i\eta_{\Sigma\Sigma}^0 \pi^{1/2} \rho_\Sigma^{1/2} (b^0)^{1/2} e^{i\lambda_\Sigma}] \Delta_0^{-1}. \tag{18}$$

We note that $T'_{\Lambda\gamma K}$, $T'_{\Sigma\gamma K}$, and $T'_{\Sigma K}$ have almost the same energy dependence in the low-energy region, where (assuming the relative parity of the hyperons to be positive) the energy dependence of $\rho_\Sigma$, $\rho_\Lambda$, $\rho_{\gamma\Lambda}$, and $\rho_{\gamma\Sigma}$ can be neglected.

Let us proceed to examine the channels with isospin $I = 1$. In this case $\gamma$ and $\beta$ are matrices,

$$\gamma = \begin{pmatrix} \gamma_{\Lambda\Lambda} & \gamma_{\Sigma\Lambda} \\ \gamma_{\Lambda\Sigma} & \gamma_{\Sigma\Sigma} \end{pmatrix}, \quad \beta = (\beta_{\Lambda K}, \beta_{\Sigma K}). \tag{19}$$

It is easy to verify that in this case X is simply a complex number

$$X = a^1 + ib^1, \tag{20}$$

where

$$a^1 = \alpha - \pi\beta\rho_Y^{1/2} \frac{1}{1 + \gamma'^2} \gamma' \rho_Y^{1/2} \beta^T, \qquad b^1 = \pi\beta\rho_Y^{1/2} \frac{1}{1 + \gamma'^2} \rho_Y^{1/2} \beta^T \tag{21}$$

From (12), (13), and (19)—(21) it follows that

$$T'_{KK} = \pi\rho_K (a^1 + ib^1) \Delta_1^{-1}, \quad T'_{\Lambda K} = \pi^{1/2} \rho_K^{1/2} (b_{\Lambda K}^1)^{1/2} e^{i\lambda_{\Lambda K}} \Delta_1^{-1},$$
$$T'_{\Sigma K} = \pi^{1/2} \rho_K^{1/2} (b_{\Sigma K}^1)^{1/2} e^{i\lambda_{\Sigma K}} \Delta_1^{-1}, \tag{22}$$

where

$$\pi^{1/2} \rho_K^{1/2} b_{\Lambda K}^{1/2} e^{i\lambda_{\Lambda K}} \equiv \langle \Lambda | (1 - i\gamma')^{-1} \beta'^T | K \rangle,$$
$$\pi^{1/2} \rho_K^{1/2} b_{\Sigma K}^{1/2} e^{i\lambda_{\Sigma K}} \equiv \langle \Sigma | (1 - i\gamma')^{-1} \beta'^T | K \rangle,$$
$$\Delta_1 = 1 - i\pi\rho_K (a^1 + ib^1), \tag{23}$$

and the quantities $b_{\Lambda K}$ and $b_{\Sigma K}$ are related with b by the equation $b_{\Lambda K} + b_{\Sigma K} = b$. If we represent the matrices $\xi$ and $\eta$ in the form

$$\xi = (\xi_{\Lambda K}, \xi_{\Sigma K}), \quad \eta = \begin{pmatrix} \eta_{\Lambda\Lambda} & \eta_{\Sigma\Lambda} \\ \eta_{\Lambda\Sigma} & \eta_{\Sigma\Sigma} \end{pmatrix}, \tag{24}$$

then the matrix elements $T'_{\gamma\Lambda K}$ and $T'_{\gamma\Sigma K}$ become

$$T'_{\gamma\Lambda K} = \pi\rho^{1/2}_{\gamma\Lambda}\rho^{1/2}_{K}\Delta_1^{-1}\,[\xi_{\Lambda K} + i\eta_{\Lambda\Lambda}\pi^{1/2}\rho^{1/2}_{\Lambda}b^{1/2}_{\Lambda K}e^{i\lambda_{\Lambda K}} + i\eta_{\Lambda\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}b^{1/2}_{\Sigma K}e^{i\lambda_{\Sigma K}}], \quad (25)$$

$$T'_{\gamma\Sigma K} = \pi\rho^{1/2}_{\gamma\Sigma}\rho^{1/2}_{K}\Delta_1^{-1}\,[\xi_{\Sigma K} + i\eta_{\Sigma\Lambda}\pi^{1/2}\rho^{1/2}_{\Lambda}b^{1/2}_{\Lambda K}e^{i\lambda_{\Lambda K}} + i\eta_{\Sigma\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}b^{1/2}_{\Sigma K}e^{i\lambda_{\Sigma K}}]. \quad (26)$$

To simplify matters we introduce new symbols

$$\alpha^0_\Lambda = \pi^{1/2}\rho^{1/2}_{\gamma\Lambda}\,[\xi^0_{\Lambda K} + i\eta^0_{\Lambda\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}(b^0)^{1/2}e^{i\lambda_\Sigma}],$$

$$\alpha^0_\Sigma = \pi^{1/2}\rho^{1/2}_{\gamma\Sigma}\,[\xi^0_{\Sigma K} + i\eta^0_{\Sigma\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}(b^0)^{1/2}e^{i\lambda_\Sigma}],$$

$$\alpha^1_\Lambda = \pi^{1/2}\rho^{1/2}_{\gamma\Lambda}\,[\xi^1_{\Lambda K} + i\eta^1_{\Lambda\Lambda}\pi^{1/2}\rho^{1/2}_{\Lambda}(b^1_{\Lambda K})^{1/2}e^{i\lambda_{\Lambda K}} + i\eta^1_{\Lambda\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}(b^1_{\Sigma K})^{1/2}e^{i\lambda_{\Sigma K}}],$$

$$\alpha^1_\Sigma = \pi^{1/2}\rho^{1/2}_{\gamma\Sigma}\,[\xi^1_{\Sigma K} + i\eta^1_{\Sigma\Lambda}\pi^{1/2}\rho^{1/2}_{\Lambda}(b^1_{\Lambda K})^{1/2}e^{i\lambda_{\Lambda K}} + i\eta^1_{\Sigma\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}(b^1_{\Sigma K})^{1/2}e^{i\lambda_{\Sigma K}}],$$

$$\alpha'^1_\Sigma = \pi^{1/2}\rho^{1/2}_{\gamma\Sigma}\,[\xi'^1_{\Sigma K} + i\eta'^1_{\Sigma\Lambda}\pi^{1/2}\rho^{1/2}_{\Lambda}(b^1_{\Lambda K})^{1/2}e^{i\lambda_{\Lambda K}} + i\eta'^1_{\Sigma\Sigma}\pi^{1/2}\rho^{1/2}_{\Sigma}(b^1_{\Sigma K})^{1/2}e^{i\lambda_{\Sigma K}}], \quad (27)$$

with which the cross sections of the processes (1) can be written in the following form:

| Process: | Cross section: |
|---|---|
| $K^- + p \to \Lambda^0 + \gamma$, $\overline{K}^0 + n \to \Lambda^0 + \gamma$ | $\dfrac{2\pi m_K}{E_K k}\left\|\dfrac{\alpha^0_\Lambda}{\Delta_0} \pm \dfrac{\alpha^1_\Lambda}{\Delta_1}\right\|^2,$ |
| $K^- + p \to \Sigma^0 + \gamma$, $\overline{K}^0 + n \to \Sigma^0 + \gamma$ | $\dfrac{2\pi m_K}{E_K k}\left\|-\dfrac{\alpha^0_\Sigma/\sqrt{3}}{\Delta_0} \pm \dfrac{\alpha^1_\Sigma}{\Delta_1}\right\|^2,$ |
| $K^- + n \to \Sigma^- + \gamma$, $\overline{K}^0 + p \to \Sigma^+ + \gamma$ | $\dfrac{2\pi m_K}{E_K k}\left\|\dfrac{\alpha^1_\Sigma \pm \alpha'^1_\Sigma/\sqrt{2}}{\Delta_1}\right\|^2.$ |

Thus, the experimental investigation of the processes $\overline{K} + N \to \Lambda(\Sigma) + \gamma$ in $\overline{K}p$ and $\overline{K}d$ collisions can yield certain information on the matrix elements $\alpha_\Lambda$ and $\alpha_\Sigma$. Naturally, this information is not sufficient to reconstitute the matrix elements $\xi$ and $\eta$, which describe the photoproduction of mesons on hyperons. Nonetheless they may prove useful for a study of the interaction between hyperons and mesons or photons.

**4.** A powerful method for the analysis of strong interactions is the method of dispersion relations (d.r.), the use of which yields in many cases interesting results in the low-energy region. It can be assumed that the d.r. method is applicable to the photoproduction of mesons and hyperons. In the present paper we confine ourselves to a generalization of the Kroll-Ruderman theorem for photoproduction of pions near threshold.[4]

Let us assume that the $\Lambda$ and $\Sigma$ hyperons have a positive relative parity and that the K meson is pseudoscalar. If the created particles have low energies account of the electric dipole radiation is sufficient. The generalized Kroll-Ruderman theorem states that, accurate to $m_\pi/M \approx 15\%$, the matrix for the electric dipole transition is determined completely by the pion-hyperon coupling constant.

Let us write the Hamiltonian of the pion-hyperon interaction in the form

$$\mathscr{H} = ig_{\Sigma\Lambda}\overline{\psi}_\Sigma\gamma_5\psi_\Lambda\psi_\pi + ig_{\Sigma\Sigma}([\overline{\psi}_\Sigma\gamma_5\psi_\Sigma]\,\psi_\pi) + \text{Herm. conj.} \quad (28)$$

Following Low's method[5] we can obtain

$$\eta^0_{\Lambda\Sigma} \sim m_\pi/M, \quad \eta^1_{\Sigma\Lambda} \sim m_\pi/M, \quad \eta^1_{\Sigma\Sigma} \sim m_\pi/M,$$

$$\eta'^1_{\Sigma\Lambda} = \eta'^1_{\Lambda\Sigma} = \sqrt{2}\,\alpha^{1/2}f_{\Sigma\Lambda}\,[1 + O(m_\pi/M)],$$

$$\eta'^1_{\Sigma\Sigma} = \alpha^{1/2}f_{\Sigma\Sigma}\,[1 + O(m_\pi/M)], \quad \eta^0_{\Sigma\Sigma} \approx m_\pi/M.$$

Here $m_\pi$ is the pion mass, M is the hyperon mass, $\alpha = \epsilon^2/4\pi = 1/137$, and $f^2 = g^2/8\pi M$.

[1] L. I. Lapidus and Chou Kuang-chao, JETP **37**, 283 (1959), Soviet Phys. JETP **10**, 199 (1960).

[2] Jackson, Ravenhall, and Wyld, Nuovo cimento **9**, 834 (1958). R. H. Dalitz and S. F. Tuan, Ann. of Physics **8**, 100 (1959). M. Ross and G. Snow, Phys. Rev. **115**, 1773 (1959).

[3] R. H. Dalitz and S. F. Tuan, Ann. of Physics **10**, 307 (1960). P. T. Mathews and A. Salam, Nuovo cimento **13**, 382 (1959). J. D. Jackson and H. Wyld, Nuovo cimento **13**, 84 (1959).

[4] K. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

[5] F. E. Low, Phys. Rev. **97**, 1392 (1955).

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