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NUCLEAR RECOIL IN THE EQUIVALENT PHOTON METHOD

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The region of nuclear recoil q^2, in which the Weizsäcker-Williams relation is valid, is investigated by considering the bremsstrahlung of an electron in the nuclear field as an example. It is shown that the region of permissible values of q^2 depends on whether the cross section is averaged over the directions of nuclear recoil or not. It is also demonstrated that the Weizsäcker-Williams relation is violated in a narrow region of q^2 corresponding to strictly forward radiation by the electron. The treatment is of a completely covariant nature.

1. INTRODUCTION

THE method of equivalent photons was proposed initially [1,2] for the calculation of several electromagnetic processes. The justification of the method was semi-qualitative and its field of application was limited to the calculation of total cross sections. In several recent papers [3-5] this method is used to solve a new group of problems, including those involving processes with strong interactions [4] and the calculation of radiative corrections. [5] In these papers, however, the method is applied to cross sections averaged over the polarizations and integrated over the nuclear recoil directions. The recent results [6] on the covariant form of the equivalent photon spectrum, for an arbitrary process in the field of a heavy particle, make it possible for the first time to take covariant account of the dependence of the cross section on the direction of the nuclear recoil. The approximations used in the equivalent-photon method have likewise not been examined before.

We show in the present paper that the derivation of the fundamental relation of the method (the Weiszäcker-Williams relation), connecting the cross section $d\sigma_b$ of the considered process* and the cross section $d\sigma_C$ of the photo process,

$$d\sigma_b = N d\sigma_C, \tag{1.1}$$

entails two approximations.

1) Separation of the equivalent-photon spectrum N in $d\sigma_b$. It is shown in [6] that the separation of the spectrum N is connected only with a new gauge for the field of the heavy particle. This means that by using the gauge-invariance condition we can rewrite (1.1) in the form

$$d\sigma_b = N d\sigma_q, \tag{1.2}$$

where the cross section $d\sigma_q$ describes a process that differs from the photoprocess in that $q^2 \neq 0$ for the incoming photon (q^2 is the invariant square of the nuclear recoil momentum). The polarization $\tilde{e}$ of the incoming photon is determined here by the direction of the nuclear recoil (Sec. 4).

2) Replacement of the cross section $d\sigma_q$ by its value at the "pole" when $q^2 = 0$, i.e., by $d\sigma_C$. The conditions under which such extrapolation is possible were not made clear and this problem can apparently not be analyzed in general form. Furthermore, the limit of applicability of this method for the given process will depend, for example, on whether the cross section under consideration is averaged over the polarization or not.

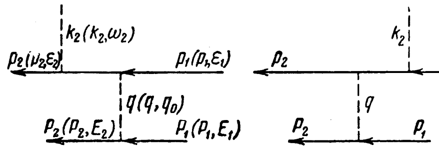
We investigate here the nuclear recoil region where relation (1.1) holds, using as an example the bremsstrahlung of an electron in the field of the nucleus. We consider two cases: a) $d\sigma_b$ depends on the direction of the nuclear recoil (Secs. 2–6), in which case the incident photon is polarized in the corresponding Compton effect; b) $d\sigma_b$ is averaged over the nuclear recoil directions (Appendix), and then the corresponding cross section $d\sigma_C$ is averaged over the polarization of the incident photon. We shall show that 1) if the term in $d\sigma_b$ dependent on the direction of the nuclear recoil is small, the limits of applicability of the method are the same in cases a and b; 2) if this term in $d\sigma_b$ is appreciable, the limits of applicability of (1.1) change, the region of permissible values of q^2 becoming narrower; 3) for a relativistic electron when the frequency of the radiated photon is of the same order as the energy of the incoming electron, relation (1.1) is violated in a narrow region corresponding to strictly forward radiation.

*We consider a certain process which includes exchange of one virtual photon upon collision of two particles, of which one is heavy (nucleus).

The entire analysis in the present paper is completely covariant.

2. BREMSSTRAHLUNG CROSS SECTION

The bremsstrahlung of an electron in the field of the nucleus corresponds to two diagrams (see the figure). If the nuclear spin is assumed to be zero, then the differential cross section of the



process (after averaging over the electron spin before and after the scattering and polarization of the photon) has the following form (see the figure for notation)

$$d\sigma_b = \frac{Z^2\alpha^3}{\pi^2} \frac{F_{\alpha\beta}P_\alpha P_\beta}{q^4} \frac{1}{(J\varepsilon_1 E_1)} \frac{d\mathbf{p}_2}{2\varepsilon_2} \frac{d\mathbf{k}_2}{2\omega_2} \frac{d\mathbf{P}_2}{2E_2} \delta^4(q + p_1 - k_2 - p_2), \quad (2.1)$$

$$q = P_1 - P_2. \quad (2.2)$$

$J\epsilon_1 E_1$ assumes the role of invariant current density of the colliding particles (with masses m and M):

$$J\varepsilon_1 E_1 = [(p_1 P_1)^2 - m^2 M^2]^{1/2} \quad (2.3)$$

(the density of the electrons and nuclei in their rest system is assumed equal to unity). In the coordinate system in which one of the particle is at rest (prior to collision) J is the velocity of the second particle.

The 4-vector P, namely

$$P = P_1 + P_2, \quad (2.4)$$

defines the polarization of the pseudophoton (virtual quantum). The Lorentz condition is satisfied here:

$$qP = 0. \quad (2.5)$$

We note, however, that $P^2 < 0$ and therefore the normalization condition $P^2 = 1$, which usually applies to polarization vectors, cannot be imposed on this vector.

We introduce the usual invariant variables

$$m^2\varkappa_1 = q^2 + 2qp_1 = 2k_2p_2, \quad m^2\varkappa_2 = q^2 - 2qp_2 = -2k_2p_1,$$
$$m^2\varkappa_1 = -(w^2 - m^2) < 0, \quad (2.6)$$

where w is the total energy of the final photon and electron ($ab = \mathbf{a} \cdot \mathbf{b} - a_0 b_0$). In these variables the matrix $F_{\alpha\beta}$ is written in the form

$$F_{\alpha\beta} = -\frac{\varkappa_1}{\varkappa_2}\left(1 - \frac{q^2}{m^2\varkappa_1}\right)^2 F_{\alpha\beta}^{(1)} - \frac{\varkappa_2}{\varkappa_1}\left(1 - \frac{q^2}{m^3\varkappa_2}\right)^2 F_{\alpha\beta}^{(2)}$$
$$- \frac{8}{\varkappa_1^2\varkappa_2^2}\xi_\alpha\xi_\beta + \frac{q^2}{m^2}\left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right) F_{\alpha\beta}^{(0)}; \quad (2.7)$$

$$F_{\alpha\beta}^{(1,2)} = \delta_{\alpha\beta} - (qp_i)^{-1}(q_\alpha p_{i\beta} + q_\beta p_{i\alpha})$$
$$+ q^2 (qp_i)^{-2} p_{i\alpha} p_{i\beta}, \quad i = 1, 2,$$

$$\xi_\alpha = m^{-1}\left(\varkappa_2 p_{1\alpha} + \varkappa_1 p_{2\alpha} - \frac{1}{2} q_\alpha (\varkappa_1 - \varkappa_2)\right),$$

$$F^{(0)} = -2(1/\varkappa_1 + 1/\varkappa_2)^2. \quad (2.8)$$

The symmetrical matrix $F_{\alpha\beta}$ has the gauge-invariance property

$$F_{\alpha\beta}q_\beta = 0. \quad (2.9)$$

The same property is possessed by all the quantities in (2.7). If we use the Lorentz condition (2.5), then some of the terms in the cross section vanish after multiplication by $P_\alpha P_\beta$.

3. COMPTON-EFFECT CROSS SECTION

The formula for the differential cross section of the Compton scattering can be written in a form similar to (2.1) (with averaging over the spins of the electron and polarization of the final photon)

$$d\sigma_C = \frac{4\alpha^2}{m^2|\varkappa_1|}\Phi_{\alpha\beta}e_\alpha e_\beta \frac{d\mathbf{p}_2}{2\varepsilon_2}\frac{d\mathbf{k}_2}{2\omega_2}\delta^4(k_1 + p_1 - k_2 - p_2). \quad (3.1)$$

The notation here is analogous to that of (2.6)—(2.9). The expression for $\Phi_{\alpha\beta}$ is obtained from (2.8) with $q^2 = 0$:

$$\Phi_{\alpha\beta} = -\frac{\varkappa_1}{\varkappa_2}\Phi_{\alpha\beta}^{(1)} - \frac{\varkappa_2}{\varkappa_1}\Phi_{\alpha\beta}^{(2)} - \frac{8}{\varkappa_1^2\varkappa_2^2}\xi_\alpha\xi_\beta;$$

$$\Phi_{\alpha\beta}^{(1,2)} = \delta_{\alpha\beta} - (k_1 p_i)^{-1}(k_{1\alpha}p_{i\beta} + k_{1\beta}p_{i\beta}), \quad i = 1, 2,$$
$$\xi_\alpha = m^{-1}(\varkappa_2 p_{1\alpha} + \varkappa_1 p_{2\alpha}), \quad (3.2)$$

where k_1 is the wave vector of the incoming quantum, $-m^2\kappa_1/2 = J\epsilon_1\omega_1$ is in this case an invariant current obtained from (2.3) with M = 0, and e is the polarization of the incoming quantum ($e^2 = 1$).

The usual expression for the Compton effect is obtained by leaving out of (3.1) the terms that do not contribute to the cross section by virtue of the Lorentz gauge. We then have, in particular,

$$\Phi_{\alpha\beta}e_\alpha e_\beta = -\frac{\varkappa_1}{\varkappa_2} - \frac{\varkappa_2}{\varkappa_1} - 8\frac{[\varkappa_2(p_1 e) + \varkappa_1(p_2 e)]^2}{m^2\varkappa_1^2\varkappa_2^2}. \quad (3.3)$$

4. EQUIVALENT-PHOTON SPECTRUM

The gauge-invariance condition (2.1) enables us to introduce in place of the time-like polarization vector P a new space-like vector $\tilde{e}$. Following[6], we define this vector in such a way that its scalar component vanishes in the frame where the electron is at rest, i.e., $\tilde{e}$ is subject to the condition

$$\tilde{e}p_1 = 0, \quad (4.1)$$

$$\tilde{e} = f^{-1}(P - q(p_1P/p_1q)) \quad (\tilde{e}^2 = 1), \quad (4.2)$$

$$f = [P^2 + q^2 (p_1P/p_1q)^2]^{1/2}. \quad (4.3)$$

The pseudophoton polarization vector $\tilde{e}$ no longer satisfies the Lorentz condition (2.5)

$$\tilde{e}q \neq 0. \quad (4.4)$$

We therefore separate from q a vector q_1 orthogonal to $\tilde{e}$:

$$q = q_1 + q_2, \quad q_1 = q - \tilde{e}(\tilde{e}q), \quad (4.5)$$

$$q_1\tilde{e} = 0. \quad (4.6)$$

The vector q_1 is time-like

$$q_1^2 = q^2P^2/f^2 < 0. \quad (4.7)$$

If along with making the gauge transformation $P \rightarrow f\tilde{e}$, we replace P_2 in the bremsstrahlung cross section (2.1) by q and make the substitution ($P = 2P_1 - q$)

$$d\mathbf{P}_2/2E_2 \rightarrow d^4P_2\delta(P_2^2 + M^2) \rightarrow d^4q\delta(qP),$$

it becomes convenient to rewrite (2.1) in the form of (1.2), with

$$d\sigma_q = \frac{2\alpha^2}{|qp_1|} F_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta \frac{d\mathbf{p}_2}{2\varepsilon_2}\frac{d\mathbf{k}_2}{2\omega_2}\delta^4(q + p_1 - p_2 - k_2), \quad (4.8)$$

$$N = \frac{Z^2\alpha}{2\pi^2}\frac{|qp_1|}{(J\varepsilon_1E_1)}\frac{f^2}{q^4}d^4q\delta(qP). \quad (4.9)$$

$d\sigma_q$ describes a process that differs from the Compton effect only in that $q^2 \neq 0$ for the incoming photon, whose polarization does not satisfy the Lorentz condition. N is the differential spectrum of the pseudophotons, written in fully covariant form. Formula (4.9) coincides in fact with the expression given in [6] for the spectrum [(formula (6)]. They differ only in the normalization of the cross section and of the square of the matrix element.

We note that (4.9) coincides with the conventional expression for the spectrum [see, for example,[4] Eq. (2.16)] if the following conditions are satisfied

$$m^2M^2 \ll (p_1P_1)^2, \quad |qp_1| \ll |p_1P_1|. \quad (4.10)$$

The first condition in (4.10) denotes* that the velocity of the electron relative to the nucleus is $v \sim 1$, while the second condition is that of the external field. We shall not repeat here the known expressions for the spectrum.

*Conditions (4.10) can apply not only to the case of a heavy nucleus ($M \rightarrow \infty$), but also to the analogous process occurring upon collision of particles of equal mass, say two electrons (if $v \rightarrow 1$).

5. LIMITS OF APPLICABILITY OF THE WEIZSÄCKER-WILLIAMS RELATION

Relation (1.2) is exact, while (1.1) is approximate. It holds only in that region of nuclear recoil (q^2) where $d\sigma_q$ coincides with $d\sigma_C$. We shall determine below the region of such values of q^2 and show that in this region $d\sigma_q$ determines the cross section for Compton scattering of a photon with wave vector q_1 and polarization $\tilde{e}$.

The Compton-effect cross section [see (3.3)] has two terms, one independent of the photon polarization

$$-\varkappa_1/\varkappa_2 - \varkappa_2/\varkappa_1 \equiv F_{12}, \quad (5.1)$$

and another determined by e:

$$-8[\varkappa_2(ep_1) + \varkappa_1(ep_2)]^2/m^2\varkappa_2^2\varkappa_1^2 \equiv -F_3. \quad (5.2)$$

We consider the Compton scattering photons with $e = \tilde{e}$. Therefore, by virtue of (4.1),

$$F_3 = 8(\tilde{e}p_2)^2/m^2\varkappa_2^2. \quad (5.3)$$

The conditions under which $d\sigma_q$ coincides with $d\sigma_C$ will be considered separately for the case when a) $F_3 \ll F_{12}$ and b) $F_3 \sim F_{12}$. Only in case b) can the dependence of the cross section on the direction of the nuclear recoil $\tilde{e}$ be accounted for in the method of equivalent photons.

The cross section $d\sigma_q$ [cf. (2.7) and (3.2)] contains, in addition to the corresponding Compton terms, the term

$$F^{(0)}m^{-2}(q^2 - (\tilde{e}q)^2) \equiv F^{(0)}q_1^2/m^2, \quad (5.4)$$

which is missing from $d\sigma_C$. It is obvious that $d\sigma_q$ and $d\sigma_C$ can be regarded as equal only in that region of q^2 where this term is small compared with the other terms that determine $d\sigma_q$. This condition is explained in Sec. 6.

Before we proceed to cases a) and b) we note that by considering the limits of variation of the parameters κ_1 and κ_2,

$$\varkappa_{2\,max} = -\varkappa_1, \quad \varkappa_{2\,min} = \varkappa_1/(\varkappa_1 - 1), \quad (5.5)$$

we can readily show that

$$F_{12} \geqslant 2, \quad F_3 \leqslant 2.$$

It also follows from (5.5) that

$$\varkappa_2 \sim |\varkappa_1| \quad |\varkappa_1| \lesssim 1,$$
$$1 \lesssim \varkappa_2 \leqslant |\varkappa_1| \quad |\varkappa_1| \gg 1. \quad (5.6)$$

We shall assume further that $\kappa_2 \sim |\kappa_1|$ and consequently $F_{12} \sim 1$, after which we shall consider the case

$$\varkappa_2 \ll |\varkappa_1|, \tag{5.7}$$

which is possible when $|\kappa_1| \gg 1$.

It follows from (5.5) and (5.6) that

$$\text{a) } F_3 \ll 1, \quad |\tilde{e}p_2| \ll m\varkappa_2, \tag{5.8}$$

$$\text{b) } F_3 \sim F_{12} \sim 1, \quad |\tilde{e}p_2| \sim m\varkappa_2. \tag{5.9}$$

Case a). Comparing the coefficients preceding $F^{(1,2)}_{\alpha\beta}$ and $\Phi^{(1,2)}_{\alpha\beta}$ in (2.7) and (3.2), we obtain the first condition for q^2:

$$q^2 \ll m^2 |\varkappa_1|. \tag{I'}$$

(We shall therefore no longer distinguish between $m^2\kappa_1$ and $2qp_1$ or $m^2\kappa_2$ and $-2qp_2$.)

It is obvious that by virtue of condition (4.1) and the Lorentz condition ($\tilde{e}k_1 = 0$) we have

$$\Phi^{(1,\,2)}_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta = F^{(1)}_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta = 1. \tag{5.10}$$

But

$$F^{(2)}_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta = 1 + (\tilde{e}q)(\tilde{e}p_2)/qp_2 + q^2(\tilde{e}p_2)^2/(qp_2)^2. \tag{5.11}$$

Equations (5.10) and (5.11) can be regarded as equivalent by stipulating that the second and third terms (5.11) be much smaller than unity.

Let us estimate the absolute value of $\tilde{e}q$ in (5.11):

$$\tilde{e}q = -q^2 f^{-1}(p_1 P)/p_1 q. \tag{5.12}$$

If we assume [see (4.3)]*

$$f^2 \approx q^2 (p_1 P)^2/(p_1 q)^2, \tag{5.13}$$

then

$$|\tilde{e}q| \approx \sqrt{q^2} \tag{5.14}$$

and the third term in (5.11) can be regarded as the square of the second term. It is therefore sufficient to require that

$$\sqrt{q^2}\,|\tilde{e}p_2/qp_2| \ll 1. \tag{II'}$$

By virtue of (5.8), condition (II') is satisfied if (I') is satisfied, and we can assume that the first two terms in (2.8) coincide with F_{12}.

Let us consider the third term in (2.7):

$$8\,\frac{(\tilde{e}\xi)^2}{m^2\varkappa_1^2\varkappa_2^2} = 8\,\frac{\left[\varkappa_1(\tilde{e}p_2) - \frac{1}{2}(\tilde{e}q)(\varkappa_1 - \varkappa_2)\right]^2}{m^2\varkappa_1^2\varkappa_2^2}. \tag{5.15}$$

According to condition (5.8), the term with $(\tilde{e}p_2)^2$ is small compared with F_{12}. The other two terms with $\tilde{e}q$ in (5.15) can be neglected if

$$q^2 \ll m^2\varkappa_1^2, \tag{I''}$$

$$\sqrt{q^2}\,|\tilde{e}p_2| \ll m^2\varkappa_1^2. \tag{II''}$$

Condition (II″) is weaker than (I″). Condition (I″) supplements (I′) if $|\kappa_1| \ll 1$. Combining (I′) and (I″) we conclude that in case a) the equivalent-photon method is applicable in the nuclear-recoil region defined by the condition

$$q^2 \ll \begin{cases} m^2\varkappa_1^2, & |\varkappa_1| \lesssim 1 \\ m^2|\varkappa_1|, & |\varkappa_1| \gg 1 \end{cases}. \tag{Ia}$$

The Appendix contains similar conditions for the case of cross sections integrated over the nuclear-recoil directions. It follows from (Ia) and (A.1) that these conditions coincide in case a).

Case b). In this case, conditions (I′)—(II′) and (I″)—(II″) obviously remain in force. However, unlike case a), fulfillment of (I′) and (I″) does not guarantee fulfillment of (II′)—(II″), since $|\tilde{e}p_2| \sim m\kappa_2$. We must stipulate in addition that the term with $\tilde{e}q$ in (5.15) be small compared with F_3, i.e.,

$$\sqrt{q^2} \ll |\tilde{e}p_2| \sim m\varkappa_2, \quad \varkappa_2 \sim -\varkappa_1,$$

or

$$\sqrt{q^2} \ll m|\varkappa_1|. \tag{5.16}$$

Combining conditions (I) and (II) with (5.8) and (5.16), we find that in case b) relation (1.1) is valid if

$$\sqrt{q^2} \ll \begin{cases} m|\varkappa_1|, & |\varkappa_1| \leqslant 1 \\ m, & |\varkappa_1| \gg 1 \end{cases}. \tag{Ib}$$

Thus, in case b) there exists a nuclear-recoil region (Ib) where on the one hand we can neglect the nuclear recoil, i.e., q^2, and on the other hand we can take into account the nuclear-recoil direction, which is characterized by the polarization vector $\tilde{e}$. However, region (Ib) of q^2 is found to be narrower than in case a), where the nuclear recoil is not considered at all.

We have not considered as yet the range $\kappa_2 \ll |\kappa_1|$. It is obvious from the form of $F_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta$ that in this case the main contribution to the cross section is due to the term with $F^{(1)}_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta$, the value of which is

$$-\varkappa_1/\varkappa_2 \gg 1.$$

Then in the entire permissible range of q^2 we have

$$q^2 \ll m^2|\varkappa_1| \tag{5.17}$$

or, since $|\kappa_1| \gg 1$,

$$q^2 \lesssim m^2.$$

*It will be shown below that assumption (5.13) excludes only a narrow region of values of q^2, corresponding to strictly forward radiation of a relativistic electron.

we can neglect the contributions of the other terms in $d\sigma_q$ (and $d\sigma_C$). Condition (5.17) is weaker than (Ia).

6. VIOLATION OF THE WEIZSÄCKER-WILLIAMS RELATIONS IN FORWARD RADIATION OF A RELATIVISTIC ELECTRON

The requirement that the term

$$F^{(0)}q_1^2/m^2, \qquad F^{(0)} = -2\,(1/\varkappa_1 + 1/\varkappa_2)^2,$$

in $d\sigma_q$, a term missing from the Compton-effect cross section, be small compared with the other terms of $d\sigma_2$, leads to another limitation, in addition to (Ia) [or (b)], on the limits of applicability of the Weizsäcker-Williams relation (1.1).

The following relation will be useful in the subsequent estimates:

$$f^2\,(qp_1)^2 \geqslant -q^2P^2m^2. \qquad (6.1)$$

(This inequality can be readily proved by changing to a frame in which the electron is at rest; (6.1) then goes into the condition $q_\perp^2 \geq 0$, where $q_\perp$ is the q component perpendicular to the vector p_1). The equality sign in (6.1) corresponds to the minimum and maximum values of q^2 for a specified qp_1. The quantities q^2_{max} that follow from (6.1) do not satisfy condition (Ia) and are of no interest to us.

Using (4.3) and introducing $1 - v^2 \equiv -m^2P^2/(p_1P)^2$, we rewrite (6.1) in the form

$$m^2q^2v^2/(1-v^2) \geqslant (qp_1)^2. \qquad (6.2)$$

Condition (6.2) imposes an upper limit on the permissible range of $|qp_1|$. We note that conditions (Ia) and (6.2) are compatible if

$$1 - v^2 \ll 1 \text{ for } |\varkappa_1| \lesssim 1 \quad \text{or } |\varkappa_1|\,(1-v^2) \ll 1 \text{ for } |\varkappa_1| \gg 1. \qquad (6.3)$$

The equality in (6.1) determines the minimum value of q^2 corresponding to forward radiation. We obtain from (6.1)

$$|q_1|^2 = -q^2P^2/f^2 \leqslant (qp_1)^2/m^2 \qquad (6.4)$$

$$F^{(0)}q_1^2/m^2 \lesssim (\varkappa_1 + \varkappa_2)^2/2\varkappa_1^2, \qquad m^2\varkappa_1 \approx 2qp_1. \qquad (6.5)$$

When this term is small, there are two possibilities.

$$1)\ |\varkappa_1 + \varkappa_2| \ll |\varkappa_1|, \qquad \varkappa_2 \sim -\varkappa_1.$$

It follows from (6.5) and (5.5) that in this case $F^{(0)}q_1^2/m^2 \ll F_{12}$, and can be neglected, regardless of the value of q_1^2. Condition (6.6) holds for radiation of a nonrelativistic electron* and when the relativistic electron radiates photons with frequency $\omega \ll \epsilon_1$.

$$2)\ |\varkappa_1 + \varkappa_2| \sim |\varkappa_1|. \qquad (6.7)$$

This condition corresponds to an emitted photon with energy ω (in the laboratory system) of the same order as the energy ϵ_1 of the initial electron ($\epsilon_1 \gg m$). Then

$$F^{(0)}q_1^2/m^2 \sim 2\,|q_1^2|/\varkappa_2^2m^2. \qquad (6.8)$$

For forward radiation, i.e., when $q^2 = q^2_{min}$, it follows from (4.7) that

$$F^{(0)}q_1^2/m^2 \sim \varkappa_1^2/\varkappa_2^2. \qquad (6.9)$$

Since $|\kappa_1| > \kappa_2$, this term is either greater than or of the same order as F_{12}, and cannot be neglected. The region where the term $F^{(0)}q_1^2/m^2$ is significant is very narrow

$$\Delta q^2 \approx q^2\,(1 - v^2).$$

Outside this region we can assume $f^2 \approx q^2(p_1P)^2/(p_1q)^2$ and obtain from (4.7)

$$q_1^2 \approx -(qp_1)^2\,(1 - v^2)/m^2. \qquad (6.10)$$

Substituting (6.10) in (6.8) we obtain

$$F^{(0)}q_1^2/m^2 \approx \varkappa_1^2\,(1 - v^2)/\varkappa_2^2. \qquad (6.11)$$

This term is small compared with F_{12} if [see (6.6) and (6.7)]

$$1 - v^2 \ll \begin{cases} 1, & |\varkappa_1| \lesssim 1 \\ 1/|\varkappa_1|, & |\varkappa_1| \gg 1 \end{cases}. \qquad \text{(III)}$$

[Conditions (III) coincide with (6.3)].

Thus, when conditions (III) are satisfied, we can replace for a relativistic electron $d\sigma_q$ by the Compton scattering cross section $d\sigma_C$, in the entire range of q^2 allowed by conditions (Ia) [or (Ib)], except for the narrow region corresponding to forward radiation.

In conclusion, the author is deeply grateful to Ya. A. Smorodinskii for constant interest in the work and for valuable comments.

APPENDIX

We give here an invariant expression for the bremsstrahlung cross section $d\bar{\sigma}_b$, integrated

*It must be noted that the limitations considered earlier (Sec. 5) are not connected with any conditions whatever on the electron velocity. Therefore, if we take for the spectrum N in (1.1) its exact expression (4.9), we can speak of the equivalent-photon method in the nonrelativistic region for the case of q^2 defined by conditions (1b) [or (1a)].

over the nuclear recoil directions [we have assumed that $1 - v^2 \equiv -m^2P^2/(p_1P)^2 \approx m^2M^2/(p_1P_1)^2$]:

$$d\bar{\sigma}_b = N d\bar{\sigma}_q, \tag{A.1}$$

$$N = \frac{Z^2\alpha}{\pi}\frac{dq^2}{q^4}\frac{d(qp_1)}{qp_1}\{q^2 - (qp_1)^2(1 - v^2)\}, \tag{A.2}$$

$$d\bar{\sigma}_q = 2\pi r_0^2 \frac{m^2}{2|qp_1|}\frac{d\varkappa_2}{W^{5/2}}\bar{F}_{\alpha\beta}\tilde{e}_\alpha\tilde{e}_\beta, \tag{A.3}$$

where

$$W = [4m^{-2}q^2 + (\varkappa_1 - m^{-2}q^2)^2],$$

$$\bar{F}_{\alpha\beta} = -m^{-2}q^2(\delta_{\alpha\beta} - q_\alpha q_\beta/q^2)\varphi_1(q^2, \varkappa_1, \varkappa_2) + (\varkappa_1 - m^{-2}q^2)^2 \times\left(\delta_{\alpha\beta} - \frac{p_{1\alpha}q_\beta + p_{1\beta}q_\alpha}{qp_1} + q^2\frac{p_{1\alpha}p_{1\beta}}{(qp_1)^2}\right)\varphi_2(q^2, \varkappa_1, \varkappa_2),$$

$$\begin{aligned}\varphi_1 = {}& 4\varkappa_1^{-1}\varkappa_2^{-2}\{4\varkappa_1^4 - 2\varkappa_1^4\varkappa_2 - \varkappa_1^4\varkappa_2^2 - 12\varkappa_1^3 + 20\varkappa_1^3\varkappa_2 - 2\varkappa_1^3\varkappa_2^2 \\ & - \varkappa_1^3\varkappa_2^3 - 24\varkappa_1^2\varkappa_2 + 16\varkappa_1^2\varkappa_2^2 + 2\varkappa_1^2\varkappa_2^3 - 12\varkappa_1\varkappa_2^2 \\ & + q^2m^{-2}(-8\varkappa_1^3 + 8\varkappa_1^3\varkappa_2 - 2\varkappa_2^3\varkappa_1 + 3\varkappa_1^3\varkappa_2^2 + 16\varkappa_1^2 \\ & - 42\varkappa_1^2\varkappa_2 + 6\varkappa_1^2\varkappa_2^2 + 2\varkappa_1^2\varkappa_2^3 + 32\varkappa_1\varkappa_2 - 28\varkappa_1\varkappa_2^2 \\ & + 16\varkappa_2^2 + 2\varkappa_2^3) + q^4m^{-4}(4\varkappa_1^2 - 10\varkappa_1^2\varkappa_2 - 3\varkappa_1^2\varkappa_2^2 + 24\varkappa_1\varkappa_2 \\ & - 4\varkappa_1\varkappa_2^2 - \varkappa_1\varkappa_2^3 + 4\varkappa_2^2 + 2\varkappa_2^3) + q^6m^{-6}(4\varkappa_1\varkappa_2 + \varkappa_1\varkappa_2^2)\},\end{aligned}$$

$$\begin{aligned}\varphi_2 = {}& \varkappa_1^{-2}\varkappa_2^{-2}\{\varkappa_1^2(-\varkappa_1^3\varkappa_2 + 4\varkappa_1^2 - 4\varkappa_1^2\varkappa_2 + 8\varkappa_1\varkappa_2 - 4\varkappa_1\varkappa_2^2 \\ & - \varkappa_1\varkappa_2^3 + 4\varkappa_2^2) + 2q^2m^{-2}(-\varkappa_1^4 + 2\varkappa_1^4\varkappa_2 + 8\varkappa_1^3 \\ & - 7\varkappa_1^3\varkappa_2 - 2\varkappa_1^3\varkappa_2^2 - 16\varkappa_1^2 + 32\varkappa_1^2\varkappa_2 - 3\varkappa_1^2\varkappa_2^2 \\ & - 2\varkappa_1^2\varkappa_2^3 - 32\varkappa_1\varkappa_2 + 24\varkappa_1\varkappa_2^2 + \varkappa_1\varkappa_2^3 - 16\varkappa_2^2) \\ & + q^4m^{-4}(4\varkappa_1^3 - 7\varkappa_1^3\varkappa_2 - 16\varkappa_1^2 + 40\varkappa_1^2\varkappa_2 + 2\varkappa_1^2\varkappa_2^2 \\ & - 64\varkappa_1\varkappa_2 + 20\varkappa_1\varkappa_2^2 - \varkappa_1\varkappa_2^3 - 16\varkappa_2^2) \\ & + 2q^6m^{-6}(-\varkappa_1^2 + 3\varkappa_1^2\varkappa_2 \\ & - 10\varkappa_1\varkappa_2 + \varkappa_1\varkappa_2^2 - \varkappa_2^2) - 2\varkappa_1\varkappa_2q^8m^{-8}\}.\end{aligned}$$

We cite without derivation the conditions under which $d\bar{\sigma}_q$ (A.3) coincides with the Compton-scattering cross section $d\bar{\sigma}_C$, averaged over the polarizations of the initial photon,

$$d\bar{\sigma}_C = 2\pi r_0^2\frac{d\varkappa_2}{\varkappa_1^2}\left\{-\frac{\varkappa_1}{\varkappa_2} - \frac{\varkappa_2}{\varkappa_1} + 4\left(\frac{1}{\varkappa_1} + \frac{1}{\varkappa_2}\right)^2 - 4\left(\frac{1}{\varkappa_1} + \frac{1}{\varkappa_2}\right)\right\}. \tag{A.4}$$

These conditions are

$$q^2 \ll \begin{cases} m^2\varkappa_1^2, & |\varkappa_1| \lesssim 1 \\ m^2|\varkappa_1|, & |\varkappa_1| \gg 1 \end{cases}, \tag{A.I}$$

$$1 - v^2 \ll \begin{cases} 1, & |\varkappa_1| \lesssim 1. \\ 1/|\varkappa_1|, & |\varkappa_1| \gg 1 \end{cases} \tag{A.II}$$

As in the case considered earlier (Sec. 6), the Weizsäcker-Williams relation (1.1) is violated in a narrow region of q^2 near q^2_{min}.

It is easy to see that (A.I) and (A.II) coincide with conditions (Ia) and (III).

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