A MORE PRECISE DETERMINATION OF THE KINETIC COEFFICIENTS OF A PLASMA

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Kinetic coefficients containing the exact values of the Coulomb logarithm are derived for a plasma. The case $\omega\tau \ll 1$, $e^2/\hbar v \ll 1$ is considered.

THE kinetic coefficients of a fully ionized gas are known with logarithmic accuracy.[1] This is connected with the logarithmic divergence of the Coulomb bremsstrahlung cross section for large impact parameters. One usually deals with this difficulty by artificially cutting off the interaction at distances of the order of the Debye radius. With the help of the methods of quantum field theory one can consistently take account of the collective interactions and obtain an expression for the renormalized scattering probability which does not contain divergences. Such an expression for the case where the de Broglie wavelength of the electron $\hbar/mv$ is much larger than the range of the essential interaction e^2/kT (e and m are the charge and mass of the electron, v is the thermal velocity of the electron, and T is the temperature) has been obtained in a number of papers.[2-4]

Since the renormalized scattering probability automatically cuts off the interaction for large impact parameters, it is possible to obtain the exact values of the terms in the argument of the logarithm appearing in the expressions for the kinetic coefficients. This has been done for the specific energy losses of a fast particle going through a plasma,[2] for the diffusion coefficient of the plasma perpendicular to the magnetic field[5] (for the case $\omega_c\tau \gg 1$, $\omega_c \ll \omega_0$, where ω_c is the cyclotron frequency, τ is the relaxation time, and ω_0 is the plasma frequency), and for the high frequency conductivity of the plasma[6] for $\omega\tau \gg 1$, where ω is the frequency of the electromagnetic wave. In the present paper we have carried out a similar more precise determination of the coefficients which determine the particle currents and the heat in the plasma in the presence of an electric field and of temperature and concentration gradients for $\omega\tau \ll 1$ (ω is the frequency of the process).

Our starting point is the kinetic equation with the collision integral obtained earlier in [3]. In solving it we have used the usual method of expanding the distribution function in a series of Laguerre polynomials (see, for example, the paper of Landshoff[7]). The result of the calculation can be written in the form

$$\mathbf{j} = \sigma\left(\mathbf{E} - \frac{1}{ne}\nabla p\right) + \tau\nabla T, \quad \mathbf{q} = -K\nabla T - \mu\left(\mathbf{E} - \frac{1}{ne}\nabla p\right), \tag{1}$$

where **E** is the electric field intensity, p is the pressure, n is the concentration, **j** is the current density, and **q** is the density of the heat flow. The kinetic coefficients σ, τ, μ, and K have the form

$$\sigma = 1.95\frac{ne^2}{m\nu_0}\frac{1}{\lambda_\sigma}, \qquad \tau = 1.39\frac{nek}{m\nu_0}\frac{1}{\lambda_\tau},$$
$$\mu = 6.25\frac{nekT}{m\nu_0}\frac{1}{\lambda_\mu}, \qquad K = 7.62\frac{nk^2T}{m\nu_0}\frac{1}{\lambda_K}. \tag{2}$$

Here

$$\nu_0 = \tfrac{2}{3}\pi(e^2/kT)^2(8kT/\pi m)^{1/2}n.$$

The difference between these formulas and the results of Landshoff[7] consists in that in our case the logarithm λ is definite (up to a term of order λ^{-1}) and has a different value for the different kinetic coefficients:

$$\lambda_\sigma = \ln\frac{1.2}{\eta}, \qquad \lambda_\mu = \ln\frac{1.3}{\eta}, \qquad \lambda_\tau = \ln\frac{1.6}{\eta},$$
$$\lambda_K = \ln\frac{1.7}{\eta}. \tag{3}$$

Here $\eta = (e\hbar/kT)(2\pi n_0/m)^{1/2}$ is the ratio of the de Broglie wavelength of the electron to the Debye radius.

These results were obtained by considering the first three Laguerre polynomials in the expansion of the distribution function. The calculations are similar to those carried out by Landshoff, but the intermediary expressions are considerably more involved and have therefore not been quoted.

[1] L. Spitzer, The Physics of Fully Ionized Gases, Interscience Publishers, N. Y. (1956).

[2] A. I. Larkin, JETP **37**, 264 (1959), Soviet Phys. JETP **10**, 186 (1960).
[3] O. V. Konstantinov and V. I. Perel', JETP **39**, 861 (1960), Soviet Phys. JETP **12**, 597 (1960).
[4] V. P. Silin, JETP **40**, 1768 (1961), Soviet Phys. JETP **13**, 1244 (1961).
[5] V. L. Gurevich and Yu. A. Firsov, JETP (in press).
[6] V. I. Perel' and G. M. Éliashberg, JETP (in press).
[7] R. Landshoff, Phys. Rev. **76**, 904 (1949).

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