THE SIGN OF THE MASS DIFFERENCE OF K_1^0 AND K_2^0 AND THEIR LEPTONIC DECAYS

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Interference of leptonic decays of K^0 mesons during the transversal through matter of K_2^0 mesons is studied as a possible method for the determination of the sign of the K_1^0-K_2^0 mass difference.

Kobzarev and Okun'[1] have proposed a method for the determination of the sign of the mass difference $\Delta m = m_1 - m_2$ of the K_1^0 and K_2^0, based on the interference phenomena that take place when a beam of K_2^0 mesons passes through a pair of plates made of different substances. The K_1^0 waves produced in each of these plates are out of phase by an amount $\Delta\varphi$ and therefore one can, in principle, determine the sign of Δm by studying the oscillation of the two-pion mode of decay of the K_1^0 as a function of the distance between the plates. In a paper by the author[2] this phenomenon was analyzed in detail and optimum conditions for performing the experiment were found. The analysis showed that it would be advisable to use two thick plates located next to each other provided that $|\Delta\varphi|$ was not very small ($\geq \pi/8$).

However it is not clear a priori what magnitudes of $\Delta\varphi$ can be expected for various pairs of plates, not to mention the extremely complicated problem of determining the sign of $\Delta\varphi$, which is essential in this method. It is not out of the question that $\Delta\varphi$ could be so small that curves with opposite signs of Δm could not be resolved. In view of these considerations it does not seem likely that an experiment to determine the sign of Δm by the two-plates method will be carried out in the near future.

It is therefore of considerable interest to investigate the possibility of determining the sign of Δm by utilizing just one plate placed in the K_2^0 beam. This could be done by studying the number of hyperons produced in a thin plate, placed at a distance x_0 from the main plate, as a function of x_0,[1,2] and by studying the leptonic decays K_{e3}^0 and $K_{\mu 3}^0$ in the part of the beam that goes through without deviation as a function of the distance x_0 between the point of decay and the plate.[2]*

*The possibility of utilizing interference effects in leptonic decays for the purposes of determining the sign of Δm was already pointed out by Biswas[3] in the case of single scattering of the K^0 beam. In contrast to Biswas we consider phenomena arising from the transmission of the K_2^0 through matter.

The obvious drawback of these methods of determining the sign of Δm in contrast to the two-plate method lies in the fact that the effect under study (hyperon production, leptonic decays) occurs not only because and not so much because of the presence of the "regenerated" K_1^0 wave, as because of the presence of the here predominating K_2^0 component. Therefore the distribution of the number of observed events will be given by a term large in magnitude, on which there will be superimposed an oscillation of small amplitude, connected with the sign of Δm. At the same time one should keep in mind that the use of a thick plate substantially increases the amplitude of this oscillation (by increasing the fraction of "regenerated" K_1^0 mesons[2]); furthermore, in the case of leptonic decays the amplitude of oscillations can be doubled by making use of the difference effect (see below) which is absent in the hyperon case.

These considerations make it necessary to investigate in detail leptonic decays in a beam of K^0 mesons that has traversed a plate.* As was shown by the author,[2] the number of leptonic decays $N_+(N_-)$ in a beam transmitted without deviation through a plate of thickness $t = x/v$ (v — velocity of the K_2^0 beam) per unit time at a distance $t_0 = x_0/v$ from the plate, relative to the initial number of mesons in the beam, for decays into π^+ mesons and leptons (π^- mesons and leptons) is equal to

*The fact that when leptonic decays are used to determine the sign of Δm it is sufficient, in principle, to transmit the K_2^0 through one plate is directly related to the fact that the decays $K^0 \to e^+(\mu^+) + \nu + \pi^-$ and $\bar{K}^0 \to e^-(\mu^-) + \nu + \pi^+$ are allowed, whereas $K^0 \to e^-(\mu^-) + \nu + \pi^+$ and $\bar{K}^0 \to e^+(\mu^+) + \nu + \pi^-$ are forbidden by the $\Delta Q = \Delta S$ rule, where Q and S are the charge and strangeness of the strongly interacting particles.[4] We note that the magnitude of Δm can be determined through leptonic decays from the free K^0 beam, whereas its determination from two-pion decays of K_1^0 requires the transmission of the K_2^0 beam through one plate. In order to determine the sign of Δm one more plate is required (of a different substance).

$$\binom{N_+}{N_-} = \Gamma_l \exp\left(-\frac{N}{2}(\bar{\sigma}+\sigma)x\right)\left\{\frac{1}{2}\exp\left(-\frac{t_0}{\gamma\tau_2}\right) \pm r\exp\left(-\frac{t_0}{2\gamma\tau_1}-\frac{t_0}{2\gamma\tau_2}\right)\left[\sigma(t)\sin\left(\varphi-\frac{\Delta m}{\gamma}t_0\right) - \varkappa(t)\cos\left(\varphi-\frac{\Delta m}{\gamma}t_0\right)\right]\right\}\exp\left(-\frac{t}{\gamma\tau_2}\right). \quad (1)$$

The notation in Eq. (1) is the same as in [2]; in contrast to the corresponding formula (15) of [2] we have taken here into account the finite lifetime τ_2 of the K_2^0 meson; $\sigma(t)$ and $\kappa(t)$ are respectively even and odd functions of Δm, given by Eq. (16) of [2].

Equation (1), as well as the entire theory developed earlier in [2], is valid provided that the condition of preponderance of K_1^0 decays over effects due to their regeneration in matter is well satisfied [condition (4) in [2]].

Taking into account that the introduced in [2] modulus r of a quantity proportional to the amplitude for the transition $K_2^0 \rightarrow K_1^0$ may be written with the help of the optical theorem in the form $r = N(\bar{\sigma}-\sigma)/4|\sin\varphi|$ $(0 < \varphi < \pi)$ we find that the condition (4) of [2] reduces to a condition whose left side has an obvious physical meaning

$$N(\bar{\sigma}-\sigma)\gamma\tau_1 v \ll 2|\sin\varphi|. \quad (2)$$

Let us note that $\varphi = \tan^{-1}(\rho_2/\rho_1)$, where $\bar{f}(0) - f(0) \equiv \rho_1 + i\rho_2$, with $\rho_2 > 0$ in the entire so far investigated large energy interval. On the other hand, Eq. (1) is invariant under simultaneous replacements $\Delta m \rightarrow -\Delta m$ and $\varphi \rightarrow \pi - \varphi$, so that it is necessary to know the sign of ρ_1 in order to be able to determine the sign of Δm. [In the two-plate case one was faced with the more complex problem of determining the sign of the difference of the real parts of $\bar{f}(0) - f(0)$ in the two plates.]

Information on the sign of ρ_1 may be obtained from the results of the analysis of experiments on elastic scattering of K^+ and K^- mesons with energies ~ 100 Mev on photoemulsion nuclei on the basis of the optical model of the nucleus with a diffuse edge.[5,6] This analysis leads to a negative real part for the interaction potential between K^- mesons and photoemulsion nuclei (about −30 Mev) corresponding to attraction, whereas for K^+ scattering the nuclear potential corresponds to a repulsion (its real part is approximately +20 Mev for photoemulsion nuclei).

These data lead to the conclusion that $\mathrm{Re}[\bar{f}(0) - f(0)]$ is positive for K^+ and K^- scattering on photoemulsion nuclei. It is to be expected that a similar relation holds true for K^0 and $\bar{K}^0$ mesons; also it is hardly to be expected that the sign of the real part of the potential would change upon passing from one nucleus to another.

At this time, however, it is not possible to arrive at any conclusions on the sign of the difference of the real parts of $\bar{f}(0) - f(0)$ for different nuclei, which requires the precise knowledge of the actual size of the real parts of the scattering amplitudes. This problem is not likely to be solved in the near future. From this point of view methods for determining the sign of Δm using one plate have an incomparable advantage over two-plate methods.

It should be noted that the experiment is impossible if φ is close to $\pi/2$. However the data on K^+ and K^- scattering[5,6] definitely indicate that $\varphi \neq \pi/2$. At the same time they indicate that φ is not near zero,* so that the condition (2) for the applicability of Eq. (1) is always satisfied, even if not with quite the margin that would be expected from the approximate condition (4′) of [2].

Before carrying out a quantitative analysis of Eq. (1) let us indicate how it is possible, in principle, to estimate $|\sin\varphi|$, i.e. φ, given the sign of ρ_1, by studying the dependence of the number N_1 of K_1^0 decays into two pions in the undeviated beam on the distance t_0 when $|\Delta m|$, v, t, $\bar{\sigma}$ and σ are given. This dependence is given by the formula

$$N_1 = \frac{N^2(\bar{\sigma}-\sigma)^2}{16\sin^2\varphi}\exp\left\{-\frac{N}{2}(\bar{\sigma}+\sigma)x\right\}e^{-t_0/\gamma\tau_1} \times \frac{1-2\cos(\Delta m t/\gamma)\exp(-t/2\gamma\tau_1)+\exp(-t/\gamma\tau_1)}{(\Delta m/\gamma v)^2 + (1/2\,\gamma\tau_1 v)^2} \quad (3)$$

(see also [7]).

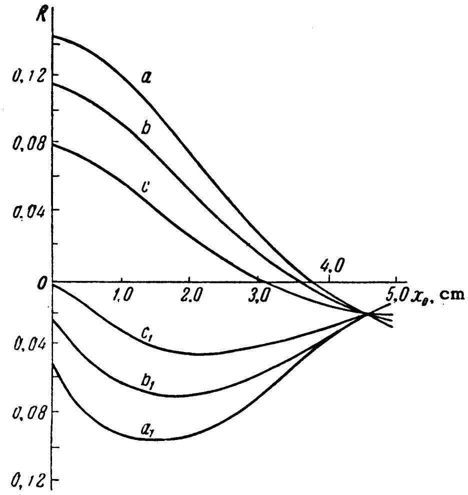
Turning to our problem with leptonic decays we introduce the ratio

$$R(t, t_0) \equiv \frac{N_+ - N_-}{N_+ + N_-} = 2r\left[\sigma(t)\sin\left(\varphi - \frac{\Delta m}{\gamma}t_0\right) - \varkappa(t)\cos\left(\varphi - \frac{\Delta m}{\gamma}t_0\right)\right]e^{-t_0/2\gamma\tau_1}. \quad (4)$$

A study of this ratio as a function of t_0 at an optimal thickness of the plate t permits the determination, in principle (given the sign of ρ_1, i.e., information about what quadrant φ is in—the first or second), of the sign of Δm.

In the figure are shown curves giving the dependence of R on $x_0 = vt_0$ for an optimal thickness of a copper plate $x \equiv vt = 3.5$ cm (the energy of the K_2^0 mesons is equal to 100 Mev; the decay length of K_1^0 mesons $\gamma\tau_1 v$ is equal to 2.2 cm). The curves a, b, c correspond to the values $\varphi = \pi/8$, $\pi/6$, $\pi/4$ for $\Delta m < 0$, and $\varphi = \pi - \pi/8$, $\pi - \pi/6$, $\pi - \pi/4$ for $\Delta m > 0$. The curves a_1, b_1

*An estimate of f(0) and $\bar{f}(0)$, based on the nuclear potentials of K^+ and K^- mesons given in[5,6], results in a value of φ of approximately $\pi/6$.



Sign of the mass difference of K_1^0 and K_2^0 mesons and their leptonic decays.

and c_1 correspond to the same choice of φ for opposite sign of Δm. The magnitude of Δm was taken to be 10^{10} sec^{-1}.[8,9]

It is seen from the figure that, given the information on the sign of ρ_1, it is possible in principle to determine the sign of Δm from the sign of R near the plate on the basis of even a single point at some arbitrary x_0. For $\varphi \approx \pi/6$ the ratio R amounts to ~ 0.08. In other words, given approximately 350 leptonic decays we should get 190 decays into leptons of one sign and 160 decays into leptons of the other sign, which insures a statistically reliable determination of the sign of R, hence of Δm.

It must, however, be kept in mind that the events of interest take place at a distance from the plate of the order of a decay length $\gamma\tau_1 v$ of the K_1^0, so that all leptonic decays in that distance will be rare. In order to obtain 350 leptonic decays in the region of interest it is necessary to transmit $\sim (\tau_2/\tau_1)\cdot 350 = 175{,}000$ K_2^0 mesons. However one should not forget that in the proposed experiment for the determination of the sign of Δm it is not necessary to observe the oscillation, it being sufficient to determine the sign of R at one point. We note that with increasing energy of the K_2^0 mesons the ordinates of the curves increase as well as the region near the plate, in which for a given φ the ratios R differ in sign for opposite signs of Δm.

If one takes into account the difficulties connected with the sign of the difference of the real parts of $\bar{f}(0) - f(0)$ for two plates and of the quantity $\Delta\varphi$, then one might conclude that the utilization of interference phenomena in leptonic decays of K^0 mesons represents a real possibility for the determination of the sign of Δm.

In order to improve the statistics one could integrate the ratio R, Eq. (4), over x_0 between 0 and $\sim\gamma\tau_1 v$, where the sign effect is most pronounced, and study the dependence of the resultant quantity on the plate thickness x. In that case the integral of R, corresponding to the curves a, b and c in the figure, will be for x less than optimal a positive increasing function of x, whereas for the curves a_1, b_1 and c_1 the integral will be negative and increasing in absolute magnitude with the plate thickness.

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