SCATTERING OF PHOTONS BY NUCLEONS

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An analysis of elastic scattering of photons with energies up to 300 Mev by protons is carried out by making use of the dispersion relations method. Six dispersion relations are utilized to estimate the real parts of the amplitudes at $Q^2 = 0$. Photoproduction of pions is taken into account in a larger energy region than was done previously. Five subtraction constants are determined from the long wavelength limit and expressed in terms of the nucleon charge and magnetic moment. Differential cross sections and polarizations of the recoil nucleons are estimated. Photon-nucleon scattering at high energies is discussed.

1. Following the work of Gell-Mann, Goldberger, and Thirring[1] dispersion relations for photon-nucleon scattering, whose validity in the e^2-approximation has been rigorously proved by Logunov,[2] have been applied to the analysis of experimental data by a number of workers.[3-7] Cini and Stroffolini[3] were the first to calculate forward scattering cross sections for photons with energies up to 210 Mev. Certain qualitative peculiarities in the energy dependence of the forward scattering cross section were indicated earlier in[1], and also in[8].

Capps[4] has considered γN scattering through an arbitrary angle by taking into account a minimal number of states. In so doing he made use of some unpublished results of Gell-Mann and J. Mathews.

Akiba and Sato[5] considered scattering through nonzero angles. In order to evaluate the subtraction constants in some of the dispersion relations they made use of perturbation theory.

The authors have previously[6] considered in detail dispersion relations for all six invariant functions, that characterize the γN-scattering amplitude, and have carried out a dispersion analysis in the energy region up to 200 Mev in an approximation in which certain recoil effects were ignored. It was shown that if photoproduction of pions in S states is taken into account significant modifications are introduced in the near threshold region. These changes are such as to improve the agreement between the dispersion analysis and experiment. Near threshold the energy dependence of the amplitudes and cross sections becomes nonmonotonic.

Aside from certain differences, connected with what assumptions were made regarding the number of subtraction constants in the dispersion relations and the maximum angular momentum of the states taken into account, all the published papers turned out to have in common the inability to obtain good agreement with experimental data in the energy region near 160—200 Mev.

In a number of papers[9,7,10] an attempt was made to eliminate this discrepancy by taking into account the contribution from Low's diagram.[11] However a direct measurement of the lifetime of the π^0 meson[12] together with an analysis of the question of the sign of the pole amplitude[13] have led to the conclusion, that the inclusion of Low's amplitude cannot substantially affect the results of the analysis. In connection with these discrepancies between the analysis and the existing experimental data we carry out in this work an analysis of γN scattering based on dispersion relation, in which we take into account in addition to photoproduction of pions in S states the contribution from the high energy regions in a more careful manner; we also analyze the question of the number of subtractions in the dispersion relations and, taking nucleon recoil fully into account, estimate the previously introduced quantities $R_i(\nu)$ at $Q^2 = 0$.

2. The connection between the invariant functions $T_i(\nu, Q^2)$ and the amplitudes $R_i(\nu, Q^2)$ in the barycentric system is given by Eq. (1) of[14] (in the following[14] will be referred to as A). For the definitions of $T_i(\nu, Q^2)$ and $R_i(\nu, Q^2)$ see[13] (in the following[13] will be referred to as B). The notation in the present paper is the same as the notation in A and B. By R_i without additional marks we will understand here the amplitudes in the barycentric frame.

Since according to the optical theorem

$$\text{Im}\,(R_1 + R_2) = \frac{\nu_c \sigma_t}{4\pi} = \frac{w^2 - M^2}{2w}\frac{\sigma_t}{4\pi} \tag{1}$$

(w is the total energy in the barycentric frame) it follows that under the assumption

$$\sigma_t(w) \to \text{const as } w \to \infty$$

we have asymptotically as $w \to \infty$

$$R_1 + R_2 \to w^2 \sim \nu. \tag{2}$$

Assuming further that as $w \to \infty$ all $R_i \sim \nu$, we get from A, Eq. (1) that as $w \to \infty$

$$T_1 - T_3 \to w^2, \quad T_1 + T_3 \to w^2, \quad T_5 \to w^2,$$

$$T_2 - T_4 \to w, \quad T_2 + T_4 \to \text{const}, \quad T_6 \to w. \tag{3}$$

Consequently, under the assumptions here made, the dispersion relations for T_1, T_3 and T_5 should contain one subtraction, whereas the dispersion relations for the quantities T_2, T_4 and T_6 may be written with no subtractions.

In order to estimate the amplitudes $R_1 + R_2$, R_3, R_4, and $R_5 + R_6$ it is sufficient to write dispersion relations for T_1, T_3 and T_5 at $Q^2 = 0$. At $Q^2 = 0$ the invariant

$$\nu = \nu_{\text{lab}} - Q^2/M$$

becomes, as is well known, the photon energy in the laboratory frame ν_{lab} (denoted in the following by ν).

3. As can be seen from A, Eq. (1), for forward scattering the functions T_5 and $T_2 + T_4$ reduce to $R_4 - R_3$, so that at $Q^2 = 0$ the dispersion relations for T_5 and $T_2 + T_4$ are equivalent.

Let us consider the functions

$$F_1(\nu_0) = \tfrac{1}{2}[T_1 - T_3 - \nu_0(T_2 - T_4)] = w_0(R_1 + R_2)/M,$$
$$F_2(\nu_0) = \nu_0 T_6 = w_0[R_3 + R_4 + 2R_5 + 2R_6]/M,$$
$$F_3(\nu_0) = \nu_0(T_1 + T_3)/2M = (w_0/M)^2(R_3 - R_4),$$
$$F_4(\nu_0) = \tfrac{1}{2}(T_1 - T_3)$$
$$= w_0^2(R_3 + R_4)/M\nu_0 - 2w_0(R_1 + R_2)/(M + w_0). \tag{4}$$

It is clear from the discussion above that the dispersion relations for the functions $F_1, \ldots, F_4$ should contain one subtraction. All quantities on the right side of Eq. (4) are in the barycentric frame. If one takes into account that (for $Q^2 = 0$) the amplitudes in the laboratory system are connected to the corresponding quantities in the barycentric system by

$$(R_1 + R_2)^n = w_0(R_1 + R_2)/M,$$
$$(R_4 - R_3)^n = (w_0/M)^2(R_4 - R_3), \tag{5}$$
$$[R_3 + R_4 + 2R_5 + 2R_6]^n = w_0[R_3 + R_4 + 2R_5 + 2R_6]/M,$$

then one obtains from the dispersion relations for $F_1, \ldots, F_4$

$$D_{1,4}^{\text{lab}}(\nu_0) - D_{1,4}(0) = \frac{2\nu_0^2}{\pi}\int_{\nu_t}^{\infty}\frac{d\nu}{\nu^2 - \nu_0^2}\frac{A_{1,4}(\nu)}{\nu},$$

$$D_{2,3}^{\text{lab}}(\nu_0) - \nu_0 D'_{2,3}(0) = \frac{2\nu_0^3}{\pi}\int_{\nu_t}^{\infty}\frac{A_{2,3}(\nu)\,d\nu}{\nu^2(\nu^2 - \nu_0^2)}, \tag{6}$$

where

$$D_1^n = \text{Re}\,(R_1 + R_2)^{\text{lab}}, \quad D_2^{\text{lab}} = \text{Re}\,[R_3 + R_4 + 2R_5 + 2R_6]^{\text{lab}},$$
$$D_3^{\text{lab}} = \text{Re}\,(R_4 - R_3)^{\text{lab}}, \quad D_4^{\text{lab}} = \text{Re}\,F_4(\nu);$$
$$D_1(0) = -e^2/M, \quad D'_2(0) = -2\mu_a^2,$$
$$D'_3(0) = -2[\mu^2 - (e/2M)^2], \quad D_4(0) = -(e^2/2M)\lambda(2 + \lambda),$$

and where $A_i(\nu_0)$ stands for the imaginary part of the corresponding amplitude; $\mu = e(1 + \lambda)/2M$ stands for the magnetic moment and μ_a for the anomalous magnetic moment of the nucleon.

If the elements of the amplitude for the photoproduction of pions in states with $J \leq 3/2$ in the barycentric frame are denoted by E_1, E_2, E_3 (electric transitions into $1/2-$, $3/2+$ and $3/2-$ respectively), M_1, M_2, M_3 (magnetic transitions into $1/2+$, $3/2-$ and $3/2+$ respectively), then the unitarity relations lead to the equalities

$$\text{Im}\,R_1 = \nu_c\left\{|E_1|^2 + 2|E_3|^2 + \tfrac{1}{3}|E_2|^2\cos\theta - \tfrac{1}{6}|M_2|^2\right\},$$
$$\text{Im}\,R_3 = \nu_c\left\{|E_1|^2 + \tfrac{1}{3}|E_2|^2\cos\theta\right.$$
$$\left. - |E_3|^2 + \tfrac{1}{12}|M_2|^2 + \text{Re}\,(E_3^* M_2)\right\},$$
$$\text{Im}\,R_5 = -\nu_c\left\{\tfrac{1}{6}|E_2|^2 + \text{Re}\,(E_2^* M_3)\right\}, \tag{7}$$

which represent the generalization of the corresponding equalities in[6]. The expressions for $\text{Im}\,R_2$ differ from those for $\text{Im}\,R_1$ by the exchange $E_i \rightleftarrows M_i$. Analogously, the expression for $\text{Im}\,R_4$ may be obtained from that for $\text{Im}\,R_3$, and for $\text{Im}\,R_6$ from $\text{Im}\,R_5$.

In Eq. (7) we mean by the modulus of the amplitude on the right side the sum of the contributions from photoproduction of π^+ and π^0 mesons. We note that if the mass differences between mesons and between nucleons are ignored then a cancellation in the interference terms, for example in $|E_{\pi^0}|^2 + |E_{\pi^+}|^2$, occurs as a consequence of isotopic symmetry in the pion photoproduction process. At that

$$A_1(\nu) = \nu\sigma_t/4\pi = \nu\left\{|E_1|^2 + |M_1|^2\right.$$
$$\left. + 2|E_3|^2 + 2|M_3|^2 + \tfrac{1}{6}|M_2|^2 + \tfrac{1}{6}|E_2|^2\right\},$$

$$A_2(\nu) = \nu\left\{|E_1|^2 + |M_1|^2 + \tfrac{1}{3}|M_2|^2 + \tfrac{1}{3}|E_2|^2\right.$$
$$\left. - |M_3 + \tfrac{1}{2}E_2|^2 - |E_3 + \tfrac{1}{2}M_2|^2\right\},$$

$$A_3(\nu) = -\nu(w/M)\{|E_1|^2 - |M_1|^2 + |M_3 - \tfrac{1}{2}E_2|^2 - |E_3 - \tfrac{1}{2}M_2|^2\},$$
$$A_4(\nu) + (w - M)\,\sigma_t/4\pi = w\{|E_1|^2 + |M_1|^2 + \tfrac{2}{3}|M_2|^2 + \tfrac{2}{3}|E_2|^2 - |E_3 - \tfrac{1}{2}M_2|^2 - |M_3 - \tfrac{1}{2}E_2|^2\}. \quad (8)$$

4. As was shown by Goldberger,[1] the sum rule that follows from the nonsubtracted dispersion relations:

$$\mathrm{Re}(R_1 + R_2) \to + \frac{1}{2\pi^2}\int_{\nu_t}^{\infty} \sigma_t(\nu)\,d\nu > 0 \quad (9)$$

is in contradiction with the long wavelength limit

$$R_1 + R_2 \to -e^2/M < 0. \quad (10)$$

Consequently, nonsubtracted dispersion relations for the amplitude $R_1 + R_2$ violate the requirements of relativistic and gauge invariance on which the long wavelength limit is based.

Let us remark that possible sum rules involving the square of the magnetic moment are not in direct contradiction with the long wavelength limit when nonsubtracted dispersion relations are assumed for $F_2(\nu)$. As can be seen from Eqs. (6) and (8), of particular importance here is the contribution of the resonant state, proportional to $|M_3|^2$. The result is unchanged if one takes into account the (numerically important) contribution from photoproduction in S states, which decreases the effective contribution of $|M_3|^2$.

The sum rule for the square of the magnetic moment is very sensitive to the ratio of the photoproduction amplitudes E_2 and M_3. For certain ratios (for example for $E_2 = M_3$[5]) one can arrive at a contradiction. At the present time, however, the analysis of photoproduction is not sufficiently precise to permit the assertion that the experimental data are in contradiction with the sum rule. An increase in the accuracy of the photoproduction analysis, aimed at obtaining information about the amplitudes E_2, M_2 and E_3, would be most welcome.

The fact that unsubtracted dispersion relations give rise to definite sum rules may be of particular interest in certain processes. Thus, in the case of $\pi\pi$ scattering analogous considerations (applied to dispersion relations at $Q^2 = 0$) lead to the conclusion that the S-state scattering lengths a_0 and a_2 are positive at low energies. The same holds for πK and KK scattering.

5. If in addition to the functions introduced previously one studies properties of the functions*

$$F_5(\nu_0) = (T_2 - T_4)', \quad (11)$$
$$F_6(\nu_0) = (T_2 + T_4)', \quad (12)$$
$$F_7(\nu_0) = T_6', \quad (13)$$

one concludes that $F_{5,6}(\nu)$ are odd functions of ν and contain no poles, whereas $F_7(\nu)$ is an even function of ν with a second order pole. As $\nu \to \infty$

$$F_{5,6,7} \to \nu^{-1/2},$$

so that the dispersion relations for these functions need no subtractions.

These dispersion relations may turn out to be useful since when photoproduction in states with $J \leq 3/2$ is taken into account the angular dependence of the amplitudes $R_i(\nu, Q^2)$ in the barycentric frame takes the form (cf.[6])

$$R_3 = \mathscr{E}_1 - \mathscr{E}_3 + 2\mathscr{E}_2\cos\theta + \tfrac{1}{2}\mathfrak{m}_2 + C(\mathscr{E}_3\mathfrak{m}_2),$$
$$R_4 = \mathfrak{m}_1 - \mathfrak{m}_3 + 2\mathfrak{m}_2\cos\theta + \tfrac{1}{2}\mathscr{E}_2 + C(\mathfrak{m}_3\mathscr{E}_2),$$
$$R_5 = -\mathscr{E}_2 - C(\mathfrak{m}_3\mathscr{E}_2), \qquad R_6 = -\mathfrak{m}_2 - C(\mathscr{E}_3\mathfrak{m}_2), \quad (14)$$

and is characterized by eight functions of energy $\mathscr{E}_{1,2,3}$, $\mathfrak{m}_{1,2,3}$, $C(\mathscr{E}_3\mathfrak{m}_2)$ $C(\mathfrak{m}_3\mathscr{E}_2)$, which can be expressed in terms of $R_i(\nu, 0)$ and $R_i'(\nu, 0)$.

It follows from Eq. (14) that if we restrict ourselves to contributions from states with $J \leq 3/2$

$$R_1' = R_3' = 2\mathscr{E}_2(\partial\cos\theta/\partial Q^2)_{Q^2=0} = -4\mathscr{E}_2 w_0^2/M^2\nu_0^2,$$
$$R_2' = R_4' = -4\mathfrak{m}_2 w_0^2/M^2\nu_0^2, \qquad R_5' = R_6' = 0,$$

so that

$$(R_1 + R_2)' = (R_3 + R_4)' = [R_3 + R_4 + 2(R_5 + R_6)]'. \quad (15)$$

In the long wavelength limit[14]

$$(R_1 + R_2)' = -2e^2/M^2\nu + O(1),$$
$$(R_3 + R_4)' = -e^2[3 + 2(1 + \lambda)^2]/2M^3 + O(\nu),$$
$$(R_3 + R_4 + 2R_5 + 2R_6)' = -e^2(2\lambda^2 - 2\lambda - 1)/2M^3 + O(\nu). \quad (16)$$

The fact that Eq. (15) is in contradiction with the long wavelength limit (16) means that the restriction to states with $J \leq 3/2$ is not a good approximation even in the low energy region. The crossing symmetry conditions introduce kinematic corrections of the order of ν/M, which corresponds to inclusion of states with higher values of J. The carrying out of the analysis with this high a precision requires the introduction of additional functions of energy and discussion of a larger number

*The prime denotes differentiation with respect to Q^2 and subsequent passage to $Q^2 = 0$.

of dispersion relations. Introduction of the Low diagram does not resolve the indicated contradiction. All estimates of the amplitudes given here were obtained with the neglect of $R'_i(\nu, 0)$.

6. The results of the calculations of the amplitudes $R_i(\nu_0)$ at $Q^2 = 0$ are shown in the figures. The energy of the photons ν_0 is given in units of the threshold energy $\nu_t = 150$ Mev, and the values of the amplitudes in units of e^2/Mc^2.

For the calculation of the forward differential scattering cross section

$$\sigma(0^\circ) = |R_1 + R_2|^2 + |R_3 + R_4 + 2R_5 + 2R_6|^2$$

the amplitudes $R_1 + R_2$ and $R_3 + R_4 + 2R_5 + 2R_6$ are sufficient.

To estimate $D_1(\nu_0)$ use was made of the data on the total cross section for the interaction of photons with protons, including the second maximum and the cross section for pion pair production. The dependence of $A_1(\nu_0)$ is shown in Fig. 1. Previously we have neglected contributions from the energy region above 500 Mev. The result of estimating the amplitude $R_1 + R_2$ is shown in Fig. 2. The main difference between this and previous results appeared in the region $1 < \nu_0 < 2$, where as a consequence of a cancellation between the long wavelength limit and dispersion terms the value of $D_1(\nu_0)$ is significantly decreased. Let us note that this is precisely the energy region that is sensitive to a change in $A_1(\nu_0)$. The second maximum in $A_1(\nu_0)$ corresponds to the second maximum in photoproduction.

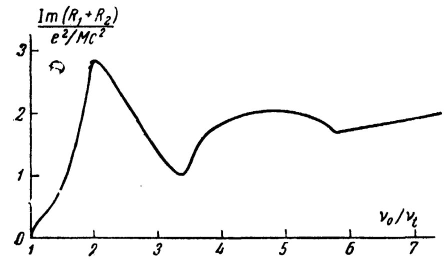

FIG. 1

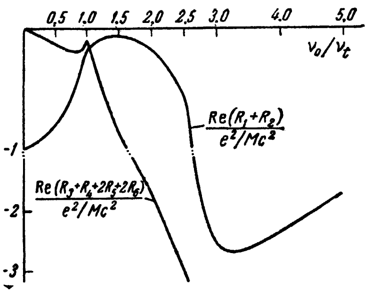

FIG. 2

For estimating real parts of the amplitudes, other than $R_1 + R_2$, which require much more detailed experimental data on photoproduction, we limit ourselves to the energy region up to 300 Mev. For the amplitude $R_1 + R_2$ it turns out to be possible to go much further, although with increasing energy the indeterminacy in the contribution from photoproduction of pairs (and larger numbers) of pions becomes appreciable.

In a number of papers[15,16] the γp scattering at 300—800 Mev has been looked upon as a diffraction process with $\mathrm{Re}\, R_i \ll \mathrm{Im}\, R_i$. The experimental study of γp scattering in the region of the second resonance is of interest as a sensitive method of investigation of the maximum itself.

If, ignoring all $\mathrm{Re}\, R_i$, we restrict ourselves to the imaginary parts of the amplitudes alone and consider only the contribution proportional to $|E_3|^2$, then we find immediately from Eq. (7) that

$$R_2 = R_4 = R_5 = R_6 = 0,$$

$$R_1 = \mathrm{Im}\, R_1 = -2\,\mathrm{Im}\, R_3 = 2\nu_c |E_3|^2,$$

whereas the differential cross section[6] is equal to

$$\sigma(\theta) = \tfrac{1}{8} R_1^2 (7 + 3\cos^2\theta) = \tfrac{1}{2} R_3^2 (7 + 3\cos^2\theta), \qquad (17)$$

in agreement with the results of Minami.[16] The same result for the form of the angular distribution remains valid if in Eq. (7) only M_3 ($R_1 \to R_2$, $R_3 \to R_4$) is different from zero. If simultaneously E_3 and M_3 (with $\mathrm{Re}\, R_i = 0$) are different from zero then we have

$$\sigma(\theta) = \tfrac{1}{2}(R_3^2 + R_4^2)(7 + 3\cos^2\theta) + 10\, R_3 R_4 \cos\theta. \qquad (18)$$

However, as our estimates indicate, the quantities $\mathrm{Re}(R_1 + R_2)$ are large in the region of the second resonance and cannot be ignored. From this point of view the second resonance differs drastically from the $3/2$, $3/2$ resonance, in whose energy region

$$\mathrm{Re}\,(R_1 + R_2) \ll \mathrm{Im}\,(R_1 + R_2).$$

The results of the calculations for $R_3 \pm R_4$, $R_3 + R_4 + 2R_5 + 2R_6$ and $R_5 + R_6$ are shown in Figs. 2—4. In the evaluation of dispersion inte-

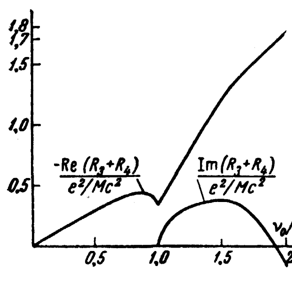

FIG. 3

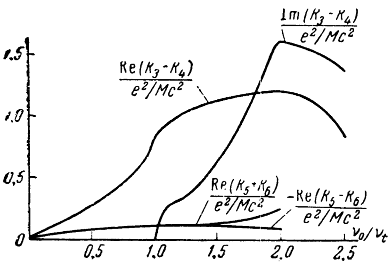

FIG. 4

grals $|E_1|^2$, $|M_3|^2$ and $|E_3|^2$ were assumed to be different from zero, and the energy dependence of $|E_1|^2$ and $|M_3|^2$ was taken from[6], whereas $|E_3|^2$ was assumed to be different from zero in the energy region $3.1 < \nu_0 < 5.8$. Let us remark that even in the absence of an imaginary part for $R_5 + R_6$ the real part of this quantity differs from its long wavelength limit, since the dispersion relations are satisfied by the invariant functions $T_i(\nu, Q^2)$.

The values of $\sigma(0°)$ are shown in Fig. 5, where we give for comparison the results of Cini and Stroffolini[3] for $\sigma_{C-S}(0°)$ in the barycentric frame. A significant difference can be seen in the near-threshold region.

7. For an estimate of $R_1 - R_2$ and $R_5 - R_6$ the

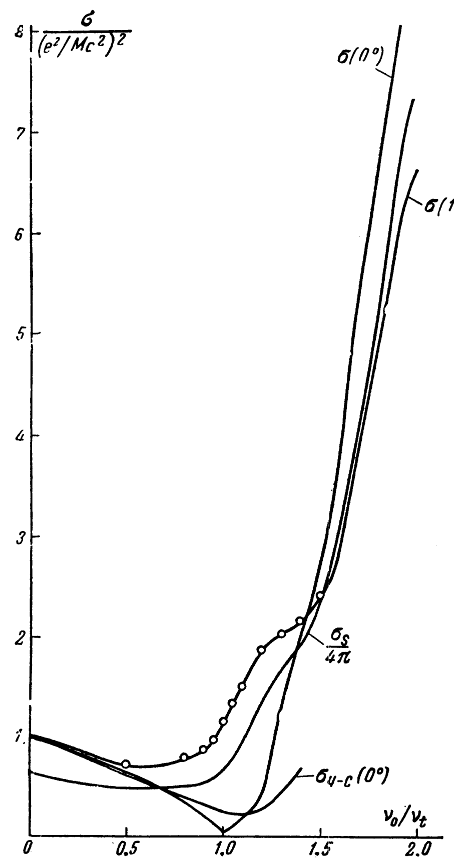

FIG. 5

dispersion relations (6) are not sufficient. Let us consider the function

$$F(\nu) = w^{-2}\varphi(\nu) = (M^2\nu^2/2w^2)\,[(T_1 + T_3)' - \nu(T_2 + T_4)']. \tag{19}$$

As can be seen from B, Eq. (4), we have

$$F(\nu) = \frac{w}{M}\left\{R_1 - R_2 - \frac{2M\nu}{w(w+M)}(R_3 - R_4)\right\}. \tag{20}$$

A study of the dispersion relation for $F(\nu)$ makes it possible to estimate $R_1 - R_2$ if $R_3 - R_4$ is known. In the energy region under consideration the coefficient of $(R_3 - R_4)$ in Eq. (20) is of the order of ν/M, however since the value of $R_3 - R_4$ is large (in comparison with $R_1 - R_2$), the second term in Eq. (20) cannot be ignored. The function $\varphi(\nu)$ introduced in Eq. (19) is an analytic function of ν with a cut along $\nu_t < \nu < \infty$, satisfying the crossing symmetry condition:

$$\varphi(\nu) = \varphi^*(-\nu). \tag{21}$$

Thus, for $\nu \ll \nu_t$ the function $\varphi(\nu)$ is a real function and

$$\varphi(\nu) \cong a + b\nu^2, \tag{22}$$

$$F(\nu) = \frac{\varphi(\nu)}{M^2 + 2M\nu} \cong \frac{a}{M^2}\left(1 - \frac{2\nu}{M}\right) + b\nu^2 + \ldots \tag{23}$$

We see that the linear term in $F(\nu)$ is fully determined by the first term in Eq. (22). It therefore follows from Eq. (20) that for small ν

$$R_1 - R_2 = -(e^2/M)(1 - 3\nu/M) + O(\nu^2)$$

and the linear term in $R_1 - R_2$ and in $F(\nu)$ are fully determined by the requirement of crossing symmetry, as is discussed in detail in B.

The function $F(\nu)$ introduced in Eq. (19) is an analytic function of ν with cuts along $\nu_t < \nu < \infty$ and $-\infty < \nu < -\nu_t$ and a (kinematic) pole at

$$w^2 = M^2 + 2M\nu = 0.$$

The requirements of crossing symmetry lead to the relation

$$F(-\nu) = \frac{M^2 + 2M\nu}{M^2 - 2M\nu}F^*(\nu),$$

and for small ν

$$F(\nu) \cong -(e^2/M)(1 - 2\nu/M) + O(\nu^2).$$

Applying the Cauchy formula to $F(\nu_0)$, for $\rho \to \infty$, along the contour shown in Fig. 6 and writing a dispersion relation with a subtraction we obtain

$$F(\nu_0) = -\frac{e^2}{M}\left(1 - \frac{2\nu_0}{M}\right) + \frac{\nu_0^2}{2\pi i}\int_C \frac{F(\nu)\,d\nu}{\nu^2(\nu - \nu_0)} = -\frac{e^2}{M}\left(1 - \frac{2\nu_0}{M}\right)$$

$$+ \frac{\nu_0^2}{\pi} P\int_{\nu_t}^{\infty} \operatorname{Im} F\left[\frac{1}{\nu - \nu_0} + \frac{M^2 + 2M\nu}{M^2 - 2M\nu}\frac{1}{\nu + \nu_0}\right]\frac{d\nu}{\nu^2}$$

$$+ \frac{\nu_0^2}{2\pi i}\int_{C_+} \frac{F(\nu + i\varepsilon)}{\nu^2(\nu - \nu_0)}d\nu + \frac{\nu_0^2}{2\pi i}\int_{C_-} \frac{F(\nu - i\varepsilon)}{\nu^2(\nu - \nu_0)}d\nu$$

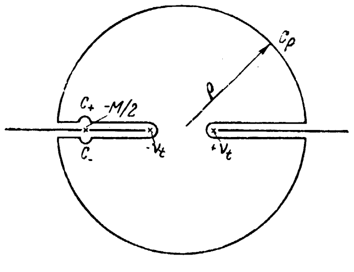

FIG. 6

and

$$\mathrm{Re}\,F(\nu_0) = -\frac{e^2}{M}\left(1-\frac{2\nu_0}{M}\right) + K(\nu_0) + \frac{4\nu_0^2\,\mathrm{Re}\,F(M/2)}{M(\nu_0+M/2)}\,; \quad (24)$$

$$K(\nu_0) = \frac{\nu_0^2}{\pi}\int_{\nu_t}^{\infty}\mathrm{Im}\,F(\nu)\left[\frac{1}{\nu-\nu_0}+\frac{M^2+2M\nu}{M^2-2M\nu}\frac{1}{\nu+\nu_0}\right]\frac{d\nu}{\nu^2}. \quad (25)$$

Since

$$K(M/2) = 0,$$

Re F(M/2) cannot be determined from Eq. (24), and this quantity enters as a free parameter, which must be determined starting from the experimental data. Under the restriction to photoproduction in the states with $J \leq 3/2$ only we get

$$\mathrm{Im}\,F(\nu) = \nu\Big\{\frac{M}{w}(|E_1|^2-|M_1|^2) + 2(|E_3|^2-|M_3|^2)\left(1+\frac{1}{2}\frac{w-M}{w}\right) + \frac{1}{2}\frac{M}{w}(|E_2|^2-|M_2|^2)\left(1+\frac{1}{6}\frac{w-M}{w}\right)\Big\}. \quad (25')$$

In Fig. 7 are shown the results of estimating Re $(R_1 - R_2)$ with the help of Eq. (24) when the contribution proportional to Re F(M/2) is ignored.

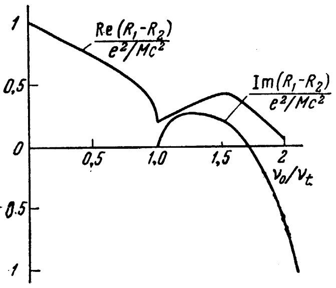

FIG. 7

For an estimate of $R_5 - R_6$ at $Q^2 = 0$, as can be seen from B, Eq. (4), it is sufficient to consider the function

$$\psi(\nu_0) = \tfrac{1}{2}\nu_0^2[T_5' + \tfrac{1}{2}(T_1+T_3)]' = \left(\frac{w_0}{M}\right)^3 \times\left\{\frac{w_0}{\nu_0}(R_5-R_6) + \frac{M}{w_0+M}[R_1-R_2-(R_3-R_4)]\right\}, \quad (26)$$

for which the dispersion relation has the form

$$\mathrm{Re}\,\psi(\nu_0) - \psi(0) = \frac{2\nu_0^2}{\pi}\,\mathrm{P}\int_{\nu_t}^{\infty}\frac{\mathrm{Im}\,\psi(\nu)\,d\nu}{\nu(\nu^2-\nu_0^2)}, \quad (27)$$

where, according to B, Eq. (2),

$$\psi(0) = -e^2(2+\lambda)/2M, \quad (28)$$

$$\mathrm{Im}\,\psi(\nu) = (w/M)^2\Big\{w\left[\tfrac{1}{6}(|E_2|^2-|M_2|^2) + \mathrm{Re}\,(E_2^*M_3 - M_2^*E_3)\right] + M\nu(M+w_0)^{-1}[3(|E_3|^2-|M_3|^2) + \tfrac{1}{4}(|E_2|^2-|M_2|^2) + \mathrm{Re}\,(E_2^*M_3-M_2^*E_3)]\Big\}. \quad (29)$$

The results of estimating Re $(R_5 - R_6)$ at $Q^2 = 0$ for Re F(M/2) = 0 are shown in Fig. 4. Estimates of the quantities $R_3 \pm R_4$ and $R_5 - R_6$, which play a dominant role in the differential cross section for $\nu_0 \gtrsim 1$, do not differ appreciably from those obtained previously.[6]

The results here obtained are of interest from the point of view of the study of the energy dependence of amplitudes near the threshold of a new reaction.[6] In that case all estimates can be carried out to the end. Let us call attention to the dependence of the amplitude Re $(R_1 + R_2)$, whose value continues to fall off also above threshold. This result indicates that a sharp energy dependence of the imaginary parts of the amplitudes above threshold may also for other processes lead to a displacement of the near-threshold minimum (or maximum) of the cross section relative to the reaction threshold.

In Figs. 5 and 8–11 are shown the results of the calculations, with the help of $R_i(\nu, 0)$, of angular distributions

$$\sigma(\theta) = \sum_{l=0}^{3} B_l \cos^l\theta$$

for the angles θ = 90, 135, 139 and 180°, and also of the total elastic scattering cross section

$$\sigma_s/4\pi = B_0 + B_2/2$$

and of the polarization of recoil nucleons for $\theta = 90°$. The experimental data are summarized in[10] and[17].

The coefficient

$$B_3(\nu_0) = 2[|R_5+R_6|^2 - |R_5-R_6|^2]$$

is near to zero in the entire energy region $\nu_0 \lesssim 2$.

The experimental data, apparently, indicate that the quantity Re $(R_5 - R_6)$ is positive. We were not able to achieve this by introducing Re F(M/2) ≠ 0. The requirement that Re $(R_5 - R_6)$ be positive leads to large (negative) values for Re F(M/2), which at the same time significantly

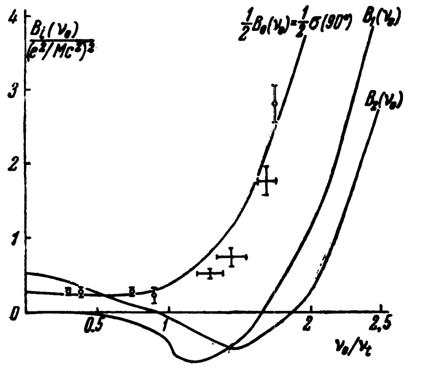

FIG. 8. Energy dependence of the coefficients in the angular distribution. The experimental points are from [9,10,17].

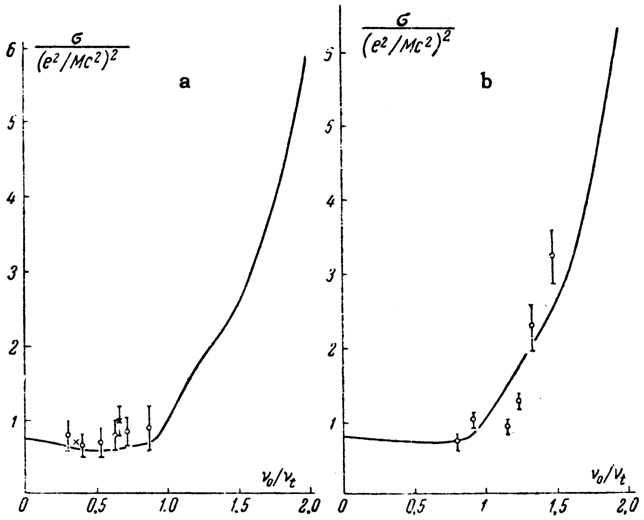

FIG. 9. Energy dependence of the scattering cross section: a – for $\theta = 135°$, b – for $\theta = 139°$. The experimental data are from [9,10,17].

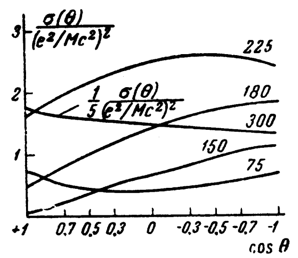

FIG. 10. Differential cross sections at different photon energies (indicated on the curves).

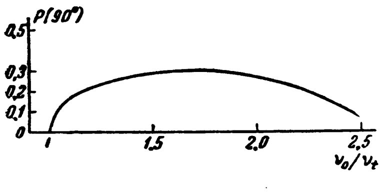

FIG. 11. Polarization of recoil protons.

increases the contribution of $|R_1 - R_2|^2$ to the cross section and does not lead to an improvement in the agreement with the experimental data.

It is necessary to remark that outside the region $1 < \nu_0 < 1.3$ a satisfactory agreement between the dispersional analysis and experimental data is obtained. In the region $1 < \nu_0 < 1.3$, which is particularly sensitive to dispersion effects, it is apparently necessary to take into account contributions from higher states, for which it is necessary to have information on pion photoproduction in a larger energy region.

[1] Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954). M. L. Goldberger, Phys. Rev. 99, 979 (1955).

[2] A. A. Logunov, Dissertation, Joint Inst. for Nucl. Research (1959).

[3] M. Cini and R. Stroffolini, Nucl. Phys. **5**, 684 (1958).

[4] R. H. Capps, Phys. Rev. **106**, 1031 (1957); **108**, 1032 (1957).

[5] T. Akiba and I. Sato, Progr. Theor. Phys. **19**, 93 (1958).

[6] L. I. Lapidus and Chou Kuang-chao, JETP **37**, 1714 (1959) and **38**, 201 (1960), Soviet Phys. JETP **10**, 1213 (1960) and **11**, 147 (1960).

[7] M. Jacob and J. Mathews, Phys. Rev. **117**, 854 (1960).

[8] M. Gell-Mann and M. L. Goldberger, Proc. 1954 Glasgow Conf. on Nucl. and Meson Physics, Pergamon Press, London-N.Y. (1954).

[9] Hyman, Ely, Frisch, and Wahlig, Phys. Rev. Lett. **3**, 93 (1959).

[10] Bernardini, Hanson, Odian, Yamagata, Auerbach, and Filosofo, Nuovo cimento **18**, 1203 (1960).

[11] F. E. Low, Proc. 1958 Ann. Intern. Conf. on High Energy Physics at CERN, p. 98.

[12] Glasser, Seeman, and Stiller, Bull. Amer. Phys. Soc. **6**, 1 (1961).

[13] L. I. Lapidus and Chou Kuang-chao, JETP **41**, 294 (1961), Soviet Phys. JETP **14**, 210 (1962).

[14] L. I. Lapidus and Chou Kuang-chao, JETP **41**, 491 (1961), Soviet Phys. JETP **14**, 352 (1962)

[15] Y. Yamaguchi, Progr. Theor. Phys. **12**, 111 (1954). S. Minami and Y. Yamaguchi, Progr. Theor. Phys. **17**, 651 (1957).

[16] S. Minami, Photon-Proton Collision at 250-800 Mev (preprint).

[17] Govorkov, Gol'danskii, Karpukhin, Kutsenko, Pavlovskaya, DAN SSSR **111**, 988 (1956), Soviet Phys. "Doklady" **1**, 735 (1957). Gol'danskii, Karpukhin, Kutsenko, and Pavlovskaya, JETP **38**, 1695 (1960), Soviet Phys. JETP **11**, 1223 (1960). V. V. Pavlovskaya, Dissertation, Phys. Inst. Acad. Sci. (1961).

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