BIMUONIUM PRODUCTION IN ELECTRON-POSITRON SCATTERING

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We investigate the formation and the properties of the bound state of a positive and negative muon (bimuonium). We discuss the probability of its observation.

1. INTRODUCTION

WHEN electron and positron beams moving in opposite directions collide, electromagnetic pair production may occur. In particular, these pairs may be produced in the bound state, or in the form of atomic systems such as $(\pi^+\pi^-)$, $(\mu^+\mu^-)$, (K^+K^-), etc. The electromagnetic decay time of such systems (i.e., the lifetime for decay into electron-positron pairs or into photons) is much shorter than the weak-interaction decay time for the same particles. The resulting decay of these systems will therefore lead to additional contributions to the electron-positron scattering and annihilation cross sections. It will be shown below that these effects can be observed under certain conditions.

The study of these bound states is not without importance, since it may lead to information on the interaction between the particles of which they are composed, especially for low energies. In particular, data on the $(\pi^+\pi^-)$ and (K^+K^-) systems may give the S-phase shifts for $\pi\pi$ (KK)-scattering.

These bound states may be thought of as unstable particles which one may treat by the apparatus of quantum field theory.[1,2] It is easily shown that the density of final states can in this case be written in the form

$$\rho(p^2)\, d^4p = \operatorname{Im} G(p^2)\, d^4 p, \qquad (1.1)$$

where $G(p^2)$ is the Green's function for the unstable particle (which for us is the bound state).

In this article we will investigate the $(\mu^+\mu^-)$ system in detail; we shall call it bimuonium. This system has the lowest production threshold (103.5 Mev) and can be calculated exactly, since one may with good accuracy consider the muon-muon interaction to be purely electromagnetic.[3]

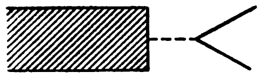

FIG. 1

2. BIMUONIUM DECAY

Bimuonium decay is more complicated than the analogous positronium decay, since positron-electron pairs can be produced as well as photons.*

Let us find the probability for bimuonium decay into an electron-positron pair in the lowest order of perturbation theory. Figure 1 gives the Feynman diagram for this process. Here and henceforth we shall use a cross-hatched rectangle to denote the bimuonium bound state, and double lines to denote free muons. In the ladder approximation the diagram of Fig. 1 is the sum of those in Fig. 2. We sum the contributions from these diagrams by the method described by Alekseev[4] and obtain the following expression for the matrix element of the $(\mu^+\mu^-) \to e^+ + e^-$ process:

$$M = e^2 \int \varphi^* (x_1, x_2)\, \gamma_\nu C^+ D(x_1 - x_2)\, [C\gamma_\nu]_{\rho_1\rho_2} \psi_{\rho_2\rho_1}(x_3, x_4) \times \delta(x_1 - x_2)\, \delta(x_3 - x_4)\, d^4x_1 d^4x_2 d^4x_3 d^4x_4. \qquad (2.1)$$

Here ψ is the bimuonium wave function, a solution of the Bethe-Salpeter equation[5] in the ladder approximation, φ is the wave function of the free electron and the free positron, and C is the charge conjugation operator (we choose the representation in which $C = \alpha_2$).†

The Bethe-Salpeter equation does not include the virtual annihilation diagrams of bimuonium, since they form part of the radiative corrections, and we shall not include these in what follows.

*Conversion of a free $\mu^+\mu^-$ pair into an electron-positron pair has been studied also by Zel'dovich.[12]

†We choose the system of units in which $\hbar = c = 1$, $e^2 = 1/137$; the metric is defined by $ab = \mathbf{a}\cdot\mathbf{b} - a_0b_0$.

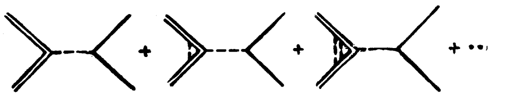

FIG. 2

Transforming (2.1) to the momentum representation and going to the rest system of the bimuonium, we obtain

$$M = -i\pi \frac{e^2}{\mu^2}\left[\bar{u}(p_-)\gamma_\mu v(p_+)\right] \mathrm{Sp} \int C\gamma_\nu \psi(p)\, d^4p\, \delta(P - p_- - p_+)$$
$$= M_1 \delta(P - p_- - p_+). \qquad (2.2)$$

Here P is the total momentum of the bimuonium, p is the relative momentum of the particles making up the bimuonium, and p_- and p_+ are the electron and positron momenta. In the photon Green's function in (2.2) we have set $P_0 = 2\mu - \delta \approx 2\mu$ ($\delta = 1.41$ kev is the bimuonium binding energy, and μ is the muon mass).

The integration over p_0 can be carried out in (2.2), if one makes use of the identity

$$\int \psi(p)\, dp_0 = 2\pi\psi(\mathbf{p}, t = 0). \qquad (2.3)$$

We proceed, carrying all calculations through to linear terms in the relative velocity v. To this accuracy the bimuonium wave function at time t = 0 is of the form [6]

$$\begin{pmatrix} \psi_{00} & \psi_{01} \\ \psi_{10} & \psi_{11} \end{pmatrix} = \begin{pmatrix} f & -f\sigma^T\mathbf{p}/2\mu \\ \sigma\mathbf{p}f/2\mu & 0 \end{pmatrix} \Phi(\mathbf{p}), \qquad (2.4)$$

where $\Phi(\mathbf{p})$ is a solution of the nonrelativistic Schrödinger equation for bimuonium, and f is the two-component spin wave function of bimuonium. To the accuracy we are working with we have

$$\int F(\mathbf{p})\, \Phi(\mathbf{p})\, d^3\mathbf{p} = (2\pi)^3 F(\mathbf{p} = 0)\, \Phi(\mathbf{x} = 0)$$
$$+ \frac{(2\pi)^3}{i} \frac{\partial F(\mathbf{p} = 0)}{\partial p_n} \frac{\partial \Phi(\mathbf{x} = 0)}{\partial x_n}, \qquad (2.5)$$

so that

$$M_1 = -i\pi (2\pi)^4 e^2 \mu^{-2} (\bar{u}\hat{B}v), \qquad (2.6)$$

where

$$B_0 = (i/2\mu)\, \mathrm{Sp}\, [\sigma_2 (f\sigma_k^T + \sigma_k f)\, \nabla_k \Phi(\mathbf{x} = 0),$$
$$\mathbf{B} = -\mathrm{Sp}\, (\sigma_2 \sigma f)\, \Phi(\mathbf{x} = 0). \qquad (2.7)$$

It follows from (2.7) that only the S state of bimuonium with a symmetric spin function (orthobimuonium) can decay into an electron-positron pair, as one may have expected from considerations of charge parity conservation. With our approximations such decay cannot take place from states with higher orbital angular momenta.

Summing over the spins of the final particles and averaging over the initial spin states, we obtain the expression

$$\Gamma_n = e^{10}\mu/6n^3; \qquad (2.8)$$

for the probability per unit time of bimuonium decay into an electron-positron pair from the nS state; for n = 1 this gives $5.6 \times 10^{11}\ \mathrm{sec}^{-1}$.

The two-photon and three-photon bimuonium decay rates can be obtained from the analogous expressions for positronium.[7] For the nS state we have

$$\Gamma_{2\gamma} = e^{10}\mu/2n^3 = 1.66 \cdot 10^{12} n^{-3}\ \mathrm{sec}^{-1}\ \text{(parabimuonium)}, \qquad (2.9)$$

$$\Gamma_{3\gamma} = \frac{4e^2}{9\pi}(\pi^2 - 9)\,\Gamma_{2\gamma} = 1.5 \cdot 10^9 n^{-3}\ \mathrm{sec}^{-1}\ \text{(orthobimuonium)}. \qquad (2.10)$$

It is thus seen that orthobimuonium decays essentially only into an electron-positron pair. We shall henceforth neglect three-photon annihilation.

3. NONRADIATIVE BIMUONIUM PRODUCTION IN ELECTRON-POSITRON COLLISIONS

This process is the inverse of the decay process treated in Sec. 2. We can therefore use the matrix element of (2.6). Recalling (1.1), we see that the cross section may be written

$$\sigma_{\mathrm{non}} = \frac{1}{2}(2\pi)^{-4} \int |M_1|^2\, \mathrm{Im}\, G(P^2)\, d^4P. \qquad (3.1)$$

In the c.m.s., we have

$$G(P^2)\, d^4P = (2\pi)^{-3} d^3P \rho(P_0)\, dP_0; \qquad (3.2)$$

where

$$\rho(P_0)\, dP_0 = \frac{\Gamma}{2\pi} \frac{dP_0}{[P_0 - (2\mu - \delta)]^2 + \Gamma^2/4}. \qquad (3.3)$$

After summing over final spins and averaging over the spins of the initial particles, we obtain*

$$\sigma_{\mathrm{non}}^{(n)} = \frac{\pi e^{10}}{8n^3\mu} \frac{\Gamma_n}{[P_0 - (2\mu - \delta)]^2 + \Gamma_n^2/4}, \qquad (3.4)$$

where E is the initial energy of the electron (or positron). At resonance (i.e., when $E = \mu - \delta/2$), we find the cross section given by

$$\sigma_{\mathrm{non}}^{(n)} = 3\pi/n^3\mu^2 = 0{,}33 \cdot 10^{-24} n^{-3}\,\mathrm{cm}^2 \qquad (3.5)$$

As is well known, when high-energy electrons and positrons interact, the radiative corrections become important, and the most important contribution comes from the so-called doubly logarithmic terms.[8,9] The radiative corrections to bimuonium production can be calculated in the way it was done by Kheĭfets with one of the present authors.[9] Assuming that the maximum energy the photons can carry off is of the order of the level width, we obtain

*The transformation of an electron-positron pair into a $\mu^+\mu^-$ pair was first discussed by Berestetskii and Pomeranchuk.[13]

$$\sigma_R^{(1)} = \sigma_{\text{non}}^{(1)} \exp\left(-\frac{4e^2}{\pi}\ln\frac{\mu}{m}\ln\frac{\mu}{\Gamma}\right) = 0.27\,\sigma_{\text{non}}^{(1)}. \quad (3.6)$$

where m is the electron mass.

4. RADIATIVE BIMUONIUM PRODUCTION IN ELECTRON-POSITRON COLLISIONS

Also of interest is bimuonium production accompanied by emission photons. It is clear that this process can be of interest only in the neighborhood of the threshold, and we therefore need consider only soft gamma radiation for which

$$\omega/\mu \ll 1. \quad (4.1)$$

The photon emission process may take place in such a way that one of the photons carries more energy than all the others. This process can be treated by perturbation theory, the criterion for the validity of which is of the form[10]

$$e^2 \ln\frac{\mu}{m} \ll 1, \qquad \frac{e^2}{\pi}\ln\frac{\mu}{m}\ln\frac{\mu}{\omega_{min}} \ll 1. \quad (4.2)$$

This equation shows that we may set ω_{min} roughly equal to the width of the 1S state of bimuonium,[8,2] namely

$$\omega_{min} = \Gamma_1 \approx 4\cdot 10^{-4}\ \text{ev} \quad (4.3)$$

and thus that we may use perturbation theory to treat the region of maximum interest, in which

$$\Gamma < \omega \ll \mu. \quad (4.4)$$

The case of softer gamma emission was treated in Sec. 3.

Diagrams for this process are shown in Figs. 3 and 4. For nonrelativistic bimuonium energies, the diagrams of Fig. 3 correspond to orthobimuonium production, while the diagrams of Fig. 4 correspond to parabimuonium. There is thus no interference between the corresponding matrix elements. Simple calculations show that parabimuonium production is less likely by a factor of $(\omega/\mu)^2$ than orthobimuonium production. Therefore we shall henceforth restrict our considerations to the diagrams of Fig. 3. In the c.m.s. the matrix element for these diagrams can be written

$$M = -ie^3(2\pi)^4\sqrt{\frac{2\pi}{\omega}\frac{4\pi}{P^2}}\,\bar{v}\left[\hat{B}^{(1)}\frac{i(\hat{p}_- - \hat{k}) - m}{(p_- - k)^2 + m^2}\hat{e} + \hat{e}\,\frac{i(\hat{p}_- - \hat{P}) - m}{(p_- - P)^2 + m^2}\hat{B}^{(1)}\right] u\,\delta(P + k - p_- - p_+), \quad (4.5)$$

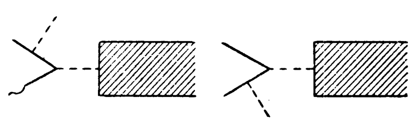

FIG. 3

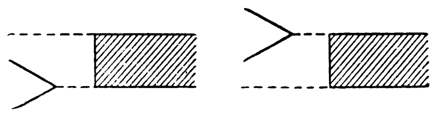

FIG. 4 .

where

$$B_\mu^{(1)} = \text{Sp}\int C\gamma_\mu \psi(p)\, d^4p. \quad (4.6)$$

In order to calculate $B^{(1)}$, one must know the bimuonium wave function in the electron-positron c.m.s. Following Alekseev[6] and making use of (4.1), one easily arrives at the conclusion that up to factors linear in $|\mathbf{P}|/\mu = \omega/\mu$ and v, and at time t = 0

$$\begin{pmatrix}\psi_{00} & \psi_{01}\\ \psi_{10} & \psi_{11}\end{pmatrix} = \begin{pmatrix} f & f\sigma^T(\mathbf{P} - \mathbf{p})/2\mu \\ \sigma(\mathbf{P} + \mathbf{p})f/2\mu & 0\end{pmatrix}\Phi(\mathbf{p}). \quad (4.7)$$

Recalling (2.3) and (2.5), averaging over the spins of the initial particles and summing over final spins, one finds that the differential cross section for radiative production of orthobimuonium in the nS state may be written*

$$d\sigma_{\text{rad}}^{(n)} = \frac{1}{4n^3}\frac{e^{12}(\mathbf{e}\mathbf{p}_-)^2}{\mu^2(E\omega - \mathbf{p}_-\mathbf{k})^2}, \quad (4.8)$$

where

$$\omega \approx 2E - (2\mu - \delta).$$

After summing over the photon polarizations and integrating over the angle of emission, one finds the total cross section to be†

$$\sigma_{\text{rad}}^{(n)} = \frac{\pi}{2n^3}\frac{e^{12}}{\mu^2}\left(2\ln\frac{\mu}{m} - 1\right)\frac{\mu}{\omega}, \quad (4.9)$$

$$\sigma_{\text{rad}} = \sum_n \sigma_{\text{rad}}^{(n)} = 3.6\pi\cdot 10^{-38}\frac{\mu}{\omega}\text{cm}^2. \quad (4.10)$$

As for bimuonium states with higher angular momenta, to the accuracy with which we are working they do not occur.

5. PROBABILITY OF OBSERVING BIMUONIUM

Since electrons and positrons in accelerator beams have some spread in energy, all the expressions obtained must be averaged over their energy spectra. If the energy distribution function of the electrons is $\rho_1(E_1)$, while that of the positrons is $\rho_2(E_2)$ and the cross section for the process is $\sigma(E_1, E_2)$, the averaged cross section becomes

*This same equation can be obtained by an elementary calculation in which the final muons are described by plane waves and the production of the bound state is accounted for by including the factor $|\Phi(0)|^2$.[11]

†One need not take into account the instability of the final state due to (1.1).

$$\langle\sigma\rangle = \int \sigma(E_1, E_2)\rho_1(E_1)\rho_2(E_2)\,dE_1\,dE_2. \quad (5.1)$$

Assuming for simplicity that the electrons and positrons are uniformly distributed in energy over some interval ΔE close to threshold, we obtain

$$\langle\sigma_{\mathrm{rad}}\rangle = 3.6\,\pi\cdot 10^{-38}\frac{\mu}{\Delta E}\left(\ln\frac{\Delta E}{\omega_{min}} - 1\right)\mathrm{cm}^2, \quad (5.2)$$

$$\langle\sigma_R\rangle = 0.16\,\pi^2\frac{e^{10}}{\mu^2}\frac{\mu}{\Delta E}. \quad (5.3)$$

It is seen from this that measurable cross sections (of the order of 10^{-31} cm^2) can be obtained only with narrow energy distributions in the beam ($\Delta E \sim 1$ kev), which is quite difficult to do with modern equipment.

As orthobimuonium decays into an electron-positron pair, its existence will manifest itself as an essentially isotropic supplementary contribution to the cross section for ordinary elastic electron-positron scattering, namely

$$\frac{d\sigma_{\mathrm{sup}}}{d\Omega} = \frac{\langle\sigma_R\rangle + \langle\sigma_{\mathrm{rad}}\rangle}{4\pi}. \quad (5.4)$$

setting $\Delta E = 1$ kev, we obtain

$$d\sigma_{\mathrm{sup}}/d\Omega = 1.6\cdot 10^{-32}\ \mathrm{cm}^2/\mathrm{sr}; \quad (5.5)$$

for comparison we mention that for $\theta = 90°$, the elastic scattering cross section in the neighborhood of the threshold for bimuonium production is 1.1×10^{-30} cm^2/sr.

We note that the energy gap corresponding to $\Delta\nu = 4.2 \times 10^{13}$ cycles between the orthobimuonium and parabimuonium ground states makes it possible to induce transitions between them by means of electromagnetic radiation of this frequency, and thus to attenuate decay from the orthobimuonium state (parabimuonium decays into two photons). Calculations show that the decay rate of the order of 10^{11} sec^{-1} requires a radiation density of about 1 erg/cm^3-cycle.

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265