DISPERSION THEORY OF DIRECT NUCLEAR REACTIONS

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It is assumed that the main contribution to the amplitude of a direct nuclear process comes from Feynman diagrams whose singularities are closest to the physical region of the variables. Using energy dispersion relations, a singular integral equation is derived which takes account of interaction in the initial and final states. This equation admits of a simple iteration procedure, the first iteration yielding the distorted wave method. It is found that, apart from pole diagrams corresponding to the Butler mechanism and also to exchange stripping and heavy pick-up reactions, more complex diagrams can also give a significant contribution to the direct process mechanism. This result is illustrated by the reactions Be^9 (d, n) B^{10}, Be^9 (α, t) B^{10}, and C^{12} (d, p) C^{13}. The mechanism of some reactions of the (x, yz) type, and in particular, of reactions in which clusters are knocked out, is considered from the same viewpoint.

1. INTRODUCTION

1. Formulation of the problem. A large amount of experimental data indicates that the direct reactions of the type

$$A + x \to B + y, \tag{1a}$$

$$A + x \to B + y + z \tag{1b}$$

are well described by Feynman diagrams with a small number of internal lines. The simplest pole diagram for deuteron stripping has been considered by Amado,[1] and corresponds to the Butler theory of stripping. The present paper is devoted to a further application of dispersion relations to the theory of direct processes. We shall mainly be concerned with processes of the type (1a). In this section we explain the notation and formulate the basic theorems of our work.

2. Kinematical relations. Reaction (1a) is characterized by two independent kinematic variables. These can be chosen as any two of the following three quantities: a) the kinetic energy E, of the colliding particles, b) the square of the momentum transfer,

$$q^2 = (\mathbf{p}_y - \mathbf{p}_x)^2, \tag{2}$$

where $\mathbf{p}_x$ and $\mathbf{p}_y$ are the momenta of the particles x and y, and c) the square of the sum of the momenta of particles x and y,

$$p^2 = (\mathbf{p}_x + \mathbf{p}_y)^2. \tag{3}$$

In the center-of-mass system of the colliding particles, which will be used throughout the following considerations, these three variables are connected through the simple relation

$$q^2 + p^2 = 4(m_{xA} + m_{yB})E + 4m_{yB}Q. \tag{4}$$

Here m_{xA} and m_{yB} are the reduced masses of particles x and y, and Q is the energy released in the reaction, or its threshold:

$$Q = m_A + m_x - m_B - m_y \tag{5}$$

(we assume throughout $\hbar = c = 1$).

The variables q^2 and E will be chosen most frequently as independent variables. However, in certain cases (exchange stripping, heavy pick-up) it is convenient to use p^2 instead of q^2.

3. Unitarity and analyticity. The unitarity condition

$$SS^+ = 1 \tag{6}$$

for the S matrix written in the form

$$S = 1 + i(2\pi)^4 T; \tag{7}$$

$$T = \mathcal{B} + i\mathcal{A}, \quad \mathcal{A} = \mathcal{A}^+, \quad \mathcal{B} = \mathcal{B}^+, \tag{8}$$

leads to the well known formula

$$\mathcal{A}_{if} = \frac{(2\pi)^4}{2}\sum_n T_{in}T^+_{nf}. \tag{9}$$

The summation (integration) in (9) goes over all intermediate states n for which the transitions $i \to n$, $n \to f$ are allowed by the conservation laws. The matrices T and $\mathcal{A}$ are of the form

$$T_{kl}(q^2, E) = M_{kl}(q^2, E)\,\delta_{\lambda_k\lambda_l}\delta^4(l-m), \quad (10)$$

$$A_{kl}(q^2, E) = A_{kl}(q^2, E)\,\delta_{\lambda_k\lambda_l}\delta^4(l-m). \quad (11)$$

The arguments of the δ functions in (10) and (11) are the momenta and energies of the states k and l; the index λ denotes the set of discrete quantum numbers. The quantity A_{kl} is the absorptive part of the amplitude M_{kl}. The basic postulate of the theory of dispersion relations is the assumption that the amplitudes $M_{kl}(q^2, E)$ are analytic functions of their arguments. The amplitude $M_{kl}(q^2, E)$ has singular points (poles or branch points) and is, therefore, in general a function defined on several sheets. In the theory of dispersion relations one is concerned with only one of these sheets, called the physical sheet. $M_{kl}(z)$ satisfies the relation

$$M_{kl}(Z^*) = M^+_{kl}(Z), \quad (12)$$

where

$$Z = q^2, E.$$

2. THE REACTION A + x → B + y

1. Pole diagrams. The part of the sum (9) which corresponds to transitions f → n in which one particle b of those emitted in the transition i → n is absorbed, can be written in the form

$$A_{if} = 2\pi m_b \delta(p_b^2 - 2m_bE_b)\sum_{s_b} M_{ib}M^+_{bf}. \quad (13)$$

The summation in (13) goes over the spin variable s_b of particle b; m_b, p_b, and E_b are the mass, momentum, and energy of the intermediate particle b. Using (13), it is easy to show that the amplitude M_{if} has a pole at

$$p_b^2 = 2m_bE_b$$

and is near this pole of the form

$$M_{if} = 2m_b\frac{\Sigma s_b M_{ib}M^+_{bf}}{p_b^2 - 2m_bE_b - i\eta}, \qquad \eta \to +0. \quad (14)$$

The pole amplitude (14) is described by a Feynman diagram with a single internal line. Figure 1a shows Amado's diagram corresponding to the Butler theory of stripping. For example, in the reaction (d, p) we have b = n, in the reaction (He^3, p), b = d, etc. The diagram of Fig. 1b corresponds to a pick-up process. For the reaction (p, d) we have b = n, for the reaction (n, α), b = He^3, etc.

Figure 1c shows a diagram corresponding to exchange stripping and heavy pick-up. For example, in the reaction B^{11}(d, n)C^{12} we have

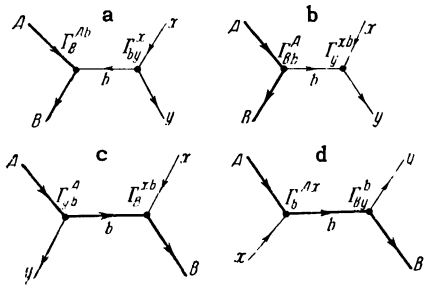

FIG. 1. a – pole diagram for the stripping reaction, b – pole diagram for the pick-up reaction, c – pole diagram for the exchange stripping and heavy pick-up reaction, d – quasi-compound process.

b = B^{10}. The diagram of Fig. 1d is reminiscent of the formation and decay of a compound nucleus, but the actual situation is somewhat more complicated. The point is that the compound nucleus is related to complex poles situated on the nonphysical sheet. The Feynman diagrams corresponding to the compound nucleus on the physical sheet are complicated and their singular points are branch points, not poles. We emphasize in this connection that the poles of the diagrams of Fig. 1 a to d lie on the real axis and correspond to such states of the nuclei b which can not decay under emission of nuclear particles (β decay or radiative transitions are not excluded). In this sense the diagram of Fig. 1d corresponds to a direct reaction which may be called a quasi-compound process. The diagrams of Fig. 1 a to d describe all possible pole terms in the amplitudes of direct reactions of the type (1a). From formula (14), which corresponds to these diagrams, one can immediately derive a number of important features of direct processes of the type (1).

For the ordinary stripping reaction (Fig. 1a) we have according to the energy and momentum conservation laws

$$p_b^2 - 2m_bE_b = q^2 + 2m_b[\mu_{yB}\varepsilon^B_{AB} + (1-\mu_{yB})\varepsilon^x_{yb} + (\mu_{yB} - \mu_{xA})E]. \quad (15)$$

Here

$$\varepsilon^\alpha_{\beta\gamma} = m_\beta + m_\gamma - m_\alpha \quad (16)$$

is the binding energy of particles β and γ in the nucleus α, and

$$\mu_{\alpha\beta} = m_{\alpha\beta}/m_\alpha. \quad (17)$$

If $m_{\alpha\beta} \approx m_\alpha$, formula (15) goes over into the simple relation

$$p_b^2 - 2m_bE_b \approx q^2 + 2m_b\varepsilon^B_{Ab}. \quad (18)$$

It is seen from (15) and (18) that the amplitude

for the stripping reaction has a pole in the variable q^2 for nonphysical values of this variable ($q^2 < 0$). It follows that the amplitude assumes its largest value at the smallest physically admissible value of q^2. This occurs when the particles y are emitted in the same direction as that of the incoming particles x.

In the pick-up reaction (Fig. 1b) the pole also lies in the region of nonphysical values of q^2:

$$q^2 = -2m_b[\mu_{yB}\varepsilon^A_{Bb} + (1-\mu_{yB})\varepsilon^y_{xb} + (\mu_{xA}-\mu_{yB})E]. \quad (19)$$

The best approximation to the pole is given by the smallest physically possible values of q^2. The angular distribution must have a maximum in the forward direction.

In exchange stripping and heavy pick-up reactions (Fig. 1c), the pole occurs at nonphysical values of the variable p^2:

$$p^2 = -2m_b[(1-\mu_{By})\varepsilon^B_{xb} + \mu_{By}\varepsilon^A_{yB} + (\mu_{xA}-\mu_{By})E]. \quad (20)$$

If m_A, $m_B \gg m_x$, m_y, then $\mu_{By} \ll 1$, $\mu_{xA} \approx 1$ and

$$p^2 = -2m_b(E + \varepsilon^B_{xb}). \quad (21)$$

It follows from (20) and (21) that exchange stripping and heavy pick-up should be most important in exothermal reactions, where E can be small. It is also clear that the angular distribution should have a backward minimum.

In the quasi-compound process (Fig. 1d) the pole occurs at negative values of the energy E:

$$E = -\varepsilon^b_{xA}. \quad (22)$$

It follows that the angular distribution will be isotropic for the quasi-compound process. For this reason it is difficult to separate the contribution of the quasi-compound process from that of reactions which go through the compound nucleus stage.

The assertions made above on the angular distributions would be exact if the amplitudes $M_{if}(q^2, E)$ had no other singular points besides the pole corresponding to the particle b [in this case the numerator of formula (14), as the residue of an analytic function, is a constant]. If there are other singular points, we can still write the amplitude in the form (14), but the numerator will not be a constant any more. It will be a slowly varying function of the variables q^2 and E if the other singularities are much farther removed from the boundary of the physical region than the pole under consideration. In the opposite case the variation of the numerator of (14) with q^2 and E in the physical region can be just as or even more important than the variation of the denominator.

It should be emphasized here that we must consider not only the singular points on the physical sheet but also those on the other sheets (for example, if we are near a pole corresponding to a compound nucleus level, the amplitude of the reaction will be a sensitive function of E although the singular points on the physical sheet may lie far away from the considered region of values of E). Since the nucleus has a radius R, the point $|q^2| = 1/R^2$ is of special physical significance. In terms of the theory of dispersion relations, this means that the reaction amplitude has a singularity in this region of values of the variable q^2. On the other hand, it is very difficult to tell immediately which Feynman diagram corresponds to this singularity. This can be done only in the case of the deuteron. The dimensions of heavier nuclei are, apparently, determined by singularities in the variable q^2 which lie on the nonphysical sheet.* We emphasize that this last assertion is to be regarded as a hypothesis. It is important mainly for the derivation of dispersion relations in q^2 on the basis of the diagrams considered in part 3 of this section. In dealing with pole diagrams it is sufficient to have in mind, first, that the singularity which determines the dimensions of the nucleus is not a pole on the physical sheet and, second, that the numerator of (14) must be regarded as a function of qR. This corresponds to the Butler theory, which is therefore described by the pole diagrams of Figs. 1a and 1b.

Formula (14) can be rewritten appropriately by introducing the vertex parts Γ representing averages over the spin variable s_b. Thus we write for the diagram of Fig. 1a, for example,

$$\overline{M_{ib}M^+_{bf}} \equiv \pi m_b^{-1}\Gamma^{\nu}_{by}(qR_x, s_x, s_y)\,\Gamma^{Ab}_{B}(qR_B, s_A, s_B). \quad (23)$$

The upper indices of Γ denote the variables of the particles coming into the vertex and the lower indices correspond to the particles coming out of the vertex. R_x and R_B are the effective radii of the vertices, and s_A, s_B, s_x, and s_y are the spin variables of the particles A, B, x, and y.

*It was noted by V. N. Gribov that the singularity corresponding to the radius of the nucleus may be due to the finite range of the nucleon-nucleon forces. It is, therefore, possible that this singularity corresponds to complicated Feynman diagrams with internal π-meson lines. The main motivation for this point of view comes from the fact that, contrary to the deuteron case, the nuclear three body problem has no solution in the approximation of δ-function forces between the nucleons.

With the new notation, formula (14) for the diagram of Fig. 1a takes the form

$$M_{if} = 2\pi\Gamma_B^{Ab}\,\Gamma_{by}^{x}/(q^2 + \varkappa_b^2), \tag{24}$$

where the quantity κ_b^2 is defined by formula (15). The differential cross section has the form

$$\frac{d\sigma}{d\Omega_y} = m_{xA}m_{yB}\,\frac{(2J_B+1)(2J_y+1)}{(2J_A+1)(2J_x+1)}\,\frac{p_y\,\overline{(\Gamma_{Ab}^{B})^2}\;\overline{(\Gamma_{by}^{x})^2}}{p_x\,(q^2+\varkappa_b^2)^2}. \tag{25}$$

Here*

$$\overline{(\Gamma_{b\beta}^{\alpha})^2} = \overline{(\Gamma_{\alpha}^{b\beta})^2} = \sum_{s_\alpha, s_\beta} |\Gamma_{b\beta}^{\alpha}|^2, \tag{26}$$

and $d\Omega_y$ is the element of solid angle in the momentum space of particle y.

The values of the quantities $\left[\overline{(\Gamma_{b\beta}^{\alpha})^2}\right]^{1/2}$ at the pole (i.e., for $q^2 = -\kappa_b^2$) are called the reduced vertex parts and are denoted by $\gamma_{b\beta}^{\alpha}$:

$$\gamma_{b\beta}^{\alpha} = \left[\overline{(\Gamma_{b\beta}^{\alpha})^2}\right]^{1/2}_{q^2=-\varkappa_b^2}. \tag{27}$$

The reduced vertex part γ_{Ab}^{B} in deuteron stripping or pick-up reactions (b = p, n) is given in terms of the reduced width of the reaction and the reduced vertex part γ_{pn}^{d} represents the normalization factor of the internal deuteron wave function.

Through the introduction of the reduced vertex parts we have achieved a uniform parametrization of the theory of direct processes. What is important here is that the cross sections of the various processes can involve the same reduced vertex parts (for example, the cross sections for the reactions A(d, n)B and A(α, t)B contain the same reduced vertex part γ_{Ap}^{B}). It is therefore possible to establish a quantitative connection between different types of direct nuclear reactions. It should be emphasized, however, that it is not easy to carry through such a program, since the pole type mechanism of direct processes is not the only and not even always the most important mechanism (see part 3 of this section). In cases where the pole diagram gives the dominating contribution to the reaction amplitude, the reduced vertex parts can be determined from the experimental data by extrapolating formula (25).†

In formula (25) we have summed over the spin variables of the emitted particles y. It is, however, easy to convince oneself that the polarization of the particles y vanishes in the pole approximation if the beam of particles x and the target nucleus A are unpolarized and the polarization of the residual nuclei B is not fixed in the experiment. This result has a simple physical interpretation. It is seen from (25) that the whole process can be regarded as consisting of two independent processes characterized by the two vertices. One of these vertices represents the decay of an unpolarized particle. There will thus be a correlation between the polarizations of the particles into which the decay takes place, and the summation over the spin variable of one of these particles (b) causes the vanishing of the polarization of the other particle (y or B).

We note further that the effect of the interference between different pole diagrams should be small as a rule, since the poles corresponding to interfering diagrams lie in different regions of the variables (an exception is the interference between the diagrams of Figs. 1c and 1d).

2. Interaction in the initial and final states. The interaction in the initial and final states is described by terms in the unitarity condition which correspond to the transitions

$$i \to n = A + x \to A' + x', \qquad n \to f = A' + x' \to B + y;$$
$$i \to n = A + x \to B' + y', \qquad n \to f = B' + y' \to B + y.$$

The diagrams corresponding to these transitions are shown in Figs. 2a and b. In the first case the particle x is scattered by the nucleus A, after which the nuclear reaction takes place. In the second case (Fig. 2b) the first transition is the nuclear reaction, while the last transition is a scattering of particle y by nucleus B. In both cases the intermediate states contain two virtual particles. Integration over momenta and energy of the intermediate particles leads to the formula

$$A_{xy} = \frac{p_x}{4\pi}\int f_{x'x}^{*} M_{x'y}\,d\Omega_{x'} + \frac{p_y}{4\pi}\int M_{y'x}^{*} f_{y'y}\,d\Omega_{y'}, \tag{28}$$

where $A_{xy} \equiv A_{if}$, $M_{xy} \equiv M_{if}$; $f_{x'x}$ and $f_{y'y}$ are the scattering amplitudes

* By definition

$$\Gamma_{\beta\cdot\gamma}^{\alpha} = (\Gamma_{\alpha}^{\beta\cdot\gamma})^{*}.$$

†To carry out the extrapolation, we must first "weaken" the dependence of the vertex parts on qR. Experience indicates then that one should divide $d\sigma/d\Omega_y$ by $[j_{l_B}(qR_B) \times j_{l_x}(qR_x)]^2$. The order of the spherical Bessel functions l_B and l_x and the radii R_B and R_x are chosen such that the angular distribution is "smoothed out." The extrapolation into the nonphysical region can then be carried out immediately with the help of (25). The guiding consideration in choosing R_B and R_x is the circumstance that these quantities must be close to the radii of the nuclei B and x. The numbers l_B and l_x can be interpreted as the orbital angular momenta of the particle b in the nuclei B and x. We emphasize that this interpretation is essential for finding the angular momenta and parities of the states of the nucleus B, but for the extrapolation procedure itself this identification only serves as a guiding principle in choosing the smoothing factor.

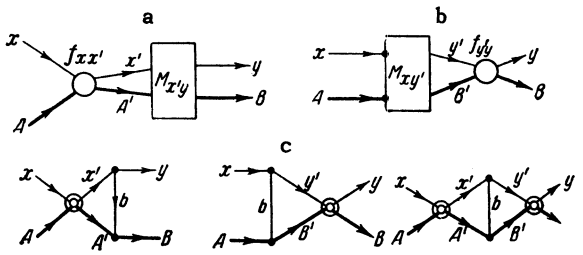

FIG. 2. General diagrams taking account of the interaction in the initial (2a) and final (2b) states, c – inclusion of the initial and final state interactions for the simplest (pole type) reaction mechanism.

$$f_{x'x} = \frac{m_{xA}}{2\pi} M_{x'x}, \qquad f_{y'y} = \frac{m_{yB}}{2\pi} M_{y'y}; \tag{29}$$

$$p_x = \sqrt{2m_{xA}E}, \qquad p_y = \sqrt{2m_{yB}(E+Q)}; \tag{30}$$

$d\Omega_{x'}$ and $d\Omega_{y'}$ are elements of solid angle in the momentum space of the intermediate particles x' and y'. The integration sign in (28) also implies summation over the spins of the intermediate particles.

If the inelastic process $A + B \rightleftarrows B + y$ gives a small contribution to the scattering amplitudes $f_{x'x}$ and $f_{y'y}$, so that the latter can be regarded as independent quantities, the following relation holds:

$$\int f^*_{x'x} M_{x'x}\, d\Omega_{x'} = \int f_{xx'} M^*_{yx'}\, d\Omega_{x'}. \tag{28a}$$

[Under these assumptions this equation is a consequence of the fact that one can interchange the positions of the matrices T and T^+ in formula (9)].

In order to construct the entire amplitude from the absorptive part of (28), we must know the position of the branch points of the amplitude $M_{xy}(E)$ in the variable E. We shall assume that these branch points are determined by the simplest Feynman diagrams shown in Fig. 2c. The first diagram gives the branch point E = 0, the second, E = −Q, and the third gives two branch points, E = 0, and E = −Q. It follows that for Q < 0 the amplitude is analytic in the complex E plane with a cut along the real axis from 0 to ∞. If Q > 0, the cut will start at the point E = −Q. Taking this into account, we have the following dispersion relation in the variable E:

$$M_{xy}(E) = M^0_{xy}(E) + \frac{1}{\pi}\int_{E_0}^{\infty} \frac{A_{xy}(E')}{E' - E - i\eta}\, dE'. \tag{31}$$

Here $M^0_{xy}(E)$ is the sum of the two pole terms and

$$E_0 = \begin{cases} 0, & Q<0 \\ -Q, & Q>0 \end{cases}. \tag{32}$$

Formula (31) represents an integral equation for $M_{xy}(E)$ whose kernel is given in terms of the scattering amplitudes $f_{x'x}$ and $f_{y'y}$. The latter can be taken from experiment or calculated on the basis of the optical model. Equations of the type (31) have been considered by Omnes,[2] who indicated methods for an exact solution. In order to establish the connection between Eq. (31) and the distorted wave method (DWM) usually employed in the theory of direct reactions, we must consider the iterations of this equation. The zeroth iteration,

$$M_{xy} = M^0_{xy} \tag{33}$$

gives the Butler theory. The first iteration leads to terms which correspond to the distorted wave method:

$$M^{(1)}_{xy} = M^0_{xy} + \frac{1}{4\pi^2}\int_{E_0}^{\infty}\int \frac{dE'\, d\Omega_{x'}}{E' - E - i\eta} p_x(E')\, M^0_{x'y}(E')\, f_{xx'}(E') + \frac{1}{4\pi^2}\int_{E_0}^{\infty}\int \frac{dE'\, d\Omega_{y'}}{E' - E - i\eta} p_y(E')\, f^{(E')}_{y'y} M^0_{xy'}(E'). \tag{34}$$

In deriving (34) we have made use of the reality of the pole term M^0_{xy} in (28a). The reaction amplitude in the approximation of the DWM can be written in the form

$$M_{xy}\ (\text{DWM}) = \int \psi^+_y(\mathbf{p}_y, \mathbf{p}'_y)\, M^0_{x'y'}(\mathbf{p}'_x, \mathbf{p}'_y)\, \psi_x(\mathbf{p}_x, \mathbf{p}'_x)\, d^3p'_x\, d^3p'_y, \tag{35}$$

where ψ_x and ψ_y are the wave functions of the particles x and y.

The identity of (34) with the corresponding terms of (35) is easily established by using

$$\psi_x = \delta(\mathbf{p}_x - \mathbf{p}'_x) + \frac{1}{2\pi^2}\frac{f_{xx'}(\mathbf{p}_x, \mathbf{p}'_x)}{p'^2_x - p^2_x - i\eta}, \tag{36}$$

$$\psi_y = \delta(\mathbf{p}_y - \mathbf{p}'_y) + \frac{1}{2\pi^2}\frac{f^*_{yy'}(\mathbf{p}_y, \mathbf{p}'_y)}{p'^2_y - p^2_y + i\eta}. \tag{36a}$$

Besides the terms contained in (34), the amplitude (35) also has a term in which the product of the scattering amplitudes $f_{xx'}$ and $f_{y'y}$ enters. This term is obtained from (31) in the second iteration. However, in this order still more terms appear which contain products of like amplitudes, $f_{xx'}f_{x'x''}$ and $f_{y'y''}f_{y''y}$, and correspond, therefore, to "double scattering" of the particles x and y by the nuclei A and B. Since these terms do not appear in (35), it follows from our analysis that the account of the product $f_{xx'}f_{y'y}$ in the DWM is in excess of the accuracy of the approximation. Reliable information on the accuracy of the DWM can be obtained by comparing the numerical solution of (31) with the amplitude (35).

Let us restrict ourselves to the case where the iteration procedure converges rapidly (this con-

dition is probably too stringent). This means that

$$\bar{k}|\bar{f}|/4\pi^2 \ll 1, \tag{37}$$

where $\bar{k}$ and $\bar{f}$ are certain effective values of the wave number and the scattering amplitude giving the most important contribution to the integrals in (31) and (34). Expressing $|\bar{f}|$ in terms of the scattering cross section $\bar{\sigma}_s$, we obtain the inequality

$$\bar{k}\sqrt{\bar{\sigma}_s}/4\pi^2 \ll 1. \tag{38}$$

For an estimate we set, in the case of neutrons, $\bar{\sigma}_s \approx \pi R^2$, and find from (38)

$$\bar{k}R/4\pi^{3/2} \ll 1, \tag{38a}$$

which is equivalent to the long wavelength approximation.

We can tell by looking at (31) and (28) under what conditions the initial and final state interactions do not appreciably distort the angular distribution of the emitted particles. This will be the case when the scattering amplitudes $f_{xx'}$ and $f_{y'y}$ are nearly δ functions in the scattering angles, i.e., if the scattering goes mainly into small angles. This situation obtains approximately in the case of nucleons with energies higher than 10 Mev.

Of the experimentally observed quantities, the polarization of the emitted particles is the most sensitive to the inclusion of the initial and final state interactions. It should be kept in mind, however, that the interaction in the initial and final states is not the only source of the polarization in direct processes. A significant contribution to the polarization of the outgoing particles may come from more complicated diagrams, which will be considered in part 3 of this section.

3. Triangular singularities. The pole mechanism of direct processes considered in the preceding discussion can predominate if the amplitude $M_{xy}(q^2)$ has, besides the poles, no other singular points lying close to the physical region or in the neighborhood of the poles. This situation, however, is realized only seldom, so that the direct reaction mechanism can be quite different from the pole mechanism and in particular, from the Butler mechanism. Let us consider here some Feynman diagrams which correspond to branch points in q^2.

We restrict our considerations to diagrams with three internal lines. The general type of such diagrams is shown in Fig. 3. The direct mechanism corresponding to the diagram of Fig. 3a, for example, consists in the following. The

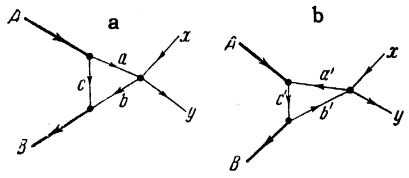

FIG. 3. The simplest triangular diagrams for the reaction A(x, y)B.

nucleus A emits particle a, which collides with particle x. As a result of the reaction

$$a + x \to y + b. \tag{39}$$

the particle b is formed, which is then captured by the nucleus c with formation of the final nucleus B. The three line vertices in this diagram are functions of q'R (as in the case of pole diagrams), where q' are the momenta transferred in the vertex. The four line vertex is the amplitude for the process (39), which also depends in general on q^2.

The singularities of the diagram of Fig. 3 considered below have nothing to do with the dependence of the vertices on q^2 and are determined only by the masses of the particles A, B, x, y, a, b, and c. The branch point closest to the physical region corresponding to the diagram of Fig. 3 is found by the general rules formulated by Landau[3] and developed in the paper of Okun' and Rudik.[4] In our case these rules lead to the formula

$$\tfrac{1}{2}q^2 = -(m_y - m_x)[Q' - \mu_{yB}Q + (\mu_{yB} - \mu_{xA})E] - m_a m_b (\sqrt{\varepsilon^A_{ac}/m_{ac}} + \sqrt{\varepsilon^B_{bc}/m_{bc}})^2. \tag{40}$$

Here

$$Q' = m_a + m_x - m_b - m_y. \tag{41}$$

The analogous formula for the diagrams of the type of Fig. 3b is obtained from (40) by making the interchanges $x \rightleftarrows A$, $y \rightleftarrows B$.

The singularity defined by (40) belongs to the class of the so-called "anomalous thresholds," which were first considered by Karplus, Sommerfield, and Wichmann.[5] As is known, these singularities occur in those cases where the following inequalities are satisfied for the vertices, e.g., the vertex $A \to a + c$:

$$m_A < m_a + m_c, \tag{42}$$

$$m_A^2 > m_a^2 + m_c^2. \tag{42a}$$

This is precisely the situation realized in nuclear reactions. The appearance of the branch points (40) is connected with the fact that for the values q^2 given by (40) the relation between the energy and the momentum of the intermediate particles is the same as for free particles. Since, on the other hand, the real decay $A \to a + c$ is impossi-

Singular points of the diagram of Fig. 3a with various virtual particles a, b for the reaction $Be^9(d, t)B^{10}$

a		n	p	d	t
b	p	d	p+p	He^3	α
$-q^2$, Mev × a.m.u.	2306 (pole)	62.8	299	467	432

ble because of the inequality (42), the singular point (40) lies always in the region of nonphysical (negative) values of q^2.

Let us now consider some examples of nuclear reactions on the basis of the foregoing discussion. In the table we have listed the values of $-q^2$ for the reaction $Be^9(\alpha, t)B^{10}$, computed according to (40) with E = 27.7 Mev. The first number in the table is the value of $-q^2$ at the pole [Fig. 1a, formula (15), b = p]. The other numbers in the table give the branch points of the graph of Fig. 3a for various virtual particles a, b, and c. All data in the table refer to the ground states of the nuclei A, B, and c (the singular points corresponding to excited states of the virtual nucleus c lie in the region of higher values of $-q^2$).

The table shows that the branch point closest to the pole is given by the graph with a = n, b = d, and c = Be^8. The presence of a branch point so close to the pole implies that the mechanism of the reaction $Be^9(\alpha, t)B^{10}$ is certainly not of the pure Butler type. The graph of Fig. 3a will give a contribution to the reaction amplitude which is comparable with that of the pole diagram. Indeed, the boundary of the physical region lies at q^2 = 16.8 Mev × a.m.u.* Thus the ratio of the square of the pole amplitude and the interference term due to the presence of the diagram of Fig. 3a can be of order unity, since the part of the amplitude corresponding to this diagram drops off roughly like $1/(q^2 + q_0^2)$, where q_0^2 is the singular point determined by (40).

The situation is similar in the case of the deuteron stripping reaction $Be^9(d, n)B^{10}$. Here the scattering amplitude has a pole at $q^2 = -13.2$ Mev × a.m.u. and the branch point corresponding to the diagram of Fig. 3a with a = n, b = d, and c = Be^8 lies at $q^2 = -48.4$ Mev × a.m.u. There is hence no reason to expect that the Butler mechanism plays the most important role in the reaction $Be^9(d, n)B^{10}$. In the light of our discussion, the results of Vlasov and co-workers [6] now become perfectly understandable. These results are that the ratios of the reduced widths of the reactions leading to the formation of the nucleus B^{10} in various states are different in the reactions $Be^9(d, t)B^{10}$ and $Be^9(d, n)B^{10}$.

In a number of cases the closest branch points lie much farther away from the pole than in the case of the reactions $Be^9(d, t)B^{10}$ and $Be^9(d, n)B^{10}$. For example, in the reaction $C^{12}(d, p)C^{13}$ the pole lies at $q^2 \approx -12$ Mev × a.m.u. and the nearest branch point (diagram of Fig. 3a, a = b, b = d, c = B^{11}) lies at $q^2 = -200$ Mev × a.m.u. However, even in this case the relative importance of the pole diagram and the diagram of Fig. 3a can be estimated only if there are data on the reduced vertex parts and the amplitudes of the reaction (39). Thus the knowledge of the amplitudes of nuclear reactions with light nuclei is of prime importance for the understanding of the direct interaction mechanism in complex nuclei.

An essential factor in the investigation of the direct interaction mechanism is the measurement of the polarization of the reaction products. Contrary to the situation in the pole type mechanism, we have now in the case of the diagram of the type of Fig. 3 that the polarization of the particle y is determined not only by the interaction in the initial and final states, but also by the polarization due to the reaction (39). It should be emphasized that the angular distribution of the emitted particles provides a less sensitive test of the direct reaction mechanism.

3. ON THE REACTIONS A + x → B + y + z

As in the case of the reactions (1a), the simplest diagrams for the reactions (1b) are the pole type ones. These diagrams for the processes (x, dx), (π^-, 2n), and (K^-, Λ^0n) are shown in Fig. 4 a and b. A number of experimental results now available indicate that the pole type mechanism predominates, but the contributions from other diagrams cannot be excluded just on the basis of these data. Thus, for example, it was observed in the work of Ozaki and co-workers [7] that the neutrons in the reaction (π^-, 2n) are emitted preferably into opposite directions. This result corresponds to the pole

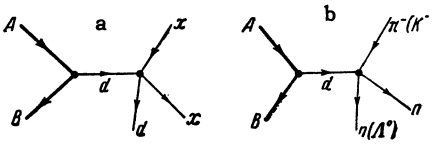

FIG. 4. a – pole diagram for the reaction (x, xd), b – pole diagram for the reactions (π^-, 2n) and (K^-, Λ^0 n).

*a.m.u. ≡ atomic mass units

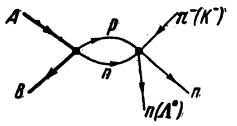

FIG. 5. The simplest non-pole-type diagram for the reactions (π^-, 2n) and (K^-, Λ°n).

diagram of Fig. 4b, but cannot be obtained from the diagram of Fig. 5, which involves the emission of a neutron and a proton in a singlet state by the nucleus A. It follows that the interpretation of the reaction mechanism requires the comparison of data on different processes.

If the pole diagrams are the most important, the cross sections for the processes corresponding to the diagrams of Figs. 4 a, b, and 5 should agree numerically (if the reduced vertex parts γ^A_{Bd} are the same). It should be noted in this connection that the "knocking-out" of deuterons from nuclei is not surprising within the framework of dispersion theory, since there is no reason to believe that the reduced vertex parts γ^A_{Bd} are equal to zero. The quantitative description of this process requires, at least, the knowledge of the scattering cross section of the particles x on a free deuteron in a wide region of energies and of the reduced vertex parts γ^A_{Bd}. For this it is necessary to compare with data on other processes (which are rather sparse at the present time).

What has been said above about the knocking out of deuterons applies equally well to other "knock-out" processes, i.e., processes of the type (x, xy). This provides us with a uniform approach to the mechanism of the knocking out of different types of complex particles ("clusters") from the nucleus. We shall not quote here the formulas for the cross sections of the processes of the type (2) in the pole approximation, since they can be found in the well-known paper of Chew and Low.[8]

In the case of reactions (2) the inclusion of diagrams more complicated than the diagrams of Figs. 4 and 5 is beset with a number of difficulties. This has to do with the fact that to each diagram with five external lines there correspond five independent kinematic variables, and the position of the singular points in one variable depends on the values of the other variables. Nevertheless, the interactions in the initial and final states can be included for many processes with nonrelativistic energies in exactly the same manner as was done in Sec. 2. Of particular interest is the study of the reactions (π^-, pn) and (K^-, Λ^0p) with the help of this method. The simplest diagram for these reactions is shown in Fig. 6. The reaction amplitude corresponding to this diagram satisfies the integral equation (31) with

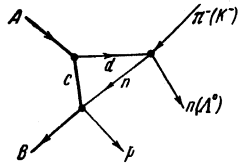

FIG. 6. The simplest diagram for the reactions (π^-, pn) and (K^-, Λ°p).

$f_{y'y}$ replaced by the amplitude M_{np} of the reaction c(n, p)B [using formula (29)]. However, to carry out this program we must require more complete experimental data than are available at the present time.

4. CONCLUDING REMARKS

The method discussed in this paper has a number of attractive features: it is illustrative, does not make use of perturbation theory, and leads to formulas that are transparent in their physical content and relates amplitudes for various processes to one another. This last circumstance leads to a definite experimental program for the verification of the original assumptions (namely, the possibility of a description of the direct processes in terms of Feynman diagrams). The formalism developed is essentially a phenomenological theory of direct processes which is completely analogous to the Breit-Wigner theory of the compound nucleus. These two theories may be called twins in that they are both dealing with the singularities on different sheets of one and the same analytic function. Like the Breit-Wigner theory of the compound nucleus, our method does not pretend to a calculation of the reduced widths of the nuclear reactions but provides a uniform procedure for extracting such levels from the experimental data and establishes a quantitative relation between nuclear reactions which, at first glance, seem to be completely different.

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ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2f^2}{4\pi^3}\omega^2(\ln\frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2f^2}{9\pi^3}\omega^2\left(\ln\frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2f^2/4\pi^3)\,\omega^2 \geqslant \ldots$	$(e^2f^2/9\pi^3)\,\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-[\frac{1}{4}(3\cos^2\theta - 1)\ldots$
14	3	Charakhchyan et al.	533		Table II, col. 6 line 1	1.9	0.9
14	3	Malakhov	550		The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.		
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2\sum_{s=1}^{\infty}K_2(bs)\right]^2$	$\ldots\left[b^2\sum_{s=1}^{\infty}(-1)^{s+1}K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	l	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	$1 < d/2 < 2$	$1 < d/r < 2$
14	5	Shapiro	1154	l	Table	2306	23.6