INCOHERENT SCATTERING OF ELECTRONS BY NUCLEI AND THE MAGNETIC RADIUS OF THE NEUTRON

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It is shown that the seemingly apparent smaller size of the neutron, compared with that of the free proton, observed in the experiments of Hofstadter on the Be^9 nucleus can be partially explained by the binding of the neutron in the given nucleus. The mean square radius of the neutron magnetic-moment distribution thus appears to be larger if the neutron motion is taken into account.

1. It has been shown in several recent papers that the curve for the cross section of scattering of fast electrons by a point-like proton does not agree with the experimental data. The situation can be rectified by assuming that the charge and the magnetic moment of the proton are distributed in space. The best agreement with experiment is attained in this case when these distributions are approximated by an exponential or near-exponential model with a mean-square radius $r_p = 0.8$ f. To study the structure of the neutrons, we used H^2 and Be^9 nuclei[1,2], which contain a weakly-bound neutron. We started from the fact that at large momentum transfers and at large scattering angles θ, one observes only incoherent scattering[3] (the momentum transferred k is much greater than the reciprocal of the mean distance between nucleons), which can be represented as the sum of the cross sections of the scattering by individual nucleons of the nucleus. Under these conditions the main contribution to the incoherent spectrum is made by scattering on the magnetic moment of the nucleon, while the scattering on the specific charge of the nucleon can be neglected.

Starting with the foregoing, we used the "area method"[1,2] to determine the magnetic form factor, assuming the nucleons of the nucleus to be free and independent. Measurements made on H^2 and Be^9 yielded values $r_n = 0.8$ f and 0.53 f for the mean-square radius of the distribution of the neutron magnetic moment (in the exponential model). It was suggested that the apparent reduction in the neutron dimensions in measurements on the Be^9 nucleus is due to the greater role of the meson-exchange effects in this nucleus and also to the interaction between the emitted neutron and the nuclear remnant.[2] Since the neutron is assumed free in the interpretation of the experimental data, it is of interest to ascertain whether an account of the neutron binding in the Be^9 nucleus leads to an increase in r_n. The present article is devoted to this question.

2. Since the electric form factor of the neutron is $F_{1n} \sim 0$[3] the energy of interaction between the electron and the neutron is determined by the interaction between the magnetic moment of the neutron with the magnetic field of the electron; therefore

$$H' = -\int \mathbf{M}[\nabla \mathbf{A}]\, d\tau, \tag{1}$$

where

$$\mathbf{M} = \frac{e\hbar}{2Mc} f_{2n}(|\mathbf{r} - \mathbf{r}_n'|)\, \chi_n \sigma_n, \tag{2}$$

f_{2n} is a structural function characterizing the distribution of the magnetic moment, χ_n is the anomalous magnetic moment of the neutron, and **A** is the Møller potential.[4]

To take into account the motion of the center of mass we can use the model of two bodies (core + neutron), the interaction between which we describe by a spherical symmetrical potential well of depth V_0 and width $R_0 = r_0 A^{1/3}$ f.

The ground state, in accordance with the shell theory for light nuclei, is described by known spin-angle functions,[5] and the final state can be determined with the aid of the Born approximation, if the energy of the free neutron is sufficiently large. Then, after several mathematical manipulations, we obtain for the cross section of electrodisintegration of a nucleus with mass number A, wherein an electron of energy q_i is scattered within a solid angle $d\Omega$ and has an energy in the interval dq_f, the following expression

$$\sigma_n(\theta, q_f) = d^2\sigma/d\Omega dq_f = \sigma_{n0} F_{2n}^2(kr_n)\,\gamma(x, r_0)\,q_i^{-1}, \tag{3}$$

where σ_{n0} is the Rosenbluth cross section for the scattering of an electron by a point-like free neutron,[6] and F_{2n} is the form factor of the neutron, the Fourier transform of the structural function; the remaining quantities, which take into account the binding of the neutron, are

$$\gamma(x, r_0) = \frac{A}{A-1}\frac{2\eta}{\pi}(a^2+b^2)\,b^2\,\frac{R_0 Mc}{\hbar}\left(1+\frac{2q_i}{Mc^2}\sin^2\frac{\theta}{2}\right)^2 \times f(x)\int_{z_1}^{z_2}\varphi(z, r_0)\,dz, \tag{4}$$

where

$$f(x) = (1 + x^2 - 2x\cos\theta)^{-1/2}x\left\{1+\frac{q_i}{Mc^2}\left[1-x\left(1+2\alpha_0\sin^2\frac{\theta}{2}\right)\right]\right\},$$

$$\varphi(z) = \left[\frac{j_l(a)\,z j_{l-1}(z) - a j_{l-1}(a)\,j_l(z)}{(b^2-z^2)(a^2+z^2)}\right]^2 z;$$

$$\alpha_0 = q_i/\mu_0, \qquad \mu = \frac{A-1}{A}M, \qquad x = q_f/q_i,$$

$$\eta = [j_{l-1}(a)\,j_{l+1}(a)]^{-1}, \qquad a = \frac{R_0}{\hbar}\sqrt{2\mu(V_0-\varepsilon_n)},$$

$$b = \frac{R_0}{\hbar}\sqrt{2\mu\varepsilon_n}, \qquad z_{2,1} = \left|\frac{A-1}{A}kR_0 \pm k_n R_0\right|, \tag{5}$$

M and μ_0 are the masses of the neutron and of the target nucleus, ε_n and k_n are the binding energy and the wave vector of the neutron, while j_l is a spherical Bessel function.

In the derivation of (3), account must be taken of the energy and the momentum conservation laws, the use of which yields

$$q_i - q_f = W + E_c, \qquad W = q_i - q_f\,[1 + \alpha_0(1-\cos\theta)], \tag{6}$$

where E_c is the kinetic energy of the scattering nucleus and W the excitation energy transferred to the emitted neutron.

Integrating (3) with respect to dq_f we obtain

$$\sigma_n(\theta) = \sigma_{n0}\xi(r_0, r_n), \tag{7}$$

where the value of ξ, which is equal to

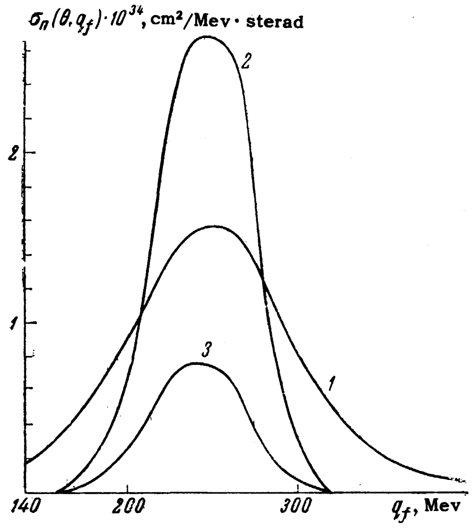
$$\xi(r_0, r_n) = \int_{x_1}^{x_2} F_{2n}^2(kr_n)\,\gamma(x, r_0)\,dx = F_{2n}^2(\zeta)\,C(r_0), \tag{8}$$

can be connected with the experimentally-measured value of $R = \sigma_n/\sigma_p$ [2] by the relation

$$R = A(\theta, q)\,\xi(r_0, r_n)\,F_p^{-2}, \tag{9}$$

with $A(\theta, q)$—ratio of the Rosenbluth cross sections for the scattering of an electron by a point-like free neutron and proton.

3. Let us apply these formulas to scattering on Be^9, the incoherent electron spectrum of which was observed by Ehrenberg and Hofstadter,[2] who



obtained R = 1.17 at q_i = 500 Mev and θ = 135°. Using these data, we obtain for $F_p^2 = 0.15$ [2]

$$\xi(r_0, r_n) = \int_{0.4}^{0.65} F_{2n}^2(kr_n)\,\gamma(x, r_0)\,dx = 0{,}39, \tag{10}$$

where the integration limits were chosen in accordance with the energy region for which R had been determined.

It is known that the neutron in Be^9 is in a $p_{3/2}$ state and its binding energy is ε_n = 1.67 Mev. Obviously, we can use in this case the Born approximation, since the emitted neutron acquires an energy not less 150 Mev. Calculations have shown that $C(r_0) = 1.12$ when $r_0 = 1$ and increases to 1.43 at $r_0 = 2.4$. Thus, an account of the neutron binding can lead to an increase in r_n. We assume $r_0 = 2.4$ because a value $R_0 = 5$ f (V_0 = 12.17 Mev) yielded good agreement with experiment at low energies.[7]

The figure shows the cross section $\sigma_n(\theta, q_f)$ for a point-like neutron with r_0 having values 1 and 2.4 (curves 1 and 2), and also for a neutron with exponential distribution of the magnetic moment at $r_0 = 2.4$ and $r_n = 0.6$ f (curve 3). All the curves have a maximum at $q_f \sim 250$ Mev, which agrees with the variation of the experimental cross section.[2] It is possible to integrate in (10)

$f_{2n}(r)$	r_n	$r_n\ (\xi = 1)$
$r^{-2}e^{-r}$	0.85	0.68
$r^{-1}e^{-r}$	0.65	0.56
e^{-r}	0.6	0.53
re^{-r}	0.58	0.51
r^2e^{-r}	0.57	0.50
e^{-r^2}	0.56	0.49
$r^2e^{-r^2}$	0.54	0.47

from zero to infinity because the function $\sigma_n(\theta, q_f)$ assumes very small values when $x < 0.4$ and $x > 0.65$.

Using formula (10) and also the table from Hofstadter's paper,[3] we can find the mean-square radius for different neutron magnetic-moment distributions. The results of the calculations are listed in the table, which also contains the values of the mean-square radius obtained from (9) under the assumption that $\xi = 1$ (free neutron).

On the other hand, research on H^2 yields values $r_n = r_p = 0.8$ f, which does not agree with the value of r_n listed in the table. The reason for this discrepancy may be, first, the inaccuracy of the experiment and second the fact that in the interpretation of the experimental data we neglected certain effects, an account of which can change r_n. It has been assumed, in particular, that 1) the electrons are scattered by stationary free nucleons, 2) the emitted nucleon does not interact with the remaining nucleons, and 3) the dimensions of the nucleon remain unchanged under the influence of the nuclear forces.

One can think that the equality of the neutron and free-proton radii, obtained in the experiments on H^2 (assuming the charge independence hypothesis to be valid), corroborates the foregoing assumptions made for the deuteron, in which the nucleons stay the greater part of the time outside the range of the nuclear forces. This conclusion may not prove to be completely true for Be^9. Indeed, as can be seen from the table, an account of the neutron bond leads to an increase in the mean-square radius. Regardless of how reliable the experimental data are, we can conclude therefore that in the interpretation of the experimental results obtained by the "area method" for nuclei such as Be^9, C^{13}, or O^{17} the effect of the binding of the nucleons in the nucleus must be taken into account.

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[1] M. Yearian and R. Hofstadter, Phys. Rev. **110**, 552 (1958); Phys. Rev. **111**, 934 (1958).

[2] H. F. Ehrenberg and R. Hofstadter, Phys. Rev. **110**, 544 (1958).

[3] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

[4] R. Hofstadter, Ann. Rev. of Nucl. Science, Stanford, (1957) Vo. 7.

[5] A. S. Davydov, Teoriya atomnogo yadra (Theory of the Atomic Nucleus), Fizmatgiz, 1958.

[6] M. Rosenbluth, Phys. Rev. **79**, 615 (1950).

[7] E. Guth, Phys. Rev. **76**, 234 (1949).

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