TRANSFORMATION OF PHOTONS INTO NEUTRINO PAIRS AND ITS SIGNIFICANCE IN STARS

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The cross section for the transformation of a γ quantum into a $\nu\bar{\nu}$ pair is calculated and the cross section for transformation of two photons into $\nu\bar{\nu}$ is estimated. The stellar neutrino luminosities corresponding to these processes are computed. Within a broad range of stellar temperatures and densities, these processes are found to predominate over the bremsstrahlung of $\nu\bar{\nu}$ pairs by electrons.

1. INTRODUCTION

GAMOV and Schoenberg [1] were apparently the first to point out the important role of the energy carried away from stars by neutrinos via K capture and β decay during the process of the stellar evolution.

The great advances in the theory of the universal weak A-V interaction have called attention to the neutrino mechanism of energy loss from stars, connected with the interaction, predicted by the Feynman and Gell-Mann theory, [2] between the electrons and the neutrino $(\bar{e}\nu)(\bar{e}\nu)^+$ in the first order in the weak interaction constant G. Pontecorvo [3] noted that the existence of such an interaction should make possible emission of a neutrino-antineutrino pair in electromagnetic processes instead of emission of a γ quantum (through the virtual pair e^+e^-).

The negligible probability of such processes, compared with that of electromagnetic processes, is obvious. At the high densities and temperatures encountered in stars, however, the energy lost by stars through neutrino-pair formation may turn out to be comparable with or even greater than the energy losses due to γ-quantum emission, owing to tremendous differences between the penetrating abilities of the γ quanta and the neutrinos. In addition, the range of the γ quanta decreases at large Z whereas the cross section for the formation of neutrino pairs increases.

Pontecorvo considered by way of an example the formation of neutrino pairs in collisions between electrons and nuclei. [3] A quantitative investigation of this process as applied to astrophysics was made by Gandel'man and Pinaev. [4]

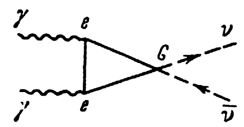

FIG. 1

They obtained the important result that, within a definite region of high temperatures and pressures, the energy carried away by the neutrinos in the aforementioned process exceeds the photon energy loss. This result indicates the need for a detailed study of all the neutrino mechanisms of energy loss from stars. Recently Chiu and Morrison [5] and Chiu and Stabler [6] considered certain possible neutrino mechanisms whereby energy is carried away from stars:*

$$e^- + e^+ \to \nu + \bar{\nu}, \qquad \gamma + e^- \to e^- + \nu + \bar{\nu},$$

$$\gamma + \gamma \to \gamma + \nu + \bar{\nu}.$$

In the present article we investigate quantitatively a new supplementary mechanism by which neutrinos can carry energy away from the stars. In this mechanism the photon breaks up in the Coulomb field of the nucleus into a neutrino-antineutrino pair. In addition, we estimate approximately the effect connected with the process of conversion of two γ quanta into a $\nu\bar{\nu}$ pair. As applied to conditions inside stars, the calculation is carried out in nonrelativistic approximation.

*E. L. Feĭnberg was kind enough to advise us that the second of these processes was considered in detail also by V. I. Ritus, who obtained a much greater value for the neutrino energy loss than indicated in the table of Chiu and Morrison.[5]

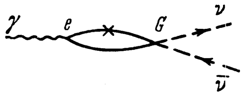

FIG. 2

2. CROSS SECTION FOR THE PRODUCTION OF A NEUTRINO PAIR BY A PHOTON IN THE FIELD OF THE NUCLEUS

As noted by Gell-Mann,[7] the amplitude of the process wherein two γ quanta are converted into a $\nu\bar{\nu}$ pair, shown in Fig. 1, vanishes in the case of local $(\bar{e}\nu)(\bar{e}\nu)^+$ interaction.

This circumstance is the result of three factors: 1) the invariance under charge conjugation, which forbids any contribution to the amplitude of the matrix element from the vector part of the electron current $e\gamma_\alpha e$, obtained from the original Lagrangian of the local A-V interaction

$$\mathcal{L} = \frac{G}{\sqrt{2}} [\bar{e}\gamma_\alpha (1+\gamma_5) \nu] [\bar{e}\gamma_\alpha (1+\gamma_5) \nu]^+$$

by a Fierz transformation; 2) the impossibility of transitions between system states with spin 1 and two real photons [8,9]; 3) the zero value of the neutrino mass.

Obviously, if one of the photons is replaced by a Coulomb field, argument 2) no longer applies and the process in the corresponding diagram of Fig. 2 should take place if the $(\bar{e}\nu)(\bar{e}\nu)^+$ interaction exists. In addition, if nonlocality (due, for example, to the intermediate charged vector boson) takes place at the vertex G, then the process $\gamma + \gamma \to \nu + \bar{\nu}$ will also occur.[7]

In this section we present an exact calculation of the cross section for Coulomb disintegration of the γ quantum into a $\nu\bar{\nu}$ pair, and obtain in addition a rough estimate of the cross section of the process $\gamma + \gamma \to \nu + \bar{\nu}$ on the assumption that an intermediate boson exists.

We consider the first process $\gamma + A \to A + \nu + \bar{\nu}$ (Fig. 2). Owing to the presence of a pseudo-vector current $e\gamma_\alpha\gamma_5 e$ along with the vector current $e\gamma_\alpha e$ this process, unlike the formally similar process of scattering of a photon in the Coulomb field of a nucleus, is of first order in Z. In addition, owing to invariance under charge conjugation, only the pseudo-scalar current contributes to the amplitude of the process. The gauge invariance of the matrix elements eliminates the divergences (both linear and logarithmic) automatically. Evaluating in standard fashion the integrals and the traces corresponding to the electron loop, we obtain for the matrix element in the non-relativistic approximation (i.e., neglecting the squares of the 4-momenta of all the external lines compared with the square of the electron mass m^2)

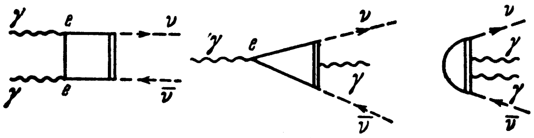

FIG. 3

$$-\frac{\alpha Z G}{2\pi\sqrt{\omega}} \frac{1}{|\mathbf{q}|^2} \varepsilon_{ikl} q_i e_k [\bar{u}(p_\nu) \gamma_l (1+\gamma_5) v(-p_{\bar{\nu}})]. \quad (1)$$

Here ϵ_{ikl} — fully antisymmetrical unit tensor of third rank $(i, k, l = 1, 2, 3)$; ω — frequency of γ quantum; q — momentum transferred to the nucleus, the action of which is usually treated like the action of an external static field; e_k — photon polarization vector; p_ν and $p_{\bar{\nu}}$ — 4-momenta of the neutrino and antineutrino: u and v — the corresponding spinors; $\alpha = 1/137$; $G = 10^{-5}/M_p^2$, M_p — proton mass $(\hbar = c = 1)$; Z = nuclear charge.

After averaging over the polarization of the γ quantum and summing over the polarizations of the neutrino and antineutrino, we obtain for the differential cross section

$$d\sigma_1 = \frac{Z^2\alpha^2 G^2}{\pi (2\pi)^6} \frac{\varepsilon_\nu^2 (\omega - \varepsilon_\nu)^2}{\omega^2} \frac{d\varepsilon_\nu}{\mathbf{q}^2} \left[1 - \frac{(\mathbf{n}_\nu \mathbf{q})(\mathbf{n}_{\bar{\nu}} \mathbf{q})}{\mathbf{q}^2}\right] d\mathbf{n}_\nu d\mathbf{n}_{\bar{\nu}}, \quad (2)$$

where ϵ_ν — energy of the neutrino, $\mathbf{n}_{\nu(\bar{\nu})}$ — unit vector along the direction of emission of $\nu(\bar{\nu})$; $\mathbf{q} = \mathbf{k} - \mathbf{p}_\nu - \mathbf{p}_{\bar{\nu}}$.

In the integration over the directions of emission of ν and $\bar{\nu}$, the second term in the square brackets drops out, while the first term yields

$$\iint \frac{d\mathbf{n}_\nu d\mathbf{n}_{\bar{\nu}}}{(\mathbf{k} - \varepsilon_\nu \mathbf{n}_\nu - \varepsilon_{\bar{\nu}} \mathbf{n}_{\bar{\nu}})^2} = \frac{4\pi^2}{\omega(\omega - \varepsilon_\nu)} \left[\ln \frac{\omega(\omega - \varepsilon_\nu)}{\varepsilon_\nu^2} + \frac{2\omega - \varepsilon_\nu}{\varepsilon_\nu} \ln \frac{\omega}{\omega - \varepsilon_\nu}\right]. \quad (3)$$

Integrating, finally, over the neutrino energy we obtain for the total cross section of the production of a neutrino pair by a photon in the field of the nucleus

$$\sigma_1 = (7/576\pi^5) Z^2\alpha^2 G^2 \omega^2. \quad (4)$$

When $\omega = 250$ kev, for example, this section is $0.4 Z^2 \times 10^{-52}$ cm^2, i.e., negligibly small. However, under conditions prevailing inside stars this cross section causes, as we shall show below, noticeable neutrino emission from very dense "hot" stars.

Let us consider briefly the process $\gamma + \gamma \to \nu + \bar{\nu}$. As mentioned above, it can occur only if a nonlocality exists in the A — V interaction. If this nonlocality is connected with a heavy intermediate vector boson, (a carrier of weak interactions), then the diagrams contributing to the amplitude of the process will be of the type shown in Fig. 3. The double line in this figure represents the intermediate vector boson of mass M.

To estimate the order of magnitude of the cross section we consider only the contribution corresponding to diagrams of type a, disregarding diagrams such as b and c, in which the γ quanta are absorbed by the heavy particle. Diagram a leads to logarithmically divergent integrals, so that the cross section of the process does not depend strongly on the cut-off momentum L. Taking L to be of the order of the mass of the intermediate meson, which in turn is taken to be of the order of the nucleon mass, we obtain the following approximate estimates for the total cross section for the production of a neutrino pair in a collision between two photons, in the nonrelativistic case (ω^2, $\omega'^2 \ll m^2$):

$$\sigma_2 \approx (\alpha^2 G^2/2\pi^5)\,\omega\omega', \tag{5}$$

where ω and ω' are the frequencies of the incoming photons in the frame fixed in the star.

3. CONTRIBUTION OF THE CONSIDERED PROCESSES TO THE NEUTRINO EMISSION FROM THE STARS

In this section we explain the role played by the elementary processes considered here in the energy loss from the stars, and compare these processes with bremstrahlung of neutrino pairs by electrons.[3,4] We are essentially interested in the conversion of a photon into a neutron pair in the field of the nucleus.

Let us find the energy $q_\nu^{(1)}$ transferred by the photons to the neutrino pairs in 1 cm^3 per second. Describing the distribution of the photons n_γ by means of Planck's formula with $kT \ll m$, we obtain for $q_\nu^{(1)}$

$$q_\nu^{(1)} = \int \omega\sigma_1 n_n\, dn_\gamma = 3.4\cdot 10^{-8}\frac{\rho}{\nu}T^6, \tag{6}$$

where n_n — number of nuclei per cm^3; ρ — density of the matter (in g/cm^3); $1/\nu = \Sigma C_i Z_i^2/A_i$ (C_i is the weight concentration of the element, Z_i its charge, and A_i its atomic weight; the summation is over all the elements contained in interstellar matter); the temperature T is given everywhere in kev.

It must be borne in mind that the rate of energy release obtained here is many times smaller than the rate of energy release in hydrogen reactions, and therefore the neutrino loss can compete with the thermonuclear loss only if the thermonuclear reactions in the star are already practically nonexistent and the star is characterized by a large value of Z.

Gandel'man and Pinaev[4] have shown that although the cross section in the Gamow-Schoenberg process[1] is proportional to G^2 and the neutrino bremsstrahlung cross section[3,4] is proportional to $\alpha^2 G^2$, nonetheless, owing to the lower abundance of the elements with low threshold for the inverse β process and to the anomalously large lifetime of certain nuclei, the process $e^- + A \to e^- + A + \nu + \bar{\nu}$ proposed by Pontecorvo[3] prevails at $T < 100$ kev over the processes considered by Gamow and Schoenberg. We shall therefore compare the effect we are considering with the effect investigated by Gandel'man and Pinaev.[4]

If q_ν is the rate of energy release per cm^3 obtained in [4], we get for a star consisting entirely* of Mg^{24}

$$q_\nu^{(1)}/q_\nu = 2.5\cdot 10^2 T^{3/2}/\rho. \tag{7}$$

When $T > 50$ kev and $\rho \approx 10^5$ we obtain $q_\nu^{(1)} > q_\nu$. We see thus that the process $\gamma + A \to A + \nu + \bar{\nu}$ can make an appreciable contribution to the neutrino emission from the stars.

Let us compare now the neutrino luminosity $L_\nu^{(1)}$, corresponding to σ_1, with the values of L_ν obtained in [4]. In addition, we compare $L_\nu^{(1)}$ with the photon luminosity L_γ. We base our calculations, as in [4], on a model of a point source. We have

$$L_\nu^{(1)} = \int q_\nu^{(1)}\, dv = 3.4\cdot 10^{-8}\frac{1}{\nu}4\pi\int_0^R \rho T^6 r^2\, dr, \tag{8}$$

where R — radius of the star, characterized by a constant central density ρ_c of a convective nucleus with radius 0.169 R[11] and temperature T_c. In the remaining part of the star which makes in practice a small contribution to L, we have

$$T = T_c\frac{R/r - 1}{1/\xi - 1}, \qquad \rho = \rho_c\left(\frac{R/r - 1}{1/\xi - 1}\right)^{3.5}$$

($\xi = 0.169$). Integrating (8) and expressing R in terms of ρ_c and T_c using the formulas

$$T_c = 0.7\cdot 10^{-22}\ \mu\frac{M}{R},$$

$$\rho = 37M\Big/\frac{4}{3}\pi R^3 \qquad \left(\frac{1}{\mu} = \sum_i C_i(Z_i + 1)/A_i\right),$$

we determine $L_\nu^{(1)}$ expressed in solar units of luminosity ($L_\odot = 3.78 \times 10^{33}$ erg/sec):

*Present-day data [10] indicate apparently that white dwarfs with mass on the order of the mass of the sun, if formed as the result of evolution of stellar remnants or stars with low initial hydrogen content, consist essentially of Mg^{24}.

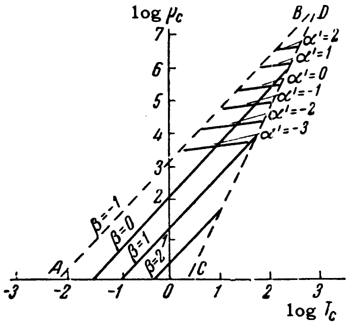

FIG. 4

$$L_\nu^{(1)} = 1.2 \cdot 10^{-8} \frac{1}{\nu \mu^{3/2}} T_c^{7.5} / \rho_c^{0.5}. \quad (9)$$

The ratios of $L_\nu^{(1)}$ to the photon luminosity [4] L_γ and to L_ν are

$$L_\nu^{(1)} / L_\gamma = 10^{-11} \rho_c^2 / \nu \mu b T_c^{0.5}, \quad (10)$$

$$L_\nu^{(1)} / L_\nu = 1.3 \cdot 10^2 \mu_e T_c^{1.5} / \rho_c, \quad (11)$$

where $1/\mu_e = \Sigma C_i Z_i / A_i$, and b is the coefficient in Kramers's formula for the free path of the photon inside the star.

Figure 4 shows on a logarithmic scale the lines $L_\nu^{(1)}/L_\gamma = 10^\alpha$ and $L_\nu^{(1)}/L_\nu = 10^\beta$ for Mg ($\mu = \mu_e = 2$, $b = 1$, $\nu = 1/6$). In the region bounded by the lines AB and CD we can compare $L_\nu^{(1)}$ with the values of L_ν and L_γ obtained in [4], assuming no degeneracy and low radiation pressure compared with matter pressure. We see that $L_\nu^{(1)} > L_\nu$ over a wide range of temperatures and densities. In addition, when $\rho_c > 10^5$ and 30 kev $< T_c <$ 100 kev, the neutrino luminosity is one order of magnitude or more greater than the photon luminosity.

Let us give, finally, the ultimate formulas corresponding to the process $\gamma + \gamma \rightarrow \nu + \bar{\nu}$, the cross section σ_2, of which was estimated by us approximately in Sec. 2.

To obtain $q_\nu^{(2)}$ we must integrate $(\omega + \omega')\sigma_2$, using a Planck distribution for the frequencies ω and ω':

$$q_\nu^{(2)} = 2 \iint (\omega + \omega') \sigma_2 \, dn_\gamma \, dn'_\gamma. \quad (12)$$

We obtain

$$q_\nu^{(2)} \approx 1.8 \cdot 10^{-8} T^9, \quad (13)$$

$$q_\nu^{(2)} / q_\nu^{(1)} \approx 3 \cdot 10^{-5} T^3. \quad (14)$$

For the neutron luminosity we obtain, in analogy with (9),

$$L_\nu^{(2)} \approx 5.8 \cdot 10^{-9} \mu^{-3/2} T_c^{10.5} / \rho_c^{1.5}. \quad (15)$$

Furthermore

$$L_\nu^{(2)} / L_\gamma \approx 5.82 \cdot 10^{-12} T_c^{2.5} \rho_c / b\mu \quad (16)$$

and

$$L_\nu^{(2)} / L_\nu^{(1)} \approx 0.48 \nu T_c^3 / \rho_c. \quad (17)$$

4. CONCLUSION

The foregoing analysis shows that at high densities and temperatures the neutrino energy loss from stars, and particularly the loss connected with the processes investigated in the present article, assume an important role in the energy release from the stars. At a density $\rho = 10^5$ and a temperature of 42 kev (5×10^8 deg K) (Z = 12), for example, the energy release from one gram of stellar matter per second due to the process $\gamma + A \rightarrow A + \nu + \bar{\nu}$ is 10^3 erg/g-sec, and is much greater than the corresponding energy due to the photons. The investigation shows that in a wide range of densities and stellar temperatures the mechanism considered here prevails over the mechanism of neutrino bremsstrahlung. [3,4]

The neutrino energy loss from stars is apparently particularly important in connection with the problem of white dwarfs and, in general, stars with low photon luminosity. It is known that the lifetime of stars with high luminosity is small compared with the age of the galaxy. On the other hand, in stars with low luminosity the evolutional time scale is very greatly increased (considerably greater than the age obtained for the stars from general cosmological considerations), if evolution due to burning up of hydrogen is taken as the base. [12] In this connection it is interesting to point out that the use of the neutrino luminosity in the analysis of the age of white dwarfs should apparently lead to a considerable reduction in their evolutional time scale.

A related question is that of the initial content of hydrogen in white dwarfs that have not passed through the nova explosion stage, and are the end products of the evolution of an entire star that has exhausted its sources of nuclear energy, so that the energy released is a result of compression.

At small amounts of radiated energy per unit mass and at a finite time scale, the white dwarfs could exhaust their hydrogen only if its initial content was small. The question of the neutrino luminosity of such stars obviously can change our notions concerning the smallness of their hydrogen content. The neutrino energy release apparently plays an important role also in the dynamics of the explosion of supernovae. [1]

No matter how attractive these considerations may be, it must be kept in mind that the question of the high neutrino luminosity of stars is connected with the still unsolved question whether the $(\bar{e}\nu)(\bar{e}\nu)^+$ interaction of first order in G exists in nature.

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