

DIRECT MEASUREMENT OF THE MOMENTUM OF CONDUCTION ELECTRONS IN A METAL

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A method for "cutting-off" cyclotron resonances is described which can be employed for direct measurement of the Fermi momentum of conduction electrons in a metal. Results of experiments on tin single crystals are presented and discussed. The electron momentum in one of the Fermi surface sections has been measured with an accuracy of ~2.5% in the range of angles ±26° from the [100] axis in the (010) plane.

A study of the phenomenon of cyclotron resonance is a direct and most reliable method for measuring the effective masses of conduction electrons.^[1-5] However, this phenomenon can also be used to study several other characteristics of the electrons in a metal. Kaner and Azbel',^[6] for example, note the possibility in principle of measuring the Fermi momentum of electrons. An experimental method for the direct measurement of the momentum of conduction electrons is described in the present paper, and the results of the first experiments carried out by this method are presented.

THEORY

Let us consider the motion of an electron along the largest closed orbit which can still be contained within a metal plate of thickness D_z situated in a constant magnetic field H_y (Fig. 1a). We integrate the x component of the equation of motion of the electron $\dot{\mathbf{p}} = (e/c)[\mathbf{v} \times \mathbf{H}]$ with respect to time, over the limits of half a period of revolution:

$$\int_0^{T/2} \dot{p}_x dt = -\frac{e}{c} H_y \int_0^{T/2} v_z dt.$$

We take $p_x = -p_{-x}$ (Fig. 1b), since the Fermi surface has central symmetry. As a result we find

$$|p_x| = H_y D_z e / 2c. \tag{1}$$

This formula gives the extremal value of the momentum component p_x of electrons belonging to a certain section of the Fermi surface, if it is known for what field H_y the diameter of their orbits becomes equal to D_z . The latter condition can be determined by observing cyclotron resonance on the electrons of the given group. If reso-

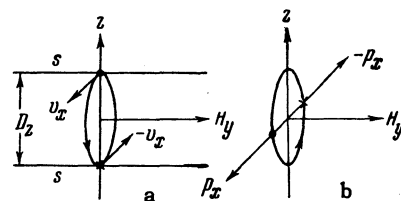


FIG. 1. The trajectory of an electron in spatial coordinates (a) and in momentum coordinates (b). Corresponding points of the trajectory are indicated by the same symbols. s is the intersection of the surfaces of the metal plate with the plane of the drawing.

nances of order 1 to n (at field H_n) are observed, but the $n + 1$ order resonance, which can be seen for a thicker plate, is missing for a given plate, then $H_n > H_y > H_{n+1}$ and the value of H_y can be determined with higher relative accuracy the higher the order n , i.e. the thicker the plate.

For reduction of experimental data, it is convenient to express p_x in terms of n , using the formula for the field H_y at cyclotron resonance:^[1]

$$H_n = m^* \omega c / ne, \tag{2}$$

where m^* is the effective mass of the electrons and ω the frequency of the high-frequency measuring field. Substituting H_n into (1) in place of H_y , we obtain

$$p_x = m^* \omega D_z / 2n. \tag{3}$$

We should regard n as the magnitude of the measuring field H_y^{-1} at which "cutting-off" of the electron orbits takes place, not necessarily having an integer value, as distinct from the meaning of n in (2).

The considerations presented indicate the means

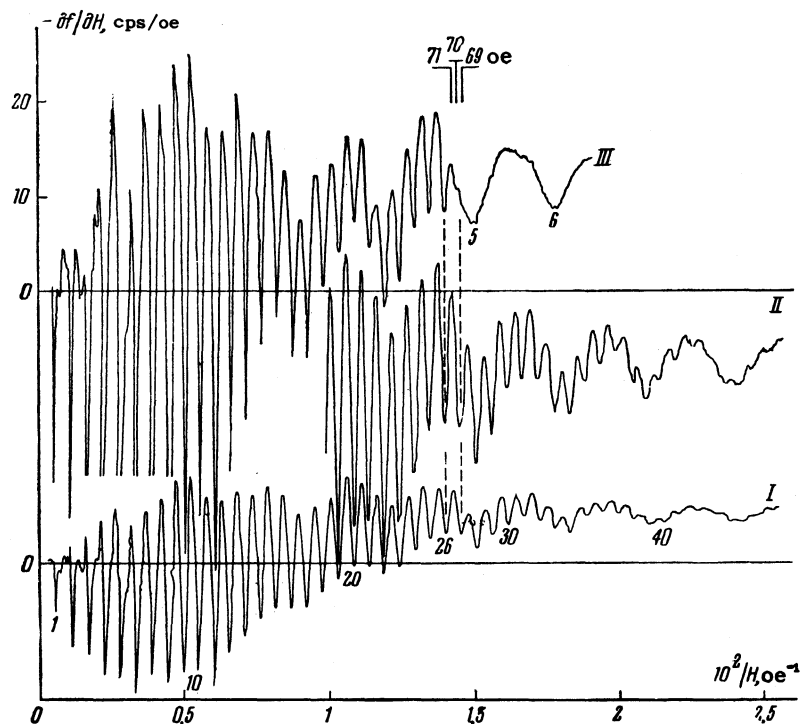


FIG. 2. Trace of cyclotron resonances on tin single crystals of thickness 2 mm (curves I and II) and of thickness 0.982 mm (curve III). It can be seen that in curve III resonances of order 27 and higher are missing. The orders of the resonances are shown under curves I and III; the traces of some deep resonances on curve III are cut off.

of a direct experimental measurement of the Fermi momentum of electrons in a metal.*

EXPERIMENT

The experimental investigations were carried out by the frequency modulation method^[7] on single crystals of extremely pure tin, characterized by the resistance ratio $\rho_{20^\circ\text{C}}/\rho_{3.75^\circ\text{K}} = 1.4 \times 10^5$ (specimen No. 6 of^[8]); at 3.75°K the electron mean free path is ~ 1 mm. The single crystals were in the form of 18 mm diameter disks of different thickness and were grown from the melt in demountable polished quartz molds; their natural surfaces received no further treatment. The plane surfaces of the disks had the (010) orientation; the disks were placed in a strip resonator^[7] so that the high-frequency currents flowed in the [100] direction.

The results of experiments at a temperature of 3.75°K with the constant magnetic field directed along the [001] axis are shown in Fig. 2, which records the derivative of the frequency of the measuring oscillator with respect to the field; this quantity is proportional to the logarithmic derivative of the reactive part of the surface impedance

*Analogous measurements on semiconductors should evidently also be possible in principle. However, in that case, in view of the existence of only first order resonance, one would have to determine the conditions for "cutting-off" the orbits both by changing the constant magnetic field and the frequency of the measuring field, which greatly complicates the experiment.

of the specimen with respect to the field.^[7] Curve I was obtained with a 2 mm thick specimen, curve II is a repeat trace of part of curve I with greater amplification. The minima of the curves are the cyclotron resonances; their order numbers are shown under curve I. The effective mass of the investigated group of electrons, belonging to the section of the Fermi surface by the (001) plane, was calculated from the period of the resonances and found to be $m_{(001)}^* = (0.558 \pm 0.005) m_e$. Resonances of longer period are also clearly visible, corresponding to a mass $m^* = 0.101 m_e$ (detailed measurements of the effective masses of electrons in tin are given elsewhere^[4]).

Curve III was obtained with a specimen of thickness $D_z(4^\circ\text{K}) = 0.982 \pm 0.003$ mm. In this case only 26 resonances are observed; the resonance of order $n = 27$ is already missing—this means that the diameter of the corresponding electron orbit is greater than the thickness of the specimen. For weaker fields only resonances for electrons of smaller effective mass are seen on curve III ($n = 5, 6, \dots$). The clearly visible phenomenon of "cutting-off" the electron orbit by the surface of the metal is also observed on a specimen of thickness 0.4 mm in the region of the most intense resonances ($n = 10-11$).

The value of n_0 corresponding to the magnetic field strength at which the electron orbit cut-off takes place can be determined from curve III in the following way. The resonance $n = 26$ is clearly visible; the next maximum of the curve, corre-

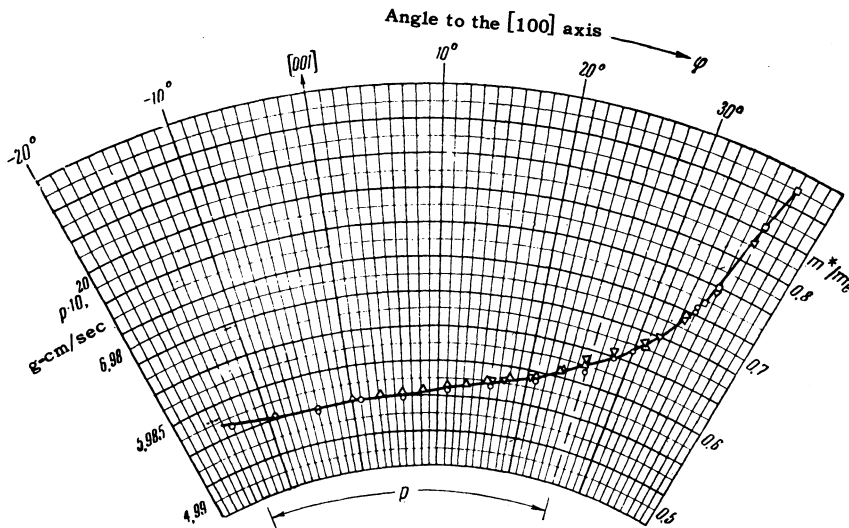


FIG. 3. The dependence of the electron momentum p [Eq. (4)] and of the effective mass m^*/m_e [Eq. (2)] on the direction φ in a tin crystal. The angle φ represents: 1) the direction of the vector p relative to the [100] axis in the (010) plane within the limits 0–26°; 2) the direction of the normal to the section, giving the mass m^*/m_e , relative to the [001] axis in the (010) plane. Different symbols refer to different experiments.

sponding to $n = 26.5$, is also observed. Resonance 27 is missing - its position is marked by the dashed line (the magnetic field strength is shown above curve III for convenience). We can thus take $n_0 = 26.75 \pm 0.25$. Substituting n_0 into (3), with $\omega = 5.967 \times 10^{10}$ cps, we find the momentum of the electrons belonging to the point on the section of the Fermi surface by the (001) plane determined by the [100] direction:

$$p_{[100]} = (5.57 \pm 0.15) \cdot 10^{-20} \text{ g-cm/sec.} \quad (4)$$

We can also calculate the mean Fermi velocity of the electrons belonging to the (001) section from the experimental data given:

$$v_{(001)} = \omega D_z / 2n = (1.10 \pm 0.01) \cdot 10^8 \text{ cm/sec.} \quad (5)$$

If the curvature of the Fermi surface changes little on circuiting along this section (which is evidently the case, as will be explained below), then the value of the electron velocity at each point of the surface must differ little from the mean value found.

The crystallographic orientation of the studied section of the Fermi surface is determined by the direction of the constant magnetic field. The dependence of the effective mass^[4,5] and momentum of the electrons on direction can be obtained by carrying out experiments with rotation of the field at different angles from the [001] axis in the (010) plane of the specimen. The results of such experiments are shown in Fig. 3. The accuracy in measuring the absolute values of the masses is about $\pm 1\%$ and of the momentum $\pm 2.5\%$; the accuracy in measuring their relative variations in one experiment on rotating the field is several times better.

It turned out that the cutting-off of the electron orbits studied in these experiments took place at a value of n_0 that remained constant, within the

accuracy of measurement, over the $\pm 26^\circ$ rotation of the field. According to (3) this indicates that for electrons of the given group the momentum is proportional to the effective mass:

$$p = (9.98 \pm 0.15) \cdot 10^{-20} m^*/m_e \text{ g-cm/sec.} \quad (6)$$

For larger angles of field rotation, resonances of sufficiently high order for observing cut-off are no longer obtained, but measurements of the effective mass can be carried out up to $\sim 37^\circ$.

DISCUSSION

Part of the section of the Fermi surface by the (010) plane and by the (100) plane equivalent to it can be constructed from the obtained dependence of electron momentum on the direction. The section of the first Brillouin zone for tin is given in Fig. 4; the lengths of the segments are shown in units of $2\pi/a$, where $a = 5.796 \text{ \AA}$ at 4°K ($a/c = 1.8470$). The experimental points represent the values of the wave vector in units of $2\pi/a$, calculated from the measured momentum values (Fig. 3). The full line shows the part of the section of the Fermi surface constructed from the experimental points and supplemented from symmetry considerations. The continuation of this section, shown by the dashed line (the angles have been smoothed out arbitrarily), takes account qualitatively of the increase in the electron effective mass with rotation of the field. It lies within 37° to the [100] axis, beyond which cyclotron resonance and, consequently, closed electron orbits cease to be observed in this section.

A comparison of these data with the results of a study of the Fermi surface of tin by the de Haas-van Alphen effect^[9] leads to the following conclusions.

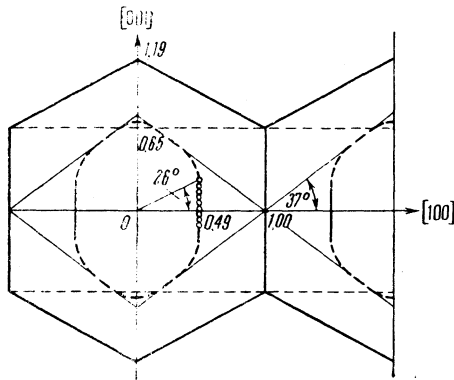


FIG. 4. Sections of the Fermi surface constructed from the experimental results. The hexagon shown by the thick line represents the section of the first Brillouin zone by the (010) plane.

The section of the Fermi surface found pertains evidently to the open hole surface of the fourth zone,^[9] represented in Fig. 5. With the magnetic field parallel to the [001] axis, the electrons describe the orbit ζ ; Gold and Priestley^[9] estimated the corresponding effective mass from the temperature dependence of the de Haas-van Alphen effect at $0.70 m_e$, and calculated it by Harrison's method,^[10] which gave $0.38 m_e$. The value $m^* = 0.558 m_e$ obtained by the method of cyclotron resonance is certainly more reliable.

The electrons go over to inclined orbits as the direction of the magnetic field is changed, and one of these, ζ' is shown in Fig. 5; the angle $\sim 37^\circ$, within which cyclotron resonance is observed, practically coincides with the angle $\sim 32^\circ$ in which the G-oscillations^[9] are observed.

The diameter of the ζ orbit in the [100] direction is $1.02(2\pi/a)$, as can be seen from Fig. 4. If it is assumed that the ζ orbit is circular, its area is $0.82(2\pi/a)^2$, while the area calculated from the period of the G-oscillations is $0.93(2\pi/a)^2$. The difference between these values does not exceed the possible errors, and the shape of the ζ orbit must, evidently, be really close to a circle.

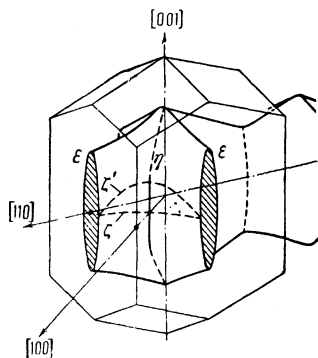


FIG. 5. The hole Fermi surface of the fourth Brillouin zone referred to the first zone of tin.^[9] The section η is constructed from the results of the present work. The shaded sections ϵ must, evidently, be lower and considerably broader in the (001) plane.

Taking into account this fact, and also the appearance of the section of the Fermi surface, shown in Fig. 4, it can be deduced that the shape of the part of the Fermi surface on which the orbits ζ and ζ' occur (Fig. 5) is close to the surface of a right cylinder. The increase in the effective mass of the electrons on rotating the field (Fig. 3) also confirms this, indicating a decrease in curvature and an increase in the area of the electron orbits on going from ζ to ζ' . This conclusion contradicts somewhat the weak dependence of the period of the G-oscillations on field direction, noted by Gold and Priestley.^[9] However, this disagreement is probably due to the low accuracy of measuring the period of the G-oscillations, as can be judged from the scatter of the experimental points in Figs. 1 and 2 of^[9].

A comparison of the results of the present work with the conclusions of Alekseevskii et al^[11] cannot be so detailed, since the method used there only made it possible to consider some topological features of the Fermi surface. It should be stated that there are no contradictions with the later communication of Alekseevskii and Gaïdukov.^[12]

The order of magnitude calculation of the electron momentum $p_{[100]} \approx 2 \times 10^{-20}$ g-cm/sec., made by Galkin, Kaner, and Korolyuk^[13] does not differ from the results of the present work or from that which should have been expected, since the orders of magnitudes of the velocity and effective mass of the electrons are known. Comparison of numerical values is, evidently, meaningless in view of the uncertainties of the data of Galkin et al^[13]. We only note that in a later work of Korolyuk^[14] the momentum for the plane (100) containing the direction [010], equivalent to [100] is calculated as $p \approx 5 \times 10^{-20}$ g-cm/sec., which practically coincides with the result of the present work. Olsen¹⁵ obtained a somewhat smaller absolute value for the electron momentum in the [100] direction, namely $(4.5 - 4.8) \times 10^{-20}$, but the form of the directional dependence of the momentum quoted by him agrees with that found here.

CONCLUSIONS

The present work is the first attempt at an experimental measurement of the momentum of electrons in a metal and a study of its Fermi surface by the new method of "cutting-off" the electron orbits in cyclotron resonance. The results obtained are evidence of the great possibilities of the method.

The only serious drawback to its wide application is the necessity of observing high order cyclotron resonances, which requires, above all, the use of single crystals of very high quality, prepared from extremely pure metals (naturally, the method of measuring the surface impedance of the specimens must also be sufficiently good).

Even when these requirements are satisfied, however, high order resonances are by no means observed for all the electron groups. But if one is content to lower the accuracy of the results, cutting-off of the cyclotron resonances of less high order can be observed on specimens of smaller thickness. In these cases the observation of the cutting-off of the orbit for cyclotron resonance in a magnetic field inclined relative to the plane of the specimen, as a function of the angle of inclination of the field, can be a useful method for the accurate measurement of the diameter of the orbit (for cyclotron resonance in the central sections of the Fermi surface).

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