PHOTOPRODUCTION OF π^0 MESONS ON DEUTERIUM AT 170–210 *Mev*

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The ratio of the π^0-meson photoproduction cross sections on deuterium and hydrogen was measured for energies in the range of 170–210 Mev. The measurements were carried out for meson emission angles of 44, 84, and 124° in the laboratory system. The experimental data are compared with the momentum approximation theory.

INTRODUCTION

A relatively large number of published experimental results [1-4] on the photoproduction of π^0 mesons on deuterium were obtained at bremsstrahlung energies > 200 Mev. Only one experiment [5] was carried out near the photoproduction threshold. However, the π^0-meson photoproduction on hydrogen at these energies has been studied rather extensively.

The energy range near the photoproduction threshold is of special interest, since the transitions into final S and P states of the meson-nucleon system are mainly involved. This greatly facilitates the interpretation of experimental results. In fact, disregarding for the time being the dependence of the corresponding transition operator T on the spin and polarization, we can write

$$T = f(E_1;\ M_1(^1/_2);\ M_1(^3/_2);\ E_2),$$

where E_1 and E_2 are the amplitudes of the electrical dipole and quadrupole transitions respectively, and $M_1(1/2)$ and $M_1(3/2)$ are the amplitudes of the magnetic dipole transitions into the state with total angular momentum of $1/2$ and $3/2$ respectively.

Baldin and Govorkov [6] have shown that the amplitudes E_2 and $M_1(1/2)$ near the photoproduction threshold can be neglected. Therefore, the main contribution to the process under consideration is due to the transition to the $3/2$—$3/2$ state and, in the energy range of interest, we have

$$T = f(E_1;\ M_1^3(^3/_2)).$$

The above considerations facilitate the comparison between the experimental results and the momentum approximation theory. A test of the applicability of that theory to the processes under consideration is interesting for its own sake.

Thus, we can expect that the study of the reactions

$$\gamma + d \begin{cases} \nearrow \pi^0 + d \\ \searrow \pi^0 + p + n \end{cases} \qquad \text{(I)}$$

in the energy range near the threshold and the comparison of their parameters with the known parameters of the process

$$\gamma + p \rightarrow \pi^0 + p \qquad \text{(II)}$$

will yield information on the photoproduction of π^0 mesons on the neutron.

For an analysis of the process (I), it is necessary to know the relative contribution of both reactions at various primary-photon energies, and also the angular distributions.

The simplest method of comparing the processes (I) and (II) is the measurement of the ratio of their differential cross sections under identical conditions.

APPARATUS

The experiments were made with the synchrotron of the Lebedev Physics Institute of the U.S.S.R. Academy of Sciences. A diagram of the setup is shown in Fig. 1. Liquid deuterium and hydrogen placed in a vacuum target (VGM-1) developed in the photon-meson laboratory of the Physics Institute of the Academy [7] were used in the experiments. The working volume of the target, using the chosen method of collimation, was 53 cm^3. The collimation system and the filtering magnet ensured the removal of electrons from the bremsstrahlung beam.

The method employed in the experiment, that of π^0-meson detection using one of the decay γ rays, has poor angular definition (details below), but permits us to obtain the results relatively rapidly

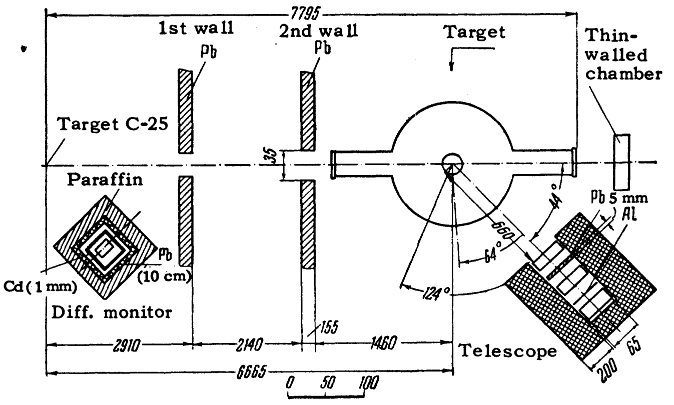

FIG. 1. Diagram of the experimental setup (geometry in the hall).

with good statistical accuracy. In experiments carried out in order to determine the most interesting range of angles and energies for further detailed investigations, such a method of π^0-meson detection is definitely acceptable. The decay γ rays were detected by a scintillation telescope consisting of three counters employing liquid scintillators and FÉU-33 photomultipliers.

The block diagram of the electronic apparatus is shown in Fig. 2. The pulses from counters 2 and 3 were fed to a fast coincidence circuit.[8] This branch of the telescope circuit forms the coincidence channel. The output pulses from counters 1 and 3 were fed to a similar coincidence circuit, and then into an anticoincidence circuit. In the coincidence channel, the pulses passed through a distributed amplifier (with gain k = 30) and a discriminator, and were then fed to an anticoincidence system. The pulses of the anticoincidence channel were fed to the second input of this system passing through a discriminator and a generator producing standard square pulses of 2×10^{-7} sec duration. The resolving times of the fast coincidence circuit and of the anticoincidence circuit were $\tau = (5-6) \times 10^{-9}$ sec and $\tau = 2 \times 10^{-7}$ sec, respectively.

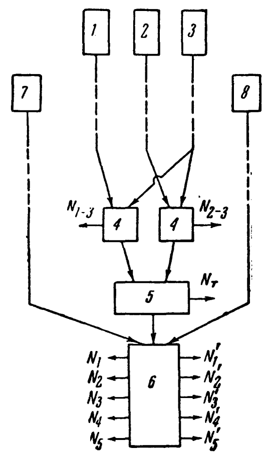

FIG. 2. Block diagram of the electronic apparatus: 1, 2, 3 – photomultipliers and output stages, 4 – coincidence circuits, 5 – anticoincidence circuits, 6 – time analyzer, 7 – trigger of the time analyzer, 8 – differential monitor, N_1–N_5 – outputs of analyzer recording the effect, N'_1–N'_5 – outputs of analyzer recording the pulses from the differential monitor, N_T – telescope count.

The electronic circuit of the telescope is such that the anticoincidence system produces, in addition to the output signal fed to the time analyzer, another fast signal with a rise time of $(1-2) \times 10^{-8}$ sec. This enabled us, where necessary, to analyze the coincidences of the telescope pulses with a signal of a single counter or an analogous telescope, obtaining a resolution better than 10^{-8} sec.

The pulse from the telescope output was fed to a time analyzer similar to the one described in[9]. In the analyzer, the signals were fed to channels, each of which corresponded to a definite range of maximum energies in the bremsstrahlung radiation spectrum from the synchrotron. The time analyzer was triggered by pulses produced in a special circuit at the moment the magnetic field of the synchrotron reached a given value. Such a method enabled us to fix the required energy range of each analyzing channel.

The effect was measured for each of the three selected angles in five channels with maximum energy equal to 178, 186, 194, 202, and 210 Mev. The time spread of the bremsstrahlung pulse from the synchrotron ensured the measurement of the upper limit of the spectrum from 160 to 220 Mev. Thus, it was possible to avoid the errors due to instability of the energy at the limits of the pulse duration. Simultaneously with the recording of the effect in the corresponding branch of the analyzer, pulses of a differential monitor (see below) were recorded in five channels of the analyzer. A special test system operating with a Co^{60} source was used for a regular check of the sensitivity of the electronics.

PRINCIPAL CHARACTERISTICS

In order to calculate the yield of reactions (I) and (II), it is first necessary to know the efficiency of the telescope for γ rays. This efficiency was measured in a series of experiments using monoenergetic electrons in which the detection probability of an electron with a given energy, produced by a photon in the i-th layer of a lead converter, was measured. The efficiency η of the γ-ray telescope determined in such a way is shown in Fig. 3. With good accuracy, it can be written in the form

$$\eta = \begin{cases} 0.0052-0.12 & E_\gamma \leqslant 110\ \text{Mev} \\ 0.42 & E_\gamma > 110\ \text{Mev} \end{cases}. \qquad (1)$$

Such an efficiency is in good agreement with the known efficiencies of analogous γ-ray telescopes described in the literature.

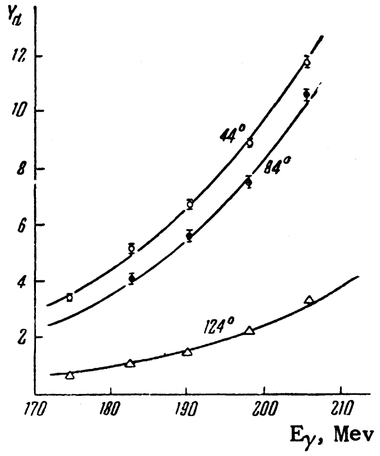

FIG. 3. Energy dependence of the γ-ray yield in the decay of π^0 mesons from deuterium (per 100 counts of the monitor).

In addition to the telescope efficiency, it is necessary to take the following facts into consideration:

1. Spurious counts due to the detection of photons propagating out from the target within the solid angle $\Delta\Omega$ of the telescope and producing a pair in the lead shielding.

2. Spurious counts due to the fact that the efficiency of the anticoincidence circuit is different from unity, since there exists a certain trigger threshold of the generator of standard pulses (see block diagram) which, at its input, receives the diffuse amplitude spectrum of the pulses after the coincidence circuit.

3. Missing counts due to the conversion of protons in the walls of the target and in the absorber before reaching the lead converter of the telescope.

These corrections, calculated as a result of a series of special experiments in which the position of converters and the operating conditions of the telescope were varied, are not greater than 10%.

Let us now consider the problem of monitoring the bremsstrahlung beam. It was mentioned above that, simultaneously with the detection of the telescope pulses in the corresponding channels of the analyzer, the pulses of a monitor counter were recorded. The analyzer channel width was chosen to be 8 Mev, which corresponds to a time of about 200 μsec. (For different channels, the time did not differ by more than 5%.) The effect recorded in each analyzer channel referred to a definite number of counts of the differential monitor.[9]

Corrections were made to account for the sensitivity of the differential monitor to the induced activity and neutron background, and also for the variation of the monitor counting rate with maximum energy of the bremsstrahlung spectrum. In order to reduce the role of these corrections, the differential monitor was used only for determining the relative intensity distribution in the channels. According to this distribution, the total count of a thin-wall monitor chamber was divided among the channels. The absolute calibration of the thin-wall ionization chamber was carried out using a thick graphite chamber and using the induced activity from the $C^{12}(\gamma, n)C^{11}$ reaction in a plate placed in the bremsstrahlung beam. The latter method was used for a regular check of the calibration. Both methods used in different series of measurements agreed to within 5 to 10%.

Having thus determined the counting response of a thin-wall chamber in Mev for the given value of the upper energy of the bremsstrahlung spectrum I^i (Mev/count), we can easily show that the number of photons per count of the thin-wall chamber with energies between the photoproduction threshold and to the maximum energy in the i-th channel (w^i_{max}) is given by the relation

$$n^i_\gamma = I^i \int_{w_t}^{w^i_{max}} f_{sp}(w, w^i_{max})\, dw \Big/ \int_0^{w^i_{av}} w f(w, w^i_{av})\, dw, \qquad (2)$$

where w_t is the energy corresponding to the photoproduction threshold, w^i_{av} is the mean value of the maximum energy in the bremsstrahlung spectrum for the i-th channel, and $f_{sp}(w, w^i_{max})$ is the energy spectrum of photons taking into account the spread of the beam from an electron with energy w^i_{max} incident upon the synchrotron target.

The integrals in Eq. (2) were calculated graphically for all channels taking the beam spread into account and using the tables of Penfold and Less.[10] The difference in the π^0-meson photoproduction thresholds on hydrogen and deuterium was taken into account in the calculations.

MEASUREMENTS

The energy dependence of the γ-ray yield from the decay of π^0 mesons produced on hydrogen and deuterium was measured for the angles of 44, 84, and 124° in the laboratory system (l.s.). The 170—210 Mev and 160—220 Mev energy ranges for an angle of 84° were investigated. A comparison of the experimental conditions at various angles of the principal γ-ray telescope was checked by a test telescope, whose position did not change throughout the experiment. The measurements were carried out alternatively with hydrogen, deuterium, and an empty target. The energy dependence of the yield decay γ rays from π^0 meson decay from deuterium is given in Fig. 3.

Using this data, we have determined the cross section for the emission of decay γ rays in the l.s. by the photon difference method. In order to check the efficiency of the array, the monitoring, and the other parameters entering into the cross section, the results obtained with hydrogen were used to calculate the coefficients A, B, and C in the expression $d\sigma/d\Omega = A + B\cos\theta + C\cos^2\theta$. For the same purpose, we have determined the energy dependence of the total cross section of the process. Both results are in agreement with previously known values,[11-13] within the limits of statistical errors. This provides the basis for using the results of the experiment for further analysis.

The chance coincidence background was determined during the measurements by introducing a time delay in one of the coincidence channels. When working at an angle of 44° in the range of small energies, it was smaller than 2–3%. However, the background from the empty target was considerable, and varied from 30% in the first channel to 15% in the last one. The relatively large statistical errors are due to this fact.

The energy width of the analyzer channels was periodically reset, and, for a long period of time, the variation in the width of separate channels was less than 5%. The graph characterizing the energy resolution of the analyzer channel has a tail at small energies. It was, however, shown that the number of protons due to such a characteristic of the curve is negligibly small.

A certain contribution to the measured effect may be due to γ rays from the Compton effect on nucleons. However, in the narrow range of energies at which the experiment was carried out, the cross section for this process is small (of the order of 10^{-32} cm²/sr)[14] and varies little with the energy of the primary beam. In connection with the above, the use of the photon difference method practically removes the contribution of the Compton effect to the final result. In addition, in control experiments on channels set for energies lower than the photoproduction threshold there were practically no counts.

Throughout the present article, only statistical errors are indicated.

DISCUSSION OF THE RESULTS

The analysis of the experimental data and their comparison with theory was carried out along two lines. First, using the value of the ratio of the cross sections of processes (I) and (II) obtained by the momentum approximation theory, the ratio of the decay photon yield was calculated. Second, the angular and energy dependences of the cross sections of process (I) were used to calculate the corresponding distributions of the decay photons. The experimental results were compared with the distributions obtained in such a way. To calculate the distributions, it is necessary to know the angular resolution of the array.

It can be shown (see the Appendix) that the angular resolution function, or the probability that a neutral pion emitted in the photoproduction at an angle θ_π is detected by a γ-ray telescope placed at an angle θ_γ (the angles are reckoned with respect to the direction of the bremsstrahlung beam) is given by the equation

$$W(\theta_\pi, \beta_\pi, \theta_\gamma) = \frac{\sin\theta_\pi}{\pi}\int_0^\pi \frac{(1-\beta_\pi^2)\,\eta(E_\gamma)\,d\varphi}{[1-\beta_\pi(\cos\theta_\pi\cos\theta_\gamma+\sin\theta_\pi\sin\theta_\gamma\cos\varphi)]^2}, \quad (3)$$

where $\eta(E_\gamma)$ is the γ-ray telescope efficiency and β_π is the π^0-meson velocity. The remaining symbols θ_π, θ_γ, and φ are defined in Fig. 8 (see below). Since the angular resolution functions for given values of θ_π and θ_γ depend on the π^0-meson velocity, they should be calculated separately for each of the three investigated processes (photoproduction on hydrogen and elastic and inelastic photoproduction on deuterium).

For the photoproduction of mesons on hydrogen and the elastic photoprocess on deuterium, a unique kinematic relation exists between θ_π, β_π, and the energy for the primary photon κ. For the inelastic process, where, in the final state, three particles are produced, such a relation does not exist. Moreover, at a given angle θ_π, mesons will be emitted with a certain velocity distribution which is determined by the binding energy of the nucleons in deuterium and the interaction of nucleons in the final state. These distributions have a maximum in the range of velocities determined by the single-nucleon kinematics, and can be constructed by using the formulae and tables of[15]. Thus, for the inelastic process, it is possible to determine the range of variation of β_π for a given angle θ_π, and then to calculate the angular resolution for the limiting values of β_π.

The angular resolution function was calculated by numerical integration of Eq. (3) for the three investigated processes. Typical functions of angular resolution for $\theta_\gamma = 84°$ are shown in Fig. 4. For inelastic photoproduction on deuterium, the graphs corresponding to the limiting values of β_π are slightly different, and the average value of the angular resolution function was used for the process.

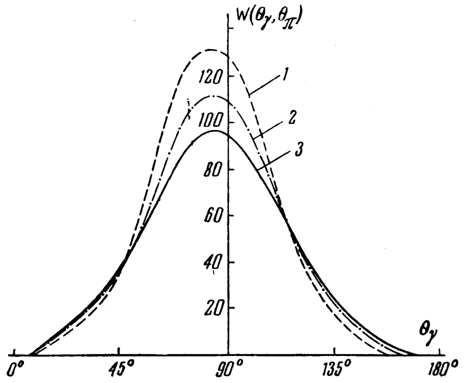

FIG. 4. Angular resolution function of the arrangement for E_γ = 210 Mev and the angle θ_π = 84°. Curve 1 – for the elastic photoproduction of π^0 on deuterium, 2 – for π^0 photoproduction on hydrogen, 3 – for inelastic π^0 photoproduction on deuterium accompanied by the emission of π^0 mesons with minimum energy.

Using the momentum approximation, we write the photoproduction cross section for π^0 mesons on deuterium [16] in the form

$$d\sigma = \left(\frac{1}{2\pi}\right)^2 \frac{k^3\, d\Omega dp}{k^2/E + (k^2 - \varkappa k)/2M} \langle M\rangle^2_{f_0}, \tag{4}$$

where **k** is the meson momentum, **p** the relative momentum of nucleons in the deuteron, κ the momentum of the primary photon, E the meson energy, M the nucleon mass, and $\langle M\rangle^2_{f_0}$ – square of the photoproduction matrix element. In order to obtain cross sections that can be compared with the experimental results, it is necessary to integrate Eq. (4) over the relative momentum of nucleons **p** within the limits from 0 to p_{max}. For this purpose, we have to know all the parameters determining the matrix element and the wave function of the final state of the nucleons. This can be avoided using the so-called completeness theorem approximation.[16,17] In this approximation, the differential cross section of the process (I) can be written as

$$(d\sigma/d\Omega)_d^{\text{compl.}} = A_p^2 + A_n^2 + 2\operatorname{Re} A_p A_n^* F(2\mathbf{q}). \tag{5}$$

where A_p and A_n are the π^0-meson photoproduction amplitudes on protons and neutrons respectively, F(2q) is the form factor of the deuteron given by the expression

$$\int e^{-2i\mathbf{qr}} \varphi_d(\mathbf{r})\, d\mathbf{r} = F(2\mathbf{q}), \tag{6}$$

where $\mathbf{q} = (\boldsymbol{\kappa} - \mathbf{k})/2$, **r** is the relative nucleon coordinate, and $\varphi_d(\mathbf{r})$ is the total function of the ground state of the deuteron.

The cross section of the elastic process in this case will have the form

$$(d\sigma/d\Omega)_d^{\text{el}} = A_p^2 F^2(\mathbf{q}) + A_n^2 F^2(\mathbf{q}) + 2\operatorname{Re} A_p A_n^* F^2(\mathbf{q}). \tag{7}$$

In this notation, the photoproduction cross section of π^0 mesons on hydrogen is

$$(d\sigma/d\Omega)_p = A_p^2. \tag{8}$$

If we assume that $A_p = A_n$, then we have

$$\left(\frac{d\sigma}{d\Omega}\right)_d^{\text{compl.}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_p = 2\,[1 + F(2\mathbf{q})], \tag{9}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_d^{\text{el}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_p = 4F^2(\mathbf{q}). \tag{10}$$

The functions F(q) and F(2q) can easily be calculated if we accept any wave function of the ground state of the deuteron. It is found that, for a relatively large class of wave functions and taking into account both S and D waves, the form factor F and its dependence on q differ relatively little.

In the range of small angles θ_π, the form factor of the deuteron tends to unity when the primary photon energies tend to the threshold value for photoproduction, i.e., when q tends to zero. The total cross section for the photoproduction of π^0 mesons on deuterons is then equal to the elastic cross section [see Eqs. (9) and (10)], and the ratio of the cross sections of the processes (I) and (II) should be equal to four.

Using the Hulthén wave function of the ground state of the deuteron, and using Eqs. (6), (9), and (10), we have calculated the energy dependence of the ratio of the cross sections of the processes (I) and (II) for three angles. The results obtained in this way are in agreement with the experimental results shown in Fig. 5.

It can be seen that, for all investigated angles of emission of π^0 mesons, the ratio of the total

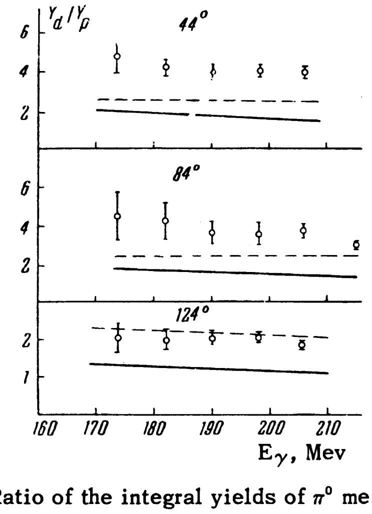

FIG. 5. Ratio of the integral yields of π^0 mesons from deuterium and hydrogen for the angles 44, 84, and 124°. Solid line – calculated ratio of the cross section of elastic processes on deuterium to the cross section on hydrogen, dashed line – calculated ratio of the total cross sections of photoproduction on deuterium and hydrogen.

and elastic cross section of process (I) to the cross section of process (II) depends very little on energy. These results are well confirmed by the experiment. However, the experimentally obtained value of this ratio for 44 and 84° is considerably greater than the theoretical value, and already in this range of angles and energies, is equal to about four. At 124° the experimental point coincides with good accuracy with the theoretical curve corresponding to the ratio of the total cross sections. For this angle, within the limits of the whole investigated energy range, the form factor F(q) is small, and the elastic photoproduction of π^0 mesons on deuterons is suppressed.

Thus, the comparison made shows that, at 170—210 Mev energies of primary photons at l.s. angles θ_π smaller than 90°, the cross section of the elastic photoproduction of π^0 mesons on deuterium is considerably greater than the value expected from the momentum approximation theory.

For a more detailed comparison of the experiment with the momentum approximation, we have used the results of Lebedev and Baldin.[14] In this reference, the cross sections of the elastic and inelastic processes in reaction (I) are obtained taking the interaction of nucleons in the final state into account. The calculations are free from the above-mentioned assumptions, but the S-wave photoproduction is not taken into account. The differential cross sections of both branches of reaction (I) in the l.s. are given by the equation

$$\left(\frac{d\sigma}{d\Omega}\right)_d^{\text{el}} = \left(\frac{1}{2\pi}\right)\frac{4}{3}(7-5\cos^2\theta_\pi)\,|M_1(^3/_2)|^2 k_s^2 \times \frac{|F((\varkappa^2+k_s^2-2\varkappa k_s\cos\theta_\pi)/2)|^2}{|k_s/\sqrt{1+k_s^2}+(k_s-\varkappa\cos\theta_\pi)/2M|}, \qquad (11)$$

where $(2\pi)^4\,|M_1(^3/_2)|^2 = (0.6 \pm 0.1)\times 10^{-4}k^2$, and k_s is the root of the equation

$$\cos\theta_\pi = \frac{(\varkappa^2-k^2)-4M(\varkappa-\sqrt{1+k^2})}{2\varkappa k}.$$

The remaining symbols have been explained above. Furthermore,

$$\left(\frac{d\sigma}{d\Omega}\right)_d^{\text{inel}} = \frac{1}{2\pi}\frac{\alpha M}{2}\{(7-5\cos^2\theta_\pi)\,[F^t(p_s, q_s) + E^t(p_s, q_s)] + (1-\cos^2\theta_\pi)\,[F^s(p_s, q_s) - E^s(p_s, q_s)]\}\,p_s\,|M_1(^3/_2)|^2; \qquad (12)$$

where

$$\mathbf{q}_s = |\boldsymbol{\varkappa} - \mathbf{k}|/2,$$

$$p_s = \{M\,[\varkappa - \varepsilon - \sqrt{1+k^2} - (\varkappa^2+k^2-2\varkappa k\cos\theta_\pi)/4M]\}^{1/2}.$$

The functions $E^{t,s}$ and $F^{t,s}$ have been tabulated in[15], ε is the binding energy of nucleons in the deuteron, and $\alpha = \sqrt{M\varepsilon}$. Equation (12) was integrated over k by a numerical method.

The angular dependence of the photon yield in π^0-meson decay was determined using the corresponding angular resolution function for each of the channels of reaction (I). The calculations carried out for primary photon energies of 174 and 206 Mev are compared with the experimental data in Fig. 6. The theoretical curves are normalized to the experimental point corresponding to θ_π = 124° and primary photon energy of 174 Mev.

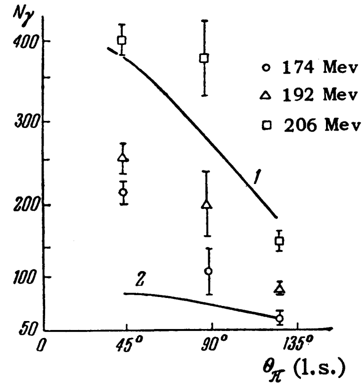

FIG. 6. Angular dependence of the production cross section of decay γ rays from π^0 mesons on deuterium. Curve 1 — calculated for E_γ = 206 Mev, curve 2 — for E_γ = 176 Mev. The calculated curves are normalized at the point E_γ = 206 Mev, θ_γ = 124°.

The character of the angular dependence of the cross section for the energy of 206 Mev is in satisfactory agreement with the experiment. It can be seen that, at these energies, the contribution of the elastic cross section is already small.

For 192 and 174 Mev, the experimentally obtained angular dependence of the cross section has a considerably steeper forward peaking than that which follows from the theory.

Thus, both methods of comparing the results of the experiment with the momentum approximation indicate the existence of a discrepancy in the region where the contribution of the elastic process of π^0 meson photoproduction on deuterium is considerable. Further experiments in the range of small energies and small angles θ_π are necessary for a more detailed analysis.

In addition to the above-described method of analyzing the results, experimental data were used to determine the ratio of differential cross sections of the processes (I) and (II) for the energy ranges from the reaction threshold to 178 Mev, and from 178 to 210 Mev. The results are shown in the table and in Fig. 7 where, for the angle of 84°, they are compared with earlier ratios of the differential cross sections.[1,4,5] It can be seen that the results of the present experiment are in good agreement with the conclusions of André.[5] All the known data show that, in the energy range

Differential Ratios of the yield of π^0 Mesons from Deuterium and Hydrogen

$\theta_{1.s.}$, deg.	E_γ	
	$E_n = 178$ Mev	178–210 Mev
44	4,8±0,8	3,7±0,3
84	4,5±1,3	3,5±0,6
124	1,95±0,2	1,7±0,1

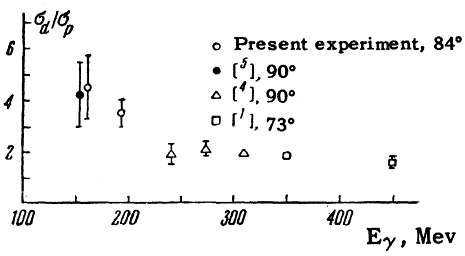

FIG. 7. Differential ratio of the cross sections for the photoproduction of π^0 mesons on deuterium and hydrogen according to the data of different authors.

of about 200 Mev, the ratio of the cross sections σ_d/σ_p increases sharply.

The experiments are at present being continued in order to measure the ratio of the cross sections of processes (I) and (II) at an angle $\theta_p \approx 0°$ in the range from the reaction threshold to 240 Mev. The π^0 mesons are, in that case, detected from the two decay photons.

In conclusion, the authors take the opportunity to express their gratitude to P. N. Shareĭko for designing the electronic apparatus and taking part in the measurements, to A. M. Baldin and A. I. Lebedev for valuable discussion, and also to the members of the synchrotron team.

APPENDIX

ANGULAR DISTRIBUTION FUNCTION OF A γ-RAY TELESCOPE

Let κ be the momentum of the primary photon, and **k** and $\mathbf{p}_\gamma$ the momenta of the π^0 meson and of the decay photon respectively moving in the direction of the telescope. The definition of the angles θ_γ, θ_π, θ_x, and φ is clear from Fig. 8.

If $d\sigma/\sigma\Omega$ is the angular distribution of π^0 mesons in the l.s., then the number of mesons propagating at an angle θ_π within a unit solid angle is

$$n_{\pi^0} = K\,(d\sigma/d\Omega)\,\sin\theta_\pi\,d\theta_\pi\,d\varphi; \qquad (A.1)$$

where K is a factor depending on the number of nuclei in the target and the primary photon flux.

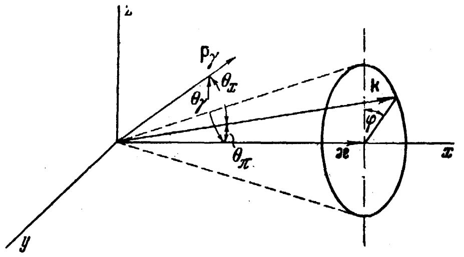

FIG. 8

The probability of the decay of a π^0 meson with velocity β_π into two photons so that one of them is emitted at an angle θ_x to the direction of motion of the π^0 meson in the l.s. can be written as

$$P(\theta_x\,\beta_\pi) = (1-\beta_\pi^2)/2\pi\,(1-\beta_\pi\cos\theta_x)^2. \qquad (A.2)$$

The number of photons propagating in the direction of the telescope and due to the decay of $n\pi^0$ mesons Eq. (A.1), will then be given by

$$n_\gamma = \frac{K}{2\pi}\frac{d\sigma}{d\Omega}\sin\theta_\pi\,d\theta_\pi\,d\varphi\,\frac{1-\beta_\pi^2}{(1-\beta_\pi\cos\theta_x)^2}. \qquad (A.3)$$

The decay photons from mesons propagating on the generator of the cone with an opening angle θ_π (the angle θ_x is variable) will also propagate in the same direction. In order to take these γ rays into account, it is necessary to integrate Eq. (A.3) over the angle φ. Moreover, as can easily be shown

$$\cos\theta_x = \cos\theta_\pi\cos\theta_\gamma + \sin\theta_\pi\sin\theta_\gamma\cos\varphi. \qquad (A.4)$$

Finally, it is necessary to take into consideration the contribution of the decay photons from π^0 mesons emitted at different angles. For this purpose, it is necessary to integrate Eq. (A.3) also over θ_π.

Taking the detection efficiency of the γ-ray telescope into account, we shall obtain the number of the decay photons detected per unit time by the telescope set at an angle θ_γ to the direction of the bremsstrahlung beam (the solid angle of the telescope enters through the factor K):

$$N_\gamma(\theta_\gamma) = \int_0^\pi\int_0^{2\pi}\frac{K}{2\pi}\frac{d\sigma}{d\Omega}\sin\theta_\pi\frac{\eta(E_\gamma)(1-\beta_\pi^2)}{(1-\beta_\pi\cos\theta_x)^2}d\varphi\,d\theta_\pi. \qquad (A.5)$$

η is given by Eq. (1), and

$$E_\gamma = \mu_0 c^2/2\gamma\,(1-\beta_\pi\cos\theta_x),$$

where $\mu_0 c^2$ is the meson rest mass and $\gamma = (1-\beta_\pi^2)^{-1/2}$. Introducing the angular resolution function of the telescope (see text) $W(\theta_\pi, \beta_\pi)$, we obtain

$$N_\gamma(\theta_\gamma) = \int_0^\pi \frac{d\sigma}{d\Omega}W(\theta_\pi\beta_\pi)\,d\theta_\pi,$$

where, taking Eq. (A.4) into account, $W(\theta_\pi, \beta_\pi)$

written in the form (5) is determined by Eq. (A.5).

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305