INVESTIGATION OF THE ENERGY DEPENDENCE OF THE CROSS SECTION FOR PHOTO-PRODUCTION OF π^+ MESONS ON HYDROGEN NEAR THRESHOLD

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The differential cross sections for photoproduction of positive pions on hydrogen have been measured in the photon energy range from 167 to 212 Mev. The measurements are carried out at an angle $\theta = \cos^{-1}(k\omega - 0.93)/kq$ (the angle for which the contribution of the nonphysical region to the dispersion integrals is zero). The results are in good agreement with the dispersion theory. A detailed analysis is made of the experimental results together with the data on the angular distribution.

INTRODUCTION

PHOTOPRODUCTION of π^+ mesons on hydrogen is one of the fundamental reactions of meson physics. Particular interest is attached to the region near the energy threshold of pion production.

Photoproduction of mesons on hydrogen was investigated in many laboratories, but only recently were experimental data obtained on the angular distribution of the mesons in a wide range of angles, for photon energies of 185 [1], 230 [2], 260, and 290 Mev [3]. All these experimental data, together with data on π^0 mesons, [4], were compared with the results of dispersion theory. The comparison has shown that experiment does not agree with theory as well as expected. For angles close to 180°, the experimental points lie below the theoretical ones, the deviation reaching several standard errors at certain energies. At low angles the discrepancy is less, but the point always lies somewhat above the theoretical curve.

As emphasized by Baldin [5], the reason for this may be that the theoretical cross sections depend on the contributions that the nonphysical region and the regions with very high energies make to the dispersion integral. Baldin also noticed that, unlike the scattering of pions on nucleons, where the contribution of the nonphysical region is small only when the angles are very small, in the case of photoproduction of pions on nucleons the contribution from the nonphysical region is equal to zero along a curve satisfying the relation

$$k\omega - kq\cos\theta = k_t = 0{,}93, \qquad (1)$$

where k — photon momentum, k_t — threshold value of the photon momentum, q, ω — the momentum and total energy of the pion (here and throughout $\hbar = c = \mu = 1$, where μ is the pion mass) and θ — angle of meson emission. All the quantities are in the center-of-mass system (c.m.s.).

An investigation of the experimental data on the photoproduction of π^0 mesons at angles satisfying relation (1) has shown that experiment agrees well with theory.[5] No such check was made on the theory for the photoproduction of charged mesons, owing to the lack of experimental data. We have therefore undertaken to measure the energy dependence of the cross section of the photoproduction of π^+ mesons on hydrogen at angles θ satisfying relation (1). The dependence of the angle θ on the photon energy E_γ in the laboratory system (l.s.), defined by this relationship, is shown in Fig. 1. The figure also shows plots of $k\omega - kq \times \cos\theta = 0.7$ and $k\omega - kq\cos\theta = 1.6$. Within the region bounded by these curves, the experimental data on the photoproduction of π^0 mesons agree with the theory within 10 or 15%.[5] It is seen from the figure that at photon energies near threshold (165 — 220 Mev) the measurements must be made in the interval 50 — 60° (c.m.s.), corresponding to 40 — 50° in the l.s.

EXPERIMENT

We measured the differential cross section of photoproduction of π^+ mesons in the photon energy interval from 167 to 212 Mev; at 42.5° (l.s.) to the photon beam. This corresponds to the c.m.s. angles indicated in Fig. 1 by the dashed line. As can be seen from the figure, Eq. (1) is rigorously satisfied for 195 Mev photons, while the value of $k\omega - kq\cos\theta$ for the boundary values of 167 and 212 Mev is respectively 0.88 and 0.99, which differs little from the value 0.93 in Eq. (1).

A photon beam from the synchrotron of the Physics Institute of the Academy of Sciences, with maximum energy 250 Mev, was guided by a system of collimators and a clearing magnet to a hydrogen target, comprising a vertical brass cylinder of 50 mm diameter and 17 μ wall thickness. The cylinder was in the center of a vacuum chamber of 519 mm diameter. Outside the vacuum chamber was placed, on a special holder, a detector comprised of a stack of 50 NIKFI type BK-400 pellicles measuring 5 × 10 cm. The stack was placed between two emulsion blocks 2 cm thick and was so secured that the mesons entered the stack from the end. A diagram of the experimental setup is shown in Fig. 2.

The pellicles, processed in the usual manner, were scanned under MBI-1 microscopes with magnifications 300 and 210. All π-μ decays were registered, as well as (for additional control) the terminations of the muon and pion tracks near the stopping points. To determine the background of the π^+ mesons produced in the target walls, we also registered the π^- mesons terminating in a star with one or more prongs. By tracing the

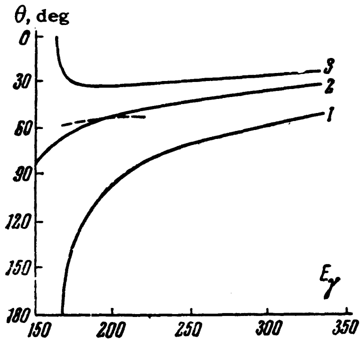

FIG. 1. Curves 1 and 3 are plots of $k\omega - kq\cos\theta = 0.7$ and $k\omega - kq\cos\theta = 1.6$. Curve 2 is the dependence of the angle of zero contribution of the nonphysical region to the dispersion integrals on the photon energy E_γ.

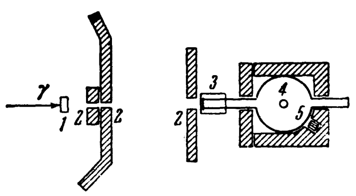

FIG. 2. Diagram of experimental setup: 1 – monitor, 2 – collimators, 3 – clearing magnet, 4 – target, 5 – pellicle stack.

Table I. Distribution of number of π^+ mesons over the photon energy intervals

E_γ interval, Mev	Number of π^+ mesons	E_γ interval, Mev	Number of π^+ mesons
164.7—169.5	403	192.0—196.6	299
169.5—173.9	329	196.6—201.3	261
173.9—178.6	357	201.3—205.7	261
178.6—183.0	319	205.7—210.2	259
183.0—187.4	287	210.2—214.8	228
187.4—192.0	319		

tracks in neighboring pellicles we were able to identify reliably each event and to eliminate the random background, and also to increase the counting efficiency. Altogether 3322 π-μ decay events and 64 π^- mesons were observed in a 340 cm^2 area.

The energy of the π^+ mesons was determined from the position of the π-μ decay in the plate. Table I lists the distribution of the number of π^+ mesons over the energy intervals. In determining the meson energy we took account of the losses in the liquid hydrogen and in the target walls. These range from 1 to 0.1 Mev, depending on the meson energy. The correction for the scattering of the meson in the emulsion did not exceed 0.1 Mev.

In the determination of the cross section, we calculated the nuclear interaction between the mesons and emulsion for each energy interval, which resulted in a correction ranging from 0 to 12.7%, and the decay in flight, which gave a correction ranging from 4.2 to 6.9%. The background due to mesons from the target walls was on the average 2%. The intensity of the photon beam was measured with a thick-wall graphite chamber. The solid angle ranged from 2.5×10^{-3} to 3.3×10^{-3} sr, depending on the interval.

The cross sections obtained are shown in Fig. 3. The errors indicated are statistical. The absolute values of the cross sections were normalized in accordance with the data of the Illinois group.[1,6]

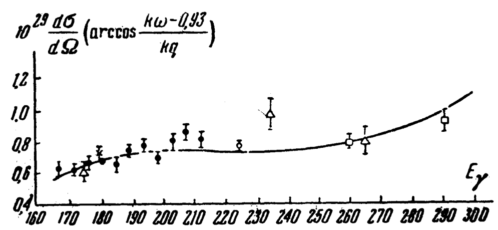

FIG. 3. Dependence of the differential cross section at an angle $\theta = \cos^{-1}(k\omega - 0.93)/kq$ on the photon energy: ● – our data, ○ – data of[2], □ – data of[3], × and Δ – data of[6].

DISCUSSION OF RESULTS

Figure 3 shows the dependence of the cross sections obtained for the photoproduction of π^+ mesons at angles $\theta = \cos^{-1}(k\omega - 0.93)/kq$ on the photon energy, and also experimental data obtained by others.[2,3,6] The solid curve is calculated by dispersion theory, with the imaginary part of the resonant amplitude taken from the experimental data (see [5]). As can be seen from the figure, the experimental data are in good agreement with the theoretical curve.

To permit a more detailed comparison of experiment with theory and to obtain the threshold value of the square of the π^+-meson photoproduction matrix element, these data were reduced by the least squares method to determine the dependence of $\chi^{-1}d\sigma/d\Omega$ on the meson momentum, where $\chi = (q/k) \times (1 + \omega/M)^{-2}$ and M is the nucleon mass. The dependence of the square of the photoproduction matrix element $\chi^{-1}d\sigma/d\Omega$ on the square of the meson momentum q^2, calculated from dispersion theory, is represented by the solid line of Fig. 4. Generally speaking, this dependence is given by

$$\chi^{-1}d\sigma/d\Omega = A + Bq + Cq^2 + Dq^3 + \ldots \quad (2)$$

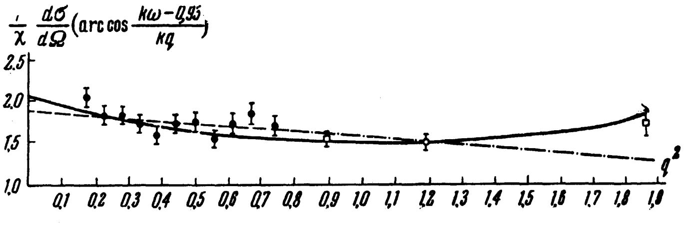

FIG. 4. Dependence of the square of the photoproduction matrix element on the square of the meson momentum.

However, for angles satisfying the relation $k\omega - kq\cos\theta = \text{const}$, this dependence simplifies to

$$\chi^{-1}d\sigma/d\Omega = a_0 + a_1q^2 + a_2q^4 + \ldots \quad (3)$$

The coefficients a_0, a_1, a_2, etc. were determined from the experimental data by the least squares method. The expression

$$S = \sum_{i=1}^{n} p_i \left[\frac{1}{\chi_i}\frac{d\sigma_i}{d\Omega} - (a_0 + a_1q_i^2 + a_2q_i^4 + \ldots)\right]^2, \quad (4)$$

where p_i is the weight of the i-th measurement of $\chi^{-1}d\sigma/d\Omega$, was minimized.

As a result of this analysis we obtained the following expressions

$$\frac{1}{\chi}\frac{d\sigma}{d\Omega}\left[10^{-29}\frac{\text{cm}^2}{\text{sr}}\right] = (1.90 \pm 0.15) - (0.34 \pm 0.22)\,q^2, \quad (5)$$

$$\frac{1}{\chi}\frac{d\sigma}{d\Omega}\left[10^{-29}\frac{\text{cm}^2}{\text{sr}}\right] = (2.39 \pm 0.21) - (2.87 \pm 0.93)\,q^2 + (2.80 \pm 1.0)\,q^4, \quad (6)$$

with almost identical Gauss criteria $S/(n - m) \approx 0.01$, where n is the number of experimental points and m is the number of parameters.

These expressions were obtained from our experimental values of q^2 in the interval from 0.17 to 0.74. In this region, both expressions describe the experimental results sufficiently well. In order to find the preferred expression, we extrapolated to the region of values $q^2 > 0.74$.

Figure 4 shows $\chi^{-1}d\sigma/d\Omega$ as a function of q^2. The dashed line shown is a plot $\chi^{-1}d\sigma/d\Omega$ vs. q^2 as obtained from (5). The extrapolation of this curve to $q^2 > 0.74$ is shown by the dash-dot line. The figure shows that the experimental data practically coincide with the theoretical values. The data of Malmberg and Robinson[2] for $q^2 = 0.89$ and of Knapp et al[3] for $q^2 = 1.18$ and 1.84 are also in good agreement with (5), and, as can be readily shown, completely disagree with (6).

For a more detailed comparison let us examine the experimental and theoretical expansions of the square of the matrix element in powers of q. It proved more convenient to represent the theoretical data in the form of an expansion

$$\left(\omega - \frac{1}{2M}\right)^3 \frac{1}{\chi}\frac{d\sigma}{d\Omega} = A_0 + A_1q^2 + A_2q^4 + \ldots \quad (7)$$

in even powers of q. The coefficients of this expansion are listed in Table II. As can be seen from the table, the coefficients A_0 and A_1 agree very well with theory. The coefficients A_2 and A_3 are subject to large errors and it is premature to compare them with theory. Furthermore, their contribution to the square of the matrix element is small in the investigated region. The third line of Table II shows the theoretical value of the coefficients, calculated under the assumption that the dispersion integrals are equal to zero.

Thus, an investigation of the energy dependence of the cross section at an angle $\theta = \cos^{-1}(k\omega - 0.93)/kq$ yields for the square of the matrix element at the pion photoproduction threshold a value $a_0 = (1.90 \pm 0.15) \times 10^{-29}$ cm²/sr, which agrees well with the theoretical value $a_0 = 2.04 \times 10^{-29}$ cm²/sr. It must be emphasized that the threshold value was obtained by extrapolating the experimental data on the basis of the dispersion theory. Our theoretical premises are free of the uncertainties connected with the possible influence of the nonphysical region. In addition, the contribution of the region of high energies along the

Table II. Coefficients of the expansion of (7) in even powers of q (in units of 10^{-29} cm^2/sr)

	A_0	A_1	A_2	A_3	A_4
Experiment	1.50±0.11	2.18±0.26	0.33±0.28	—0.23±0.08	0.14±0.01
Theory, with account of the dispersion integrals	1.62	1.92	0.14	—0.69	0.10
Theory without account of the dispersion integrals	1.62	1.96	0.35	—0.18	—0.02

curve (1) is a constant. This clarifies greatly the fundamental parameters of pion physics at low energies.

Indeed, if we use the obtained threshold value a_0, the theoretical ratio $\sigma^-/\sigma^+ = 1.28$, and the difference in the pion-nucleon scattering s-phases $|\alpha_3 - \alpha_1|/q = 0.245 \pm 0.007$,[7] we obtain the following ratio of the probabilities of scattering with charge exchange and radiation capture of negative pions in hydrogen (the Panofsky ratio):

$$P = \sigma(\pi^- + p \to n + \pi^0)/\sigma(\pi^- + p \to n + \gamma) = 1.57 \pm 0.11.$$

The weighted mean obtained from many measurements of the directly-measured Panofsky ratio[8-13] is P = 1.54 ± 0.015. However, good agreement of the two results does not mean complete agreement with theory, for possible deviations from the theoretical amplitude may influence the threshold parameters. This is illustrated by the following example.

Using P = 1.54 ± 0.015, $|\alpha_3 - \alpha_1|/q = 0.245 \pm 0.007$, and our threshold value of the square of the matrix element for positive pions $(1.90 \pm 0.15) \times 10^{-29}$, we obtain $\sigma^-/\sigma^+ = 1.34 \pm 0.11$. The theoretical value of σ^-/σ^+ with an account of the γ-3π interaction is [14]

$$R = \frac{\sigma^-}{\sigma^+} = \frac{\{1 + \mu/2M + \mu(\mu_p + \mu_n)/2M + \lambda_1 g^{-1}[{}^1/_4(\mu_p - \mu_n) + {}^1/_2\xi]\}^2}{\{1 - \mu/2M - \mu(\mu_p + \mu_n)/2M - \lambda_1 g^{-1}[{}^1/_4(\mu_p - \mu_n) + {}^1/_2\xi]\}^2}, \quad (8)$$

where the coupling constant is $g^2/4\pi = 15$, μ_p and μ_n are the anomalous magnetic moments of the proton and neutron respectively, ξ is the probability of dissociation of the nucleon, and λ_1 is the effective coupling constant of the γ-3π interaction. If $\sigma^-/\sigma^+ = 1.35$, then λ_1 is positive and lies between $\lambda_1 = 0.26$ when $\xi = 0$ and $\lambda_1 = 0.17$ when $\xi = 1$. The positive sign of λ_1 agrees with the fact that the γ-3π interaction is responsible for the isoscalar part of the electromagnetic form factor of the nucleon.[14] Thus, further study of the threshold parameters can yield interesting information on the influence of pion-pion interaction.

Unfortunately, the experimental data obtained do not enable us to calculate the photoproduction amplitudes and to compare them with theory. This is due both to the insufficient accuracy of the experimental data and to the complexity of the equations for the amplitudes. We have attempted to determine the amplitudes from our experiments, using the angular distribution of the pions at a photon energy 185 Mev.[1]

According to the theory, the differential cross section of pion photoproduction has the following dependence on the photoproduction amplitudes

$$d\sigma/d\Omega = (q/k)\{|F_1|^2 + |F_2|^2 - 2\mathrm{Re}\,F_1^*F_2\cos\theta + \tfrac{1}{2}\sin^2\theta\,[|F_3|^2 + |F_4|^2 + 2\mathrm{Re}\,F_2^*F_3 + 2\mathrm{Re}\,F_1^*F_4 + 2\mathrm{Re}\,F_3^*F_4\cos\theta]\}, \quad (9)$$

where we have for the region near threshold

$$F_1 = \sqrt{2}F_{10} - \sqrt{2}F_{11}\cos\theta, \qquad F_2 = \sqrt{2}F_{20},$$
$$F_3 = \sqrt{2}F_{30} + \sqrt{2}F_{31}/(1 - \beta\cos\theta),$$
$$F_4 = \sqrt{2}F_{41}/(1 - \beta\cos\theta);$$

β — velocity of the pion. It follows from these formulas that if we have experimental data for 0 and 180° we can determine the amplitude F_{10} and the sum of the amplitudes $F_{11} + F_{20}$.

Using our data[1] for 15° and 165° in the c.m.s., we obtain the following two sets of solutions for 185 Mev photon energy

$$(F_{10})_1 = (1.81 \pm 0.034)\cdot 10^{-2},$$
$$(F_{11} + F_{20})_1 = -(0.105 \pm 0.034)\cdot 10^{-2},$$
$$(F_{10})_2 = -(1.81 \pm 0.034)\cdot 10^{-2},$$
$$(F_{11} + F_{20})_2 = (0.105 \pm 0.034)\cdot 10^{-2}.$$

In addition to the indicated errors, we must also take into account an uncertainty on the order of 5% due to the presence of the term with $\sin^2\theta$. Theoretically, $F_{10} = 1.92 \times 10^{-2}$ and $(F_{11} + F_{20}) = 0.110 \times 10^{-2}$. The value of F_{10}, determined from the coefficient a_0 in the expansion of $\chi^{-1}d\sigma/d\Omega$ in powers of q, is $\pm(1.85 \pm 0.074) \times 10^{-2}$. If we confine ourselves only to positive F_{10}, we find that the sign of $F_{11} + F_{20}$ is the opposite of the theoretical value. An analogous conclusion follows also from the analysis of other experimental angular distribution data.[3,4,6]

Figure 5 shows the dependence of the coefficient B_0 of $\cos\theta$ in the angular distribution of the photomesons. The coefficient B_0 was calculated from the formula

$$B_0 = 2\beta\,[\beta\,(B + D) + A + C + E]/(1 - 3\beta^2), \qquad (10)$$

where A, B, C, D, and E are the coefficients in the expansion

$$(1 - \beta\cos\theta)^2 d\sigma/d\Omega = A + B\cos\theta + C\cos^2\theta + D\cos^3\theta + E\cos^4\theta,$$

obtained by reducing the experimental data by the method of least squares.[15] At photon energies $E_\gamma < 230$ Mev, we have $B_0 \sim -4F_{10}(F_{11} + F_{20})$.

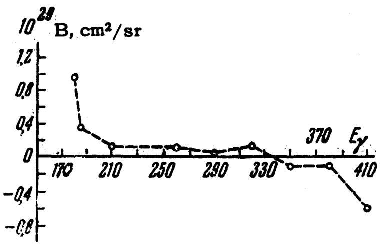

FIG. 5. Dependence of the coefficient B_0 on the photon energy E_γ.

As can be seen from the plot, B_0 is positive up to $E_\gamma = 330$ Mev, passes through zero at 330 Mev, and becomes negative when $E_\gamma > 330$ Mev. The positive value of B_0 at low energies indicates that the sum $F_{11} + F_{20}$ is a negative quantity. As was already mentioned, the theoretical value of this quantity remains positive. To be sure, it must be noted that the coefficient B_0 is subject to a very large statistical error (which reaches 200% in some cases). Disregarding the errors, so interesting a conclusion should stimulate an exhaustive study of the angular distributions of π^+ mesons near threshold ($E_\gamma < 230$ Mev).

More accurate experiments near threshold will permit in the future a more detailed comparison between experiment and theory and hence determine the pion photoproduction amplitudes.

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