CONTRIBUTION TO THE THEORY OF ELECTROMAGNETIC FLUCTUATIONS IN A NON-STEADY-STATE PLASMA

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The components of the tensor $\varphi_{\alpha\beta}(\omega, t)$ of the spectral intensity of an electric current in a non-relativistic magneto-active plasma, located in strong varying electric and magnetic fields, are calculated. From the electrodynamic point of view, such a plasma can be regarded as a medium with time-varying parameters. Some general properties of such a medium are considered.

In a previous paper[1] (cited below as I), the collision-induced fluctuations were considered for a non-relativistic, non-equilibrium plasma. In this case, only the steady state was studied, in which the kinetic parameters of the plasma are unchanging in time.* Such a case is realized, for example, when a strong constant or rapidly varying electric field E acts on the plasma. A work of Silin[3] was also devoted to the study of fluctuations in a non-uniform steady-state plasma, in which a highly rarefied plasma is considered, such that collisions in it can be neglected, and the fluctuations of the electromagnetic field are entirely determined by the Cerenkov radiation of the electrons.

The present paper, which is a direct continuation of paper I, is devoted to the consideration of the electromagnetic fluctuations in a non-relativistic, periodically non-uniform plasma—a case which exists when strong variable (periodic) electric and magnetic fields act on the plasma.

1. STATEMENT OF THE PROBLEM

Let us consider a non-relativistic plasma located in strong variable electric and magnetic fields,† and also in a homogeneous constant magnetic field H_0.

By a strong electric field, we mean a field whose amplitude E_1 satisfies the condition [4,5]

$$E_1 \gtrsim E_p \equiv [3kTm\delta(\Omega^2 + \nu_{\text{eff}}^{02})/e^2]^{1/2}, \tag{1.1}$$

where e, m are the charge and mass of the electron, k is Boltzmann's constant, T is the absolute temperature of the heavy particles of the plasma, Ω is the frequency of the field, ν^0_{eff} is the effective collision frequency of the electron with the heavy particle in the absence of the field, δ is the mean relative fraction of the energy lost by the electron in a single collision with a heavy particle: $\delta \ll 1$ (for the precise meaning of ν_{eff} and δ, see [4,5]). We shall regard the variable magnetic field $H_{\text{ext}}(t)$ as strong (and shall accordingly consider its effect on the plasma) in the case in which its amplitude H_1 and frequency Ω satisfy the condition

$$\omega_{H_1} \equiv |e| H_1/mc \gtrsim \Omega, \tag{1.2}$$

where c is the velocity of light.* It is easy to see that if the field H_{ext} is strong, then the electric field E_{ext} associated with it should generally be strong ($E_1 \gg E_p$). This follows from the fact that the amplitudes of the electric and magnetic fields at each point of space are connected by the linear equation $H_1 = \alpha E_1$, and therefore, by the condition (1.2),

$$E_p \lesssim A\alpha E_1 \qquad (\text{if } \omega_{H_1} \gtrsim \nu^0_{\text{eff}})$$

or

$$E_p \lesssim A\,(\nu^0_{\text{eff}}/\omega_{H_1})\,\alpha E_1 \qquad (\text{if } \omega_{H_1} \lesssim \nu^0_{\text{eff}}).$$

Here $A \equiv (3kT\delta/mc^2)^{1/2}$. Thus, if special measures are not taken to make the coefficient α suffi-

*The work of Bekefi, Hirshfield and Brown,[2] which is devoted to the same problem, appeared after paper I had gone to press.

†In the problem considered by us, there will always be strong external fields; therefore, we shall omit the word "external," using in certain cases the index "ext" (E_{ext} and H_{ext}).

*We note that in the kinetic theory of electrical conduction in a plasma one usually neglects the effect of the variable magnetic field. This neglect is valid when the condition $u/c \ll 1$ is satisfied (u is the mean ordered velocity of the electron).

ciently large (large relative to A^{-1} or $A^{-1}(\nu_{eff}^0/\omega_{H_1})^{-1}$,* then, in accord with (1.2), the condition $E_p \ll E_1$ is automatically satisfied. In what follows, in speaking of a strong field, we shall always have in mind that the electric field is also strong in this case. It is evident that fulfillment of condition (1.2) no longer follows from satisfaction of the condition (1.1); therefore, the plasma may be situated in a strong electric but weak magnetic field.

The effect on the plasma of strong external variable electric and magnetic fields reduces electrodynamically to the result that the plasma becomes a medium, generally speaking, with properties that are variable in time. The problem of the fluctuating electromagnetic radiation of such a plasma, set up within the framework of macroscopic electrodynamics,[6,7] reduces to the corresponding solution of the following system of macroscopic field equations:

$$\text{rot}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\int_0^\infty \hat{K}(t,\tau)\,\mathbf{E}(t-\tau)\,d\tau + \frac{4\pi}{c}\mathbf{j},$$
$$\text{rot}\,\mathbf{E} = -c^{-1}\partial\mathbf{H}/\partial t. \qquad (1.3)\dagger$$

Here $\mathbf{j} = \mathbf{j}(\mathbf{r}, t)$ is the fluctuating current density at the point $\mathbf{r}$, considered as an external current. The integral term in the first equation is the total current density induced by the field $\mathbf{E}(\mathbf{r}, t)$ at the point $\mathbf{r}$,‡ and the coupling is assumed to be space-local (in correspondence with the neglect of spatial dispersion here and everywhere in what follows). The component of the tensor $K_{ik}(t, \tau)$ takes into account the effect of the i-th component of the total current density at the time t on the δ-pulse of the k-th component of the field $\mathbf{E}$, acting at the time $(t-\tau)$; for an inhomogeneous plasma, the components $K_{ik}(t, \tau)$ also depend on $\mathbf{r}$.

Before proceeding to the fluctuation part of the problem, let us consider some electrodynamic properties of a plasma in strong variable fields.

2. SOME ELECTRODYNAMIC PROPERTIES OF A PLASMA IN STRONG ELECTRIC AND MAGNETIC FIELDS

It follows from the definition of the tensor $K_{ik}(t, \tau)$ that $K_{ik}(t, \tau) = 0$ for $\tau < 0$ (principle of causality). Furthermore, if the external influence is periodic with period $2\pi/\Omega$ (and we consider only such interactions) then the components $K_{ik}(t, \tau)$ are periodic in the variable t with period $2\pi/\Omega$:

$$K_{ik}(t, \tau) = K_{ik}(t + 2\pi/\Omega, \tau).$$

The tensor

$$\sigma_{ik}(\omega, t) = \sigma_{ik}(\omega, t + 2\pi/\Omega) = \int_0^\infty K_{ik}(t, \tau)\,e^{i\omega\tau}\,d\tau \qquad (2.1)$$

is obviously a direct generalization of the ordinary conductivity tensor (relative to the total current) to the case under consideration of media with variable parameters: for a harmonic field, $\mathbf{E} = \mathbf{E}_0 e^{-i\omega t}$, the components of the current density are equal to $\sigma_{ik}(\omega, t)E_{0k}e^{-i\omega t}$. Just as for media with constant parameters, the real and imaginary parts of the components

$$\sigma_{ik}(\omega, t) = \sigma'_{ik}(\omega, t) + i\sigma''_{ik}(\omega, t)$$

must satisfy the Kramers-Kronig relations (relative to the variable ω), which follow from the analyticity of the function $\sigma_{ik}(\omega, t)$ in the upper half plane of the complex variable ω (for arbitrary t).[7] It is easy to show that the imaginary and real parts of the coefficients of the expansion

$$\sigma_{ik}^{(n)}(\omega) = \sigma_{ik}^{(n)\prime} + i\sigma_{ik}^{(n)\prime\prime}$$

of the function $\sigma_{ik}(\omega, t)$ in the Fourier series

$$\hat{\sigma}(\omega, t) = \sum_n \hat{\sigma}^{(n)}(\omega)\,e^{in\Omega t} \qquad (2.2)$$

must satisfy these relations. The set of the tensors $\hat{\sigma}^{(n)}(\omega)$ completely determines the electrodynamic properties of the medium. It is not difficult to prove that the local absorbing properties of the medium are determined only by the "zero" tensor $\hat{\sigma}^{(0)}(\omega)$, just as they are for media with constant parameters, i.e., for example, for the harmonic field $\mathbf{E}_0 e^{-i\omega t}$, the time average of the power dissipated per unit volume is equal to

$$P = \frac{1}{4}\left[\sigma_{ik}^{(0)}(\omega)\,E_{0k}E_{0i}^* + \text{c.c.}\right]. \qquad (2.3)$$

Finally, we note the symmetry properties, which follow from (2.1) and (2.2):

$$\sigma_{ik}(\omega, t) = \sigma_{ik}^*(-\omega, t), \qquad (2.4)$$

$$\sigma_{ik}^{(n)}(\omega) = \sigma_{ik}^{(-n)*}(-\omega). \qquad (2.4')$$

We now turn to the calculation of the tensors $\hat{\sigma}(\omega, t)$ and $\hat{\sigma}^{(n)}(\omega)$ for the cases of interest to us, namely, of a plasma located in strong electric and magnetic fields. On the basis of kinetic theory,[4,5] with accuracy up to the small terms

*An example of such a special case is a system in which a sufficiently small volume of plasma is placed in a resonant cavity in the region of the antinode of the magnetic field and the node of the electric field.

†rot = curl.

‡In the case under consideration, it is useful to divide the total current into the conduction current and the polarization current.

neglected by us,* we have the relation

$$\hat{\sigma}(\omega, t)\,\mathbf{E}_0 e^{-i\omega t} = -\frac{4\pi e N}{3}\int_0^\infty \mathbf{u} v^3 \frac{\partial f_0(v, t)}{\partial v}\,dv, \qquad (2.5)$$

where $f_0(v, t)$ is the symmetric part of the electron velocity distribution function (normalized to unity):

$$f(\mathbf{v}, t) = f_0(v, t) + \mathbf{v}\,\mathbf{f}_1(v, t)/v,$$

N is the density of electrons, the time dependence of which we shall neglect in what follows;† the vector $\mathbf{u}$ is the solution of the linear equation

$$\frac{\partial \mathbf{u}}{\partial t} + \nu(v)\mathbf{u} - \frac{e}{mc}[\mathbf{u}, \mathbf{H}_0 + \mathbf{H}_{\rm ext}(t)] = \frac{e}{m}\mathbf{E}_0 e^{-i\omega t}, \qquad (2.6)‡$$

where $\nu(v) = \nu_m^{el}(v) + \nu_i(v) + \nu_m^{inel}(v)$ is the total number of heavy particle collisions of an electron having a velocity v. We note that the field E_{ext} does not appear in Eq. (2.6); on the other hand, it does enter into the kinetic equation that determines the function $f_0(v, t)$.[4,5] We shall assume that the variable field $H_{ext}(t)$ is directed along the constant field H_0, which direction is chosen for the z axis. Substituting $H_{ext}(t) = H_1 \cos \Omega t$ in (2.6), and solving the resulting equation, we obtain the following expression for the non-zero components of the tensor $\sigma_{ik}(\omega, t)$ [on the basis of Eq. (2.5)]:

$$\sigma_{zz}(\omega, t) = -\frac{4\pi e^2 N}{3m}\int_0^\infty \frac{v^3}{\nu(v) - i\omega}\frac{\partial f_0(v, t)}{\partial v}\,dv, \qquad (2.7)$$

$$\sigma_{xx}(\omega, t) = \sigma_{yy}(\omega, t)$$

$$= -\frac{2\pi e^2 N}{3m}\sum_{n,m} J_n(\Delta) J_m(\Delta) e^{i(n-m)\Omega t}$$

$$\times \int_0^\infty v^3 \{[\nu(v) - i(\omega - \omega_H + m\Omega)]^{-1}$$

$$+ [\nu(v) - i(\omega + \omega_H - n\Omega)]^{-1}\}\frac{\partial f_0(v, t)}{\partial v}\,dv; \qquad (2.7')$$

$$\sigma_{xy}(\omega, t) = -\sigma_{yx}(\omega, t) = -i\frac{2\pi e^2 N}{3m}\sum_{n,m} J_n(\Delta) J_m(\Delta) e^{i(n-m)\Omega t}$$

$$\times \int_0^\infty v^3 \{[\nu(v) - i(\omega - \omega_H + m\Omega)]^{-1}$$

$$-[\nu(v) - i(\omega + \omega_H - n\Omega)]^{-1}\}\frac{\partial f_0(v, t)}{\partial v}\,dv, \qquad (2.7'')$$

where $\omega_H = |e| H_0/mc$, $J_n(\Delta)$ is the Bessel function, $\Delta \equiv \omega_{H_1}/\Omega = |e| H_1/mc\Omega$. If the field H_{ext} is absent (more precisely, for $\Delta \ll 1$), only a single term remains of the sums appearing on the right hand sides of (2.7′) and (2.7″), corresponding to $n = m = 0$ ($J_n(0) = \delta_{0n}$). The time dependence of the components of $\sigma_{ik}(\omega, t)$ is determined in this case only by the time dependence of the distribution function $f_0(v, t)$. For a constant or rapidly varying ($\Omega \gg \delta\nu_{eff}$) field E_{ext}, even this source of time dependence vanishes,[4 5] and the electrodynamic properties of the plasma cease to be time dependent. Precisely this case was considered in I.

In the presence of a strong field $H_{ext}(t)$ ($\Delta \gtrsim 1$), the greatest interest attaches to the case in which $\Omega \gtrsim \nu_{eff}$; under this condition, the distribution function $f_0(v, t)$ does not depend on the time, with accuracy up to small terms, and the effect of the non-stationarity is entirely brought about by a parametric change in the magnetic field. In this case, for the tensor $\sigma_{ik}^{(0)}(\omega)$ which determines the absorption properties of the medium, we obtain the following from (2.7) — (2.7″):

$$\sigma_{xx}^{(0)}(\omega) = \sigma_{yy}^{(0)}(\omega)$$

$$= -\frac{2\pi e^2 N}{3m}\sum_n J_n^2(\Delta)\int_0^\infty v^3 \{[\nu(v) - i(\omega - \omega_H + n\Omega)]^{-1}$$

$$+ [\nu(v) - i(\omega + \omega_H + n\Omega)]^{-1}\}\frac{\partial f_0(v)}{\partial v}\,dv, \qquad (2.8)$$

$$\sigma_{xy}^{(0)}(\omega) = -\sigma_{yx}^{(0)}(\omega)$$

$$= -i\frac{2\pi e^2 N}{3m}\sum_n J_n^2(\Delta)\int_0^\infty v^3 \{[\nu(v) - i(\omega - \omega_H + n\Omega)]^{-1}$$

$$- [\nu(v) - i(\omega + \omega_H + n\Omega)]^{-1}\}\frac{\partial f_0(v)}{\partial v}\,dv. \qquad (2.8')$$

The expression for $\sigma_{zz}^{(0)}(\omega)$ is given by the right side of (2.7) with the replacement of $\partial f_0(v, t)/\partial v$ by $\partial f_0(v)/\partial v$. It follows from these expressions, with account of the condition $\Omega \gtrsim \nu_{eff}$, that, in the case of parametric change of the magnetic field under consideration, the local absorbing properties of the plasma have distinct maxima ("peaks") at the frequencies $\omega = \pm\omega_H + n\Omega$ ($n = 0, \pm 1, \ldots$), the "amplitudes" (heights) of which are deter-

*Let us list the chief small parameters which can appear in our problem. First, we have the quantity δ, which is small in many cases. Next, the small quantity $\delta\nu_{eff}/\Omega$ occurs for rapidly varying fields. In cases of strongly (or conversely, weakly) ionized plasma, there are also small parameters, equal to $\delta\nu/\nu_e$ and $\nu_e/\delta\nu$, respectively (ν_e is the number of interelectronic collisions). In what follows, in speaking of the neglect of small terms, we shall have in mind the neglect (in comparison with unity) of terms of the order of the small parameters enumerated.

†The change in the density $\Delta N(t) = \mathrm{div}\, E_{ext}(t)/4\pi e$ is also necessarily small if the field E_{ext} is sufficiently homogeneous.

‡$[\mathbf{u}, \mathbf{H}_0 + \mathbf{H}_{ext}] = \mathbf{u} \times (\mathbf{H}_0 + \mathbf{H}_{ext})$.

mined by the quantity $\Delta = \omega_{H_1}/\Omega$ and the number n. For a particular value of Δ, which coincides with the root of the function $J_n(x)$, the "peaks" of number n vanish. In particular, for $H_1 = x_1 mc\Omega/|e|$ [$x_1 \approx 2.5$ is the first root of the function $J_0(x)$], the zero "peak" vanishes and the plasma becomes practically locally non-absorbing at a gyromagnetic frequency $\omega = \omega_H$.* This effect of the change in the absorption of the plasma was considered briefly by Lugovoĭ[8] in connection with the discussion of the possibility of the use of cyclotron resonance for obtaining negative absorption by means of the parametric change in the external magnetic field.

3. THE SPECTRAL INTENSITY TENSOR FOR THE CURRENT FLUCTUATIONS

Just as in the case of a stationary plasma,[1-3] the spectral intensity of the fluctuation components in the case of a periodically non-stationary plasma is also of prime interest. Set within the framework of macroscopic electrodynamics, the problem reduces, in the first place, to the determination of the spectral intensity tensor of the current fluctuations $\langle j_\alpha(\mathbf{r}, \omega) j^*_\beta(\mathbf{r}', \omega')\rangle$ and in the second place to the corresponding solution of the system (1.3) which, being rewritten for the Fourier amplitudes of the field intensity $\mathbf{E}(\mathbf{r}, \omega)$ and $\mathbf{H}(\mathbf{r}, \omega)$, has the form (the $\mathbf{r}$ dependence is not shown)

$$\operatorname{rot} \mathbf{H}(\omega) = -i\frac{\omega}{c}\hat{\varepsilon}(\omega)\mathbf{E}(\omega) + \frac{4\pi}{c}\sum_n{}' \hat{\sigma}^{(n)}(\omega + n\Omega)\mathbf{E}(\omega + n\Omega) + \frac{4\pi}{c}\mathbf{j}(\omega),$$

$$\operatorname{rot} \mathbf{E}(\omega) = i\frac{\omega}{c}\mathbf{H}(\omega), \tag{3.1}$$

where

$$\varepsilon_{ik}(\omega) = \delta_{ik} + i4\pi\sigma^{(0)}_{ik}(\omega)/\omega,$$

and the prime on the summation sign indicates that the term with $n = 0$ is omitted.

Inasmuch as spatial dispersion is neglected, the spatial correlation of the current fluctuation j is local (δ correlation)†.[9,10]

*In another paper, we propose to look into the use of this effect of the local absorption change (by means of a parametric change of the magnetic field) to produce "transparency" at frequencies $\omega = \pm\omega_H + n\Omega$ for a magneto-active plasma with dimensions that are large in comparison with the wavelength.

†We note in passing that neglect of spatial dispersion in the medium itself has no effect on the value of the spectral intensity of the radiation of the medium in the external space.[6,10]

Therefore,

$$\langle j_\alpha(\mathbf{r}, \omega) j^*_\beta(\mathbf{r} + \boldsymbol{\rho}, \omega + p)\rangle = D_{\alpha\beta}(\omega, p)\,\delta(\boldsymbol{\rho}), \tag{3.2}$$

where the tensor $D_{\alpha\beta}(\omega, p)$ can be connected with the correlation function of the fluctuations in the velocity of a single electron in the plasma: $\psi_{\alpha\beta}(\tau, t) = \langle v_\alpha(t) v_\beta(t + \tau)\rangle$. Actually, on the basis of the relation

$$j_\alpha(\mathbf{r}, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} j_\alpha(\mathbf{r}, t)\, e^{i\omega t}dt$$

(see I)

$$\langle j_\alpha(\mathbf{r}, t)\, j_\beta(\mathbf{r} + \boldsymbol{\rho}, t + \tau)\rangle = Ne^2\psi_{\alpha\beta}(\tau, t)\,\delta(\boldsymbol{\rho}),$$

we get*

$$D_{\alpha\beta}(\omega, p) = \frac{Ne^2}{(2\pi)^2}\int_{-\infty}^{\infty} \psi_{\beta\alpha}(\tau, t)\, e^{-ipt} e^{i\omega\tau}dt\,d\tau = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi_{\beta\alpha}(\omega, t)\, e^{-ipt}\,dt, \tag{3.3}$$

where

$$\varphi_{\alpha\beta}(\omega, t) = \frac{Ne^2}{2\pi}\int_{-\infty}^{\infty} \psi_{\alpha\beta}(\tau, t)\, e^{i\omega\tau}d\tau. \tag{3.4}$$

The components of the tensors $\psi_{\alpha\beta}(\tau, t)$ and $\varphi_{\alpha\beta}(\omega, t)$ are periodic functions of t with period $2\pi/\Omega$. It is easy to establish the fact that the components $\varphi_{\alpha\beta}(\omega, t)$ and the coefficients $\varphi^{(n)}_{\alpha\beta}(\omega)$ of their expansions in Fourier series satisfy the same conditions of symmetry as the tensor components $\sigma_{ik}(\omega, t)$ and $\sigma^{(n)}_{ik}(\omega)$. The tensor $\psi_{\alpha\beta}(\tau, t)$ also possesses another obvious symmetry property which follows from its definition: $\psi_{\alpha\beta}(-\tau, t) = \psi_{\beta\alpha}(\tau, t - \tau)$. This property makes it possible to compute the tensor $\varphi_{\alpha\beta}(\omega, t)$ [or $D_{\alpha\beta}(\omega, p)$] from the velocity tensor $\psi_{\alpha\beta}(\tau, t)$, which is given only for $\tau > 0$. For this purpose, it suffices to transform Eq. (3.4) to the following form:

$$\varphi_{\alpha\beta}(\omega, t) = \frac{Ne^2}{2\pi}\left\{\int_0^{\infty} [\psi_{\alpha\beta}(\tau, t) + \psi_{\beta\alpha}(\tau, t - \tau)]\cos\omega\tau\,d\tau + i\int_0^{\infty} [\psi_{\alpha\beta}(\tau, t) - \psi_{\beta\alpha}(\tau, t - \tau)]\sin\omega\tau\,d\tau\right\}. \tag{3.5}$$

The components $\psi_{\alpha\beta}(\tau, t)$ can be computed for $\tau > 0$ on the basis of kinetic theory, similar to what was done in I. In the interval between collisions, the electron behaves as if it were free, and

*Some general relations of the correlation theory of periodically non-stationary random processes can be found in [11].

its velocity satisfies the equations*

$$\dot{v}_x + (\omega_H + \omega_{H_1}\cos\Omega t)\, v_y = 0,$$
$$\dot{v}_y - (\omega_H + \omega_{H_1}\cos\Omega t)\, v_x = 0, \quad \dot{v}_z = 0. \tag{3.6}$$

We introduce the notation: $\Phi(t) \equiv \omega_H t + \Delta\sin\Omega t$. The values of $v_x(t+\tau)$, $v_y(t+\tau)$ and $v_z(t+\tau)$, computed from $v_x(t)$, $v_y(t)$ and $v_z(t)$, under the condition that no collision takes place within the time interval τ, have [on the basis of (3.6)] the form

$$v_x(t+\tau) = [v_x(t)\cos\Phi(t) + v_y(t)\sin\Phi(t)]\cos\Phi(t+\tau) + [v_x(t)\sin\Phi(t) - v_y(t)\cos\Phi(t)]\sin\Phi(t+\tau),$$
$$v_y(t+\tau) = [v_x(t)\cos\Phi(t) + v_y(t)\sin\Phi(t)]\sin\Phi(t+\tau) + [-v_x(t)\sin\Phi(t) + v_y(t)\cos\Phi(t)]\cos\Phi(t+\tau);$$
$$v_z(t+\tau) = v_z(t). \tag{3.7}$$

Let $w(s, v) \equiv \exp[-s/l(v)]/l(v)$ $[l = v/\nu(v)]$ be the distribution function for the length of the mean free path of the electrons for a given velocity v. Then, for non-zero components of $\psi_{\alpha\beta}(\tau, t)$ for $\tau > 0$, and on the basis of Eq. (3.7), we obtain the equation

$$\psi_{zz}(\tau, t) = \int_{-\infty}^{\infty} v_z^2 f_0(v, t)\, d\mathbf{v} \int_0^{\infty} e^{-s/l}\,\theta\left(\frac{s}{v} - \tau\right)\frac{ds}{l},$$
$$\psi_{xx}(\tau, t) = \psi_{yy}(\tau, t) = \cos[\Phi(t+\tau) - \Phi(t)] \times \int_{-\infty}^{\infty} v_x^2 f_0(v, t)\, d\mathbf{v} \int_0^{\infty} e^{-s/l}\,\theta\left(\frac{s}{v} - \tau\right)\frac{ds}{l},$$
$$\psi_{xy}(\tau, t) = -\psi_{yx}(\tau, t) = \sin[\Phi(t+\tau) - \Phi(t)] \times \int_{-\infty}^{\infty} v_x^2 f_0(v, t)\, d\mathbf{v} \int_0^{\infty} e^{-s/l}\,\theta\left(\frac{s}{v} - \tau\right)\frac{ds}{l}, \tag{3.8}$$

where $\theta(t) = 1$ for $t > 0$ and $\theta(t) = 0$ for $t < 0$.

Further, let us first consider the case when the field $H_{ext}(t)$ is absent $(\Delta \ll 1)$; here $\Phi(t+\tau) - \Phi(t) \equiv \omega_H\tau$. Substituting (3.8) in (3.5), and carrying out the integration first over τ and then over s, we get the following expressions for the components of the tensor $\varphi^{(n)}(\omega)$:

$$\varphi_{zz}^{(n)}(\omega) = \frac{2}{3}Ne^2\int_0^{\infty} v^4\left\{\frac{\nu(v)/\omega}{\omega^2 + i\nu(v)} + \frac{\nu(v)/(\omega + n\Omega)}{\omega + n\Omega - i\nu(v)} + \frac{in\Omega}{\omega(\omega + n\Omega)}\right\} f_0^{(n)}(v)\, dv,$$

$$\varphi_{xx}^{(n)}(\omega) = \varphi_{yy}^{(n)}(\omega) = \frac{2}{3}Ne^2\int_0^{\infty} v^4\left\{\frac{1}{2}\left[\frac{\nu(v)/(\omega - \omega_H)}{\omega - \omega_H + i\nu(v)} + \frac{\nu(v)/(\omega + \omega_H)}{\omega + \omega_H + i\nu(v)} + \frac{\nu(v)/(\omega + n\Omega - \omega_H)}{\omega + n\Omega - \omega_H - i\nu(v)} + \frac{\nu(v)/(\omega + n\Omega + \omega_H)}{\omega + n\Omega + \omega_H - i\nu(v)}\right] + \frac{i\omega}{\omega^2 - \omega_H^2} - \frac{i(\omega + n\Omega)}{(\omega + n\Omega)^2 - \omega_H^2}\right\} f_0^{(n)}(v)\, dv,$$

$$\varphi_{xy}^{(n)}(\omega) = -\varphi_{yx}^{(n)}(\omega) = i\frac{2}{3}Ne^2\int_0^{\infty} v^4\left\{\frac{1}{2}\left[\frac{\nu(v)/(\omega - \omega_H)}{\omega - \omega_H + i\nu(v)} - \frac{\nu(v)/(\omega + \omega_H)}{\omega + \omega_H + i\nu(v)} + \frac{\nu(v)/(\omega + n\Omega - \omega_H)}{\omega + n\Omega - \omega_H - i\nu(v)} - \frac{\nu(v)/(\omega + n\Omega + \omega_H)}{\omega + n\Omega + \omega_H - i\nu(v)}\right] + \frac{i\omega_H}{\omega^2 - \omega_H^2} - \frac{i\omega_H}{(\omega + n\Omega)^2 - \omega_H^2}\right\} f_0^{(n)}(v)\, dv, \tag{3.9}$$

where $f_0^{(n)}(v)$ are the expansion coefficients of the distribution function $f_0(v, t)$ in a Fourier series. For a time-independent function $f_0(v)$, we have $f_0^{(n)}(v) = f_0(v)\delta_{0n}$ and Eqs. (3.9) transform into Eq. (6) in I.

In the presence of a strong magnetic field, we again obtain only the case in which $\Omega \gtrsim \nu_{eff}$; here, $\Omega \gg \delta\nu_{eff}$ and the function $f_0(v)$ is time independent (with accuracy up to small terms). Substituting (3.8) in (3.5) for this case, we get

$$\varphi_{zz}^{(n)}(\omega) = \delta_{0n}\varphi_{zz}(\omega) = \frac{4}{3}\delta_{0n}Ne^2\int_0^{\infty} v^4\nu(v)\,[\omega^2 + \nu^2(v)]^{-1} f_0(v)\, dv;$$

$$\varphi_{xx}^{(n)}(\omega) = \varphi_{yy}^{(n)}(\omega) = \frac{2}{3}Ne^2\sum_k J_k(\Delta)J_{k-n}(\Delta) \times \int_0^{\infty} v^4\nu(v)\{[(\omega - \omega_H - (k-n)\Omega)^2 + \nu^2(v)]^{-1} + (-1)^n[(\omega + \omega_H - (k-n)\Omega)^2 + \nu^2(v)]^{-1}\} f_0(v)\, dv;$$

$$\varphi_{xy}^{(n)}(\omega) = -\varphi_{yx}^{(n)}(\omega) = i\frac{2}{3}Ne^2\sum_k J_k(\Delta)J_{k-n}(\Delta) \times \int_0^{\infty} v^4\nu(v)\{[(\omega - \omega_H - (k-n)\Omega)^2 + \nu^2(v)]^{-1} - (-1)^n[(\omega + \omega_H - (k-n)\Omega)^2 + \nu^2(v)]^{-1}\} f_0(v)\, dv. \tag{3.10}$$

*The inhomogenous equations, with the right hand sides equal to $(e/m)E_{ext\,i}(t)$ are more accurate equations for the motion of the electron between collisions. However, it can be shown that account of the action of the field $E_{ext}(t)$ results in a contribution of the order δ to the quantity of interest to us; we therefore start out with the homogeneous equation (3.6).

For $\Delta \ll 1$, these expressions transform into the Eqs. (6) of I.

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