PRODUCTION OF PIONS IN WEAK INTERACTIONS

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Production of mesons due to the interaction of neutrinos or antineutrinos with nucleons is considered. Numerical estimates of cross sections are derived from electroproduction data.

1. The present paper concerns itself with the interactions of neutrinos or antineutrinos with nucleons resulting in the production of pions. These processes have been considered previously by Lee and Yang[1] in connection with the possibility of experimentally testing the local nature of the leptonic current. The following reactions are possible:

$$\nu + p \to l^- + p + \pi^+, \quad \nu + n \to l^- + n + \pi^+,$$
$$\nu + n \to l^- + p + \pi^0, \tag{1}$$
$$\bar{\nu} + n \to l^+ + n + \pi^-, \quad \bar{\nu} + p \to l^+ + p + \pi^-,$$
$$\bar{\nu} + p \to l^+ + n + \pi^0, \tag{2}$$

where $l^\pm$ stands for a charged lepton (electron or μ meson).

In what follows these reactions will be identified by the target and the sign of the meson (p+, n+, n0, etc.).

In Sec. 2 we discuss the structure of the cross sections of processes (1) and (2) assuming the validity of the V,A interaction, and in Sec. 3 we make the connection with electroproduction cross sections.

2. Accurate to first order in the weak interaction the element of the T matrix responsible for processes (1) has the form

$$\langle p_2 q s_2 | T | p_1 s_1 \rangle$$
$$= -\sqrt{2}\, G \bar{u}_l(s_2)\, \gamma^\mu \frac{1+\gamma_5}{2}\, u_\nu(s_1) \cdot \frac{1}{\sqrt{2q_0}} \bar{u}_2 H_\mu u_1, \tag{3}$$

where G is the Feynman-Gell-Mann constant,[2] s_2 and s_1 are the momenta of the final lepton and neutrino respectively, $\bar{u}_l$ and u_ν are their wave functions, q is the momentum of the meson, and p_2 and $\bar{u}_2$, p_1 and u_1 are the momenta and wave functions of the initial and final nucleons. $\bar{u}_2 H_\mu u_1/\sqrt{2q_0}$ represents the matrix element of the baryonic V,A current.

Analogously the amplitude for processes (2) is given by

$$-\sqrt{2}\, G \bar{v}_\nu(-s_1)\, \gamma^\mu \frac{1+\gamma_5}{2}\, v_l(-s_2) \frac{1}{\sqrt{2q_0}} \bar{u}_2 \widetilde{H}_\mu u_1, \tag{4}$$

where $\bar{v}_\nu$ and v_l are negative frequency spinors, and $\widetilde{H}_\mu$ and H_μ are related to each other like the ∓ components of an isotopic vector.

It follows from Eq. (3) that the differential cross section for processes (1) is given by

$$d\sigma = \frac{1}{(4\pi)^4} \frac{G^2}{2} \frac{|\mathbf{s}_2|}{s_{10} m} \frac{|\mathbf{q}|}{W} \Phi\, ds_{20}\, d\Omega\, d\cos\vartheta, \tag{5}$$

where s_{10} is the neutrino energy, s_{20} and $\mathbf{s}_2$ are the energy and momentum of the emitted lepton in the laboratory frame, Ω is the solid angle of emission of the lepton; W and **q** are the total energy and momentum of the final nucleon and meson in their barycentric frame, ϑ is the nucleon scattering angle in the same frame, and m is the nucleon mass.

The quantity Φ is given by the equation

$$\Phi = [S^\mu S^\nu - k^\mu k^\nu - (m_l^2 - k^2)\, g^{\mu\nu}$$
$$- i S_\alpha k_\beta \varepsilon^{\alpha\beta\mu\nu}]\, \mathrm{Sp}(\hat{p}_2 + m)\, H_\mu (\hat{p}_1 + m)\, \bar{H}_\nu, \tag{6}$$

where $S = s_1 + s_2$, $k = s_1 - s_2$, and $\varepsilon^{\alpha\beta\mu\nu}$ is the completely antisymmetric tensor ($\varepsilon^{0123} = 1$). It is assumed that the incident nucleon is unpolarized.

From a comparison of Eqs. (3) and (4) it follows that the cross section $d\widetilde{\sigma}$ for processes (2) is also given by Eq. (5) provided that in the definition (6) for Φ one changes the sign of the last term in the square bracket and replaces H_μ by $\widetilde{H}_\mu$.

Instead of H and $\widetilde{H}$ (for the sake of brevity we drop the subscript μ on H_μ and $\widetilde{H}_\mu$) it is convenient to introduce the amplitudes H^+ and H^- which are simply numbers in isotopic space:

$$H = \pi^m \{H^+ (\delta_{m1} + i\delta_{m2}) + H^- [\tau_m, \tau_+]/\sqrt{2}\},$$
$$\widetilde{H} = \pi^m \{H^+ (\delta_{m1} - i\delta_{m2}) + H^- [\tau_m, \tau_-]/\sqrt{2}\}, \tag{7}$$

π^m stands for the isotopic polarization vector of the pion.

The amplitudes for specific physical processes of the (1) or (2) types are very simply expressed in terms of the H^+ and H^-:

$$H_{p+} = \sqrt{2}(H^+ - H^-), \quad H_{n+} = \sqrt{2}(H^+ + H^-), \quad H_{n0} = 2H^-; \tag{8}$$

$$\tilde{H}_{n-} = \sqrt{2}(H^+ - H^-), \quad \tilde{H}_{p-} = \sqrt{2}(H^+ + H^-), \quad \tilde{H}_{p0} = -2H^-. \tag{9}$$

It is obvious that isotopic invariance makes only two of the three physical amplitudes in each group independent. Consequently there exist relations among the amplitudes

$$H_{p+} - H_{n+} + \sqrt{2}H_{n0} = 0, \quad \tilde{H}_{n-} - \tilde{H}_{p-} - \sqrt{2}\tilde{H}_{p0} = 0. \tag{10}$$

These relations give rise to inequalities for the cross sections, similar to the inequalities connecting the squares of the sides of a triangle.

The equalities

$$H_{p+} = \tilde{H}_{n-}, \quad H_{n+} = \tilde{H}_{p-}, \quad H_{n0} = -\tilde{H}_{p0}$$

are a consequence of the fact that the transition from reactions (1) to reactions (2) may be accomplished by means of the charge conjugation operation on the leptons and the charge symmetry transformation on the strongly interacting particles.

If the leptonic current is local then H^+ and H^- depend on the leptonic momenta s_1 and s_2 only through the momentum transfer $k = s_1 - s_2$. At that Eq. (6) shows that the cross section depends on the components of the vector S quadratically. The term linear in S arises from the last term in the square bracket in Eq. (6). This is precisely the term that changes sign on going from reactions (1) to reactions (2). A detailed discussion of the symmetries arising from this property was given by Shekhter.[3]

Consequently the cross sections $d\sigma_{p+}$ and $d\tilde{\sigma}_{n-}$, $d\sigma_{n+}$ and $d\tilde{\sigma}_{p-}$, $d\sigma_{n0}$ and $d\tilde{\sigma}_{p0}$ differ from each other only by the sign of the terms linear in S. This means that if we increase the components of S, at fixed values of k, the corresponding cross sections will tend to the same limit. Such a limit is obtained, for example, if for small angles of emission of the final lepton $s_{10} \gg s_{10} - s_{20}$.

The quantity Φ, Eq. (6), consists of a scalar and pseudoscalar part. The pseudoscalar part describes the correlation between the momenta of the particles partaking in the reaction, of the type $\mathbf{p}_\pi \cdot \mathbf{p}_l \times \mathbf{p}$. Such a correlation is absent if time reversal invariance holds and the interaction in the final state is ignored.

It is easy to see from the definition (6) that the interference of the V,A baryonic currents can enter the scalar part only in the terms linear in S. This means that if we take for each of the above mentioned pairs of cross sections half their sum, then we will get in the scalar part of this half-sum contributions from the V and A currents independently.

3. Additional information may be obtained on the reactions (1) and (2) if use if made of the Feynman-Gell-Mann hypothesis[2] relating the electric current and the vector part of the weak, strangeness-conserving, current. This hypothesis connects reactions (1) and (2) with electroproduction reactions

$$l^\pm + p \to l^\pm + p + \pi^0, \quad l^\pm + n \to l^\pm + n + \pi^0,$$
$$l^\pm + p \to l^\pm + n + \pi^+, \quad l^\pm + n \to l^\pm + p + \pi^-. \tag{11}$$

As is well known (see e.g., [4]) the amplitude for pion electroproduction is given by

$$-e^2 \frac{1}{k^2} \bar{u}_l(s_2)\gamma^\mu u_l(s_1) \frac{1}{\sqrt{2q_0}} \bar{u}_2 H^e_\mu u_1, \tag{12}$$

where $e^2/4\pi = \alpha$, u is the wave function of the electron (μ meson) and H^e_μ is related to the matrix element of the electric current.

From this we get for the cross section of this process $d\sigma^e$ the expression

$$d\sigma^e = \frac{1}{(4\pi)^4} \frac{e^4}{4(k^2)^2} \frac{|\mathbf{s}_2|}{|\mathbf{s}_1|\, m} \frac{|\mathbf{q}|}{W} \Phi^e ds_{20}\, d\Omega d\cos\vartheta, \tag{13}$$

where

$$\Phi^e = [S^\mu S^\nu - k^\mu k^\nu + k^2 g^{\mu\nu}]\, \mathrm{Sp}\,(\hat{p}_2 + m)\, H^e_\mu (\hat{p}_1 + m)\, \bar{H}^e_\mu. \tag{14}$$

The second term in the bracket contributes nothing as a consequence of gauge invariance.

The isotopic structure is

$$H^e = \pi^m (H^0 \tau_m + H^+_V \delta_{m3} + H^-_V [\tau_m, \tau_3]/2), \tag{15}$$

where H^+_V and H^-_V coincide with the vector parts of the corresponding amplitudes H^+ and H^- [Eq. (7)], and H^0 arises from the matrix element of the isoscalar current.

The amplitudes of the reactions (11) are simply expressed in terms of H^0, H^+_V and H^-_V:

$$H^e_{p0} = H^+_V + H^0, \quad H^e_{n0} = H^+_V - H^0, \quad H^e_{p+} = \sqrt{2}(H^0 + H^-_V), \quad H^e_{n-} = \sqrt{2}(H^0 - H^-_V). \tag{16}$$

From here it is easy to express the vector amplitudes (8) and (9) of the reactions (1) and (2) in terms of the amplitudes of the reactions (11):

$$H_{p+V}=\widetilde{H}_{n-V}=(H^e_{p0}+H^e_{n0})/\sqrt{2}-\tfrac{1}{2}(H^e_{p+}-H^e_{n-}),$$
$$H_{n+V}=\widetilde{H}_{p-V}=(H^e_{p0}+H^e_{n0})/\sqrt{2}+\tfrac{1}{2}(H^e_{p+}-H^e_{n-}),$$
$$H_{n0V}=-\widetilde{H}_{p0V}=(H^e_{p+}-H^e_{n-})/\sqrt{2}. \qquad (17)$$

Suppose now that the charged lepton involved in these reactions is the electron, and consequently its mass may be ignored. Then the first factor in the scalar part of Φ, Eq. (6), coincides with the factor in Φ^e, Eq. (14). This makes it possible to derive certain relations among the cross sections as well. It is desirable, however, to eliminate the effect of H^0. It is seen from Eq. (16) that half the sum of the cross sections $d\sigma^e_{p0}$ and $d\sigma^e_{n0}$, $d\sigma^e_{p+}$ and $d\sigma^e_{n-}$ does not contain interferences between H^0 and $H^{\pm}_V$. The terms $\sim|H^0|^2$, on the other hand, may be apparently either completely ignored or taken equal to their Born approximation for values of W near to or below the energy of the (3.3) resonance.[4] For simplicity we ignore in what follows the square of H^0 in comparison with the squares of other amplitudes. Then a comparison of Eqs. (5), (6), (8) and (9) with Eqs. (13), (14) and (16) gives rise to the following relations among the cross sections:

$$\tfrac{1}{2}(d\sigma_{n0}+d\widetilde{\sigma}_{p0})>\tfrac{1}{2}(Gk^2/2\pi\alpha)^2(d\sigma^e_{p+}+d\sigma^e_{n-}), \qquad (18)$$

$$\tfrac{1}{2}(d\sigma_{p+}+d\sigma_{n+}+d\widetilde{\sigma}_{p-}+d\widetilde{\sigma}_{n-})>(Gk^2/2\pi\alpha)^2\,[(d\sigma^e_{p0}+d\sigma^e_{n0})+\tfrac{1}{2}(d\sigma^e_{p+}+d\sigma^e_{n-})], \qquad (19)$$

$$\tfrac{1}{2}(d\sigma_{p+}+d\sigma_{n+}+d\widetilde{\sigma}_{p-}+d\widetilde{\sigma}_{n-})-\tfrac{1}{2}(d\sigma_{n0}+d\widetilde{\sigma}_{p0})>(Gk^2/2\pi\alpha)^2(d\sigma^e_{p0}+d\sigma^e_{n0}). \qquad (20)$$

If at least two particles are detected in the final state then from the experimentally obtained cross sections on the left sides of the inequalities one must subtract the terms describing the correlations of the emitted particles so as to be left with the scalar components only.

It was shown earlier that on the left sides there is no interference between the V and A variants, and on the right sides there is no interference between H^0 and $H^{\mp}_V$. The sign > has to do with the fact that the left sides also have a contribution from the axial vector current. In order to make the inequalities more precise one must, of course, subtract from the right sides the contributions due to $|H^0|^2$. Afterwards they may be used to determine the contribution of the axial vector current. The difference of the left and right sides of Eq. (18) gives the contribution $\sim|H^-_A|^2$, and of Eq. (20) the contribution $\sim|H^+_A|^2$.

In connection with these inequalities it is of interest to consider the k^2-dependence of the cross sections. If the electron mass is ignored then $k^2 = -2s_{10}s_{20}(1-\cos\theta)$, where θ is the angle of emission of the final electron. Since the amplitudes $H^{\pm}$ should not, apparently, have poles as $k^2 \to 0$, it is not hard to see that the quantity $k^{-2}\Phi$ has a finite limit as s_{20} tends to zero at fixed values of θ. It therefore follows from Eq. (5) that for nonzero angles and small k^2 the cross sections $d\sigma$ and $d\widetilde{\sigma}$ should go as $(k^2)^2$ or, what is the same, as $(s_{20})^2$.

For small k^2 ($|k^2| \ll 4m_\pi^2$) the electroproduction cross sections on the right sides of the inequalities (18)—(20) may be expressed in terms of photoproduction cross sections according to the equality (see, e.g., [5]).

$$\lim_{k^2\to 0}\frac{\partial^2\sigma^e}{\partial s_{20}\,\partial\Omega}=\frac{\alpha}{(2\pi)^2}\frac{1}{s_{10}(1-\cos\theta)}\sigma^p, \qquad (21)$$

where σ^p stands for the photoproduction cross section.

For larger values of $|k^2|$ the inequalities (18)—(20) show that the ratio of the cross sections $d\sigma$ and $d\widetilde{\sigma}$ to the electroproduction cross sections grows like $(k^2)^2$. This result is a consequence of the assumption made relating the electric and weak currents, and may be subject to experimental test. If an intermediate vector boson of mass M exists, then for $|k^2| \gg M^2$ the factor $(Gk^2/2\pi\alpha)^2$ in the inequalities (18)—(20) should be replaced by $(GM^2/2\pi\alpha)^2$.

The relations (18)—(20) become simplified when expressed in terms of cross sections for deuteron reactions. The experimental data on electroproduction on deuterons [6] indicate that the cross section is approximately equal to the sum of the cross sections on neutrons and protons. If one makes use of this additivity property of nucleon cross sections one obtains the inequalities

$$\tfrac{1}{2}(d\sigma_{D0}+d\widetilde{\sigma}_{D0})>\tfrac{1}{2}(Gk^2/2\pi\alpha)^2(d\sigma^e_{D+}+d\sigma^e_{D-}), \qquad (18a)$$

$$\tfrac{1}{2}(d\sigma_{D+}+d\widetilde{\sigma}_{D-})>(Gk^2 2\pi\alpha)^2\,[d\sigma^e_{D0}+\tfrac{1}{2}(d\sigma^e_{D+}+d\sigma^e_{D-})], \qquad (19a)$$

$$\tfrac{1}{2}(d\sigma_{D+}+d\widetilde{\sigma}_{D-})-\tfrac{1}{2}(d\sigma_{D0}+d\widetilde{\sigma}_{D0})>(Gk^2/2\pi\alpha)^2\,d\sigma^e_{D0}. \qquad (20a)$$

Here $d\sigma_{D+}$, $d\sigma_{D0}$; $d\widetilde{\sigma}_{D-}$, $d\widetilde{\sigma}_{D0}$; $d\sigma^e_{D0}$, $d\sigma^e_{D+}$, $d\sigma^e_{D-}$ refer to the cross sections for the following reactions

$$\nu+D\to e^-+p+n+\pi^+,\quad \nu+D\to e^-+2p+\pi^0; \qquad (1a)$$

$$\bar{\nu}+D\to e^++p+n+\pi^-,\quad \bar{\nu}+D\to e^++2n+\pi^0; \qquad (2a)$$

$$e^{\pm}+D\to e^{\pm}+p+n+\pi^0,\qquad e^{\pm}+D\to e^{\pm}+2n+\pi^+,$$
$$e^{\pm}+D\to e^{\pm}+2p+\pi^-. \qquad (11a)$$

From Eqs. (18a)—(20a) one obtains the following relation for the cross sections summed over the charge states of the produced meson:

$$\frac{1}{2}(d\sigma_D+d\tilde{\sigma}_D)>(Gk^2/2\pi\alpha)^2\,d\sigma_D^e. \qquad (22)$$

The existing experimental data on $\partial^2\sigma_D^e/\partial s_{20}\partial\Omega$[6] permit one to calculate the right side of the inequality (22) for the energy s_{10} in the interval from 460 to 680 Mev at angles $\theta = 90°$ and 135°. One gets in this way the value $\sim(2-6)\times 10^{-43}$ cm^2/Mev·sr.

If one integrated these values over s_{20} one could obtain an estimate for the differential cross section also. A rough numerical calculation using the trapezoidal method gives for the angle $\theta = 90°$ and the energy $s_{10} = 523$ Mev the estimate 4.4×10^{-41} cm^2/sr. By the same method one gets for $\theta = 135°$ the value 3.6×10^{-41} cm^2/sr at $s_{10} = 563$ Mev, and 4.1×10^{-41} cm^2/sr at $s_{10} = 607$ Mev. These numbers should be compared with the contribution of the vector current to the differential cross section for the "elastic" processes $\nu + n \to e^- + p$ and $\bar{\nu} + p \to e^+ + n$ at the same angles and energies. This contribution may be calculated and turns out to be for the above mentioned cases respectively 2.2×10^{-40}, 1.65×10^{-40} and 1.62×10^{-40} cm^2/sr. In this manner we find that the ratio of the differential cross sections for inelastic and "elastic" processes amounts to approximately 0.20—0.25. Unfortunately the experimental data on electroproduction on deuterons are very poor and do not allow more detailed estimates. Neither can we estimate the total cross sections for reactions (1a) and (2a). It is, however, to be expected that the ratio of the total cross sections of "elastic" and inelastic processes will be of the same order of magnitude as the ratio of the differential cross sections.

Beside the processes considered above also processes leading to the production of K mesons and Σ or Λ hyperons are possible. For reactions in which Σ hyperons are produced the inequalities (18)—(20) remain valid if one interprets the signs +, −, 0 as referring to the sign of the produced Σ hyperon.

For the reactions

$$\nu+n\to e^-+\Lambda+K^+,\qquad \bar{\nu}+p\to e^++\Lambda+K^0; \qquad (23)$$
$$e^{\pm}+p\to e^{\pm}+\Lambda+K^+,\qquad e^{\pm}+n\to e^{\pm}+\Lambda+K^0 \qquad (24)$$

it is possible to obtain in the same manner as before the relation

$$\frac{1}{2}(d\sigma_++d\tilde{\sigma}_0)>\frac{1}{2}(Gk^2/2\pi\alpha)^2(d\sigma_+^e+d\sigma_0^e), \qquad (25)$$

where the subscripts refer to the sign of the produced K meson.

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