ON THE AMBIGUITY IN THE DEFINITION OF THE INTERPOLATING FIELD

D. A. SLAVNOV and A. D. SUKHANOV

Moscow State University; Mathematical Institute, Academy of Sciences of the U.S.S.R.

Submitted to JETP editor July 14, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1940-1948 (December, 1961)

The question of the ambiguity in the definition of the interpolating field is considered; this ambiguity is shown to be connected with that in the definition of the T-product for a given S matrix, and also with the ambiguity in the determination of the S matrix outside the energy surface. The possibility of going over from one of these interpretations to the other is discussed.

1. INTRODUCTION

ONE type of axiomatic approach to the construction of quantum field theory has been expounded in a series of papers by Lehmann, Symanzik, and Zimmermann.[1] This approach uses as its fundamental quantities two complete sets of field operators* $A_{in}(x)$ and $A_{out}(x)$, which satisfy the conditions

$$(\Box_x - m^2) A_{in,\,out}(x) \equiv K_x A_{in,\,out}(x) = 0, \tag{1}$$

$$[A_{in,\,out}(x), A_{in,\,out}(y)] = 0 \quad \text{for} \quad x \sim y, \tag{2}$$

$$A_{in,\,out}(x) = A^{+}_{in,\,out}(x). \tag{3}$$

The operators $A_{out}(x)$ and $A_{in}(x)$ are connected with each other by means of a unitary operator S:

$$A_{out}(x) = S^{+} A_{in}(x) S, \tag{4}$$

which is identified with the S matrix.[2]

Besides the fields A_{in} and A_{out} one introduces a so-called interpolating field $A(x)$, which is subjected to the following asymptotic condition:

$$\lim_{t\to\pm\infty} (\Phi, A^f(t)\Psi) = (\Phi, A^f_{in,\,out}(t)\Psi). \tag{5}$$

Here Ψ and Φ are arbitrary state amplitudes, and

$$A^f(t) = i\int_{x^0=t} d\mathbf{x}\left(A(x)\frac{\partial f(x)}{\partial x^0} - f(x)\frac{\partial A(x)}{\partial x^0}\right) \equiv i\int_{x^0=t}\left[A(x)\overleftrightarrow{\frac{\partial}{\partial x^0}} f(x)\right]d\mathbf{x} \tag{6}$$

where $f(x)$ is an arbitrary normalized positive-frequency solution of the Klein-Gordon equation. $A^f_{in,\,out}$ is analogously defined and does not depend on t.

It must be noted that an expression very often used as a definition of the interpolating field is

$$A(x) = A_{in,\,out}(x) + \int D^{adv,\,ret}(x-y)\, j(y)\, dy, \tag{7}$$

understood in the sense of weak convergence. It must be emphasized, however, that the field $A(x)$ defined by Eq. (7) satisfies the condition (5) only when definite requirements are imposed on $j(x)$. For example, we can state that a sufficient condition on $j(x)$ is the convergence of an integral of the type

$$\int_{-\infty}^{\infty} dy^0 \left(\Phi, \int d\mathbf{y}\, f(y)\, j(y)\, \Psi\right).$$

All treatments ordinarily implicitly make this assumption or an analogous assumption.

Furthermore, it is said that a field $A(x)$ corresponds to a given S matrix if the fields $A_{out}(x)$ and $A_{in}(x)$ which it interpolates are connected by the formula (4).

One of the basic postulates of local field theory is that of microscopic causality. In the approach under consideration[1] this postulate is formulated as follows: an S matrix is called causal if to it there corresponds at least one interpolating field satisfying the locality condition in the form

$$[A(x), A(y)] = 0 \qquad \text{for } x \sim y. \tag{8}$$

A paper by Borchers[3] gives proofs of a number of mathematical theorems from which it follows that the local interpolating field is not unambiguously determined by the conditions enumerated above. In particular, if a field $A'(x)$ corresponds to a given causal S matrix, then a field of the form

$$B(x) = A(x) + Q(\Box_x)\, j(x), \tag{9}$$

where

$$K\, A(x) = -j(x), \tag{10}$$

and $Q(\Box_x)$ is a polynomial in $\Box_x$ with real con-

*We confine ourselves to the case of a neutral scalar field.

stant coefficients, also corresponds to this S matrix.

The problem of the ambiguity of the various quantities that form the apparatus of present quantum field theory is not encountered here for the first time; in particular, it has been discussed extensively in the book of Bogolyubov and Shirkov.[4] From this mathematical point of view these ambiguities are a result of the fact that in the definitions of the quantities we need we are forced to use expressions containing products of generalized functions, which in the general case are not unambiguously defined. Therefore it is quite clear that the ambiguity that Borchers [3] has pointed out in the determination of the interpolating field must have the same origin.

The purpose of the present paper is to bring out the connections between the arbitrariness in the definition of the interpolating field, on one hand with the ambiguity in the definition of the T-product, and on the other, with the possibilities of different definitions of the S matrix off the energy surface.

2. THE CHRONOLOGICAL PRODUCT

The problem of an arbitrariness in the T-product arises when one constructs the S matrix by going from order to order in perturbation theory, on the basis of a number of general assumptions, as is done in [4]. We are actually concerned there with a T-product of nonlinear operators, of the form $T\ (:\varphi^{k_1}(x_1):\ldots:\varphi^{k_n}(x_n):)$, and there is an arbitrariness in the definition of this quantity when some of its arguments are equal. In order to write out this arbitrariness in explicit form, it is convenient to introduce a T′-product, by which we mean a chronological product which is defined for equal arguments in some arbitary but fixed way, so that the T′-product is an integrable generalized function of all its arguments.

Then the most general form for the T-product is

$$T\ (:\varphi^{k_1}(x_1):\ldots:\varphi^{k_n}(x_n):) = \sum_{m=1}^{n}\frac{1}{m!}P\left((x_1k_1)\ldots(x_{\nu_1}k_{\nu_1})\,|\ldots\ (x_nk_n)\right) \times T'\left(\Lambda^{k_1\ldots k_{\nu_1}}(x_1\ldots x_{\nu_1})\ldots\Lambda^{\ldots k_n}(\ldots x_n)\right), \tag{11}$$

where m is the number of factors in the T′-product, and P is the symmetrization operator introduced in [4]. Here

$$\Lambda^{k_1\ldots k_{\nu_1}}(x_1\ldots x_{\nu_1}) = \sum_{l=0}^{k_1+\ldots+k_{\nu_1}} M_l^{k_1\ldots k_{\nu_1}}(x_1,\ldots x_{\nu_1}):\varphi^l(x_1):,$$
$$\Lambda^{k_i}(x_i) = :\varphi^{k_i}(x_i):, \tag{12}$$

$M_l^{k_1\ldots k_{\nu_1}}$ is a c-number function which differs from zero only for $x_1=\ldots=x_{\nu_1}$.

It may seem that if the S matrix is prescribed in the entire momentum space, and not merely on the energy surface, then the arbitrariness in the definition of the T product is entirely eliminated. Actually, however, this is not true. All that follows from the general arguments [4] is the formula

$$S = T'\exp\left\{ig\int L^0(x)\,dx\right\}, \tag{13}$$

where

$$L^0(x) = \mathcal{L}(x) + \sum_{n=2}\frac{1}{n!}g^{n-1}\int L_n^0(x, x_1\ldots x_{n-1})\,dx_1\ldots dx_{n-1}. \tag{14}$$

Here $\mathcal{L}(x)$ is the bare Lagrangian, and L_n^0 are the quasi-local operators introduced in [4].

It is not hard to verify that without changing the value of the S matrix one can go over in Eq. (13) from the T′-product to the T-product defined by Eq. (11), by changing the effective Lagrangian in a suitable way. In fact, let

$$S = T\exp\left\{ig\int L(x)\,dx\right\}, \tag{15}$$

where L(x) is given by a formula analogous to Eq. (14). To establish the connection between L(x) and $L^0(x)$, let us first consider a T-product of quasi-local operators L_n. Then using Eq. (11) we have

$$T\left(L_{\mu_1}(x_1^1\ldots x_{\mu_1}^1)\ldots L_{\mu_k}(\ldots x_n^k)\right) = T'\left(L_{\mu_1}(x_1^1\ldots x_{\mu_1}^1)\ldots L_{\mu_k}(\ldots x_n^k)\right) + \sum_{m=2}^{k-1}\frac{1}{m!}P\left((x^1\mu_1)\ldots(x^{\nu_1}\mu_{\nu_1})\,|\ldots|\ldots(x^k\mu_k)\right)T'\left(R^{\mu_1\ldots\mu_{\nu_1}}\times(x^1\ldots x^{\nu_1})\ldots R^{\ldots\mu_k}(\ldots x^k)\right) + R^{\mu_1\ldots\mu_k}(x^1\ldots x^k), \tag{16}$$

where, for example, x^1 denotes the set $(x_1^1,\ldots,x_{\mu_1}^1)$. In Eq. (16) we have introduced instead of the quasi-local operators $\Lambda(x_1\ldots x_i)$ quasi-local operators $R(x_1\ldots x_i)$ which are more convenient in the present case and which are certain combinations of the $\Lambda(x_1\ldots x_i)$, since in general the operators $L_n(x_1\ldots x_n)$ are sums of expressions of the form $:\varphi^{k_i}(x_i):$ with different values of k_i. The formula (16) can be regarded as a definition of $R(x_1\ldots x_i)$. Expanding Eqs. (13) and (15) in power series in g, equating terms with equal powers of g, and using Eq. (16), we get

$$iL_n(x_1\ldots x_n) = iL_n^0(x_1\ldots x_n) - \sum_{k=2}^{n}\frac{i^k}{k!}P(x^1\ldots x^k)\times\sum_{\mu_l}R^{\mu_1\ldots\mu_k}(x^1\ldots x^k), \tag{17}$$

where $P(x^1 \ldots x^k)$ is the operator of complete symmetrization, and the summation goes over values of $\mu_1, \ldots, \mu_k$ that obey the condition $\Sigma \mu_i = n$.

Equation (17) is a recurrence formula which enables us to express the operators L_n in terms of the operators L_k^0 and Λ. If, on the other hand, we use instead of Eq. (16) the transformation from the T'-product to the T-product, we can also get a formula which directly expresses the operators L_n in terms of L_k^0 and Λ.

Thus starting from a number of general propositions we can conclude that there are two possible interpretations of the arbitrariness that arises in the construction of the S matrix by perturbation theory. Namely, this arbitrariness can be regarded as caused either by the ambiguity in the Lagrangian for a fixed definition of the T-product, or by the ambiguity of the T-product itself for a given Lagrangian. Both of these possibilities have been noted already by Bogolyubov and Shirkov.[4] In practice, however, they have used only the first possibility, although the second appears preferable. An example of the use of the second possibility in the construction of the S matrix can be found in the situation[5] that occurs in theories with derivative couplings (or with vector fields), in which the S matrix can be expressed in terms of either the Lagrangian or the Hamiltonian, depending on the use of one or another definition of the T-product. We shall also use this same possibility here to express the indefiniteness in the interpolating field in terms of the ambiguity in the definition of the T-product.

It must only be pointed out that actually one can use for the derivation of a finite S matrix a much narrower definition of the T-product than that of Eq. (11). Namely, since the problem of deriving a finite S matrix reduces essentially to the problem of defining chronological contractions and their products for coinciding arguments, we can simplify the formula (12) by introducing different $M^{\cdots k_i \cdots}$ only for different types of products of chronological contractions in the T'-product. We shall not carry out this simplification for the general case, but shall do so in the next section for the special case we need here.

3. THE INTERPOLATING FIELD AND THE AMBIGUITY IN THE DEFINITION OF THE T-PRODUCT

In order to establish the connection between the arbitrariness in the definition of the interpolating field noted by Borchers[3] and the ambiguity in the definition of the T-product for a given S matrix, we shall use the following expression for the interpolating field:

$$A(x) = S^+ T(A_{in}(x)\, S). \tag{18}$$

Here we can understand the expression $T(A_{in}(x)S)$ if we represent the S matrix in the form

$$S = \sum_{i=0} \int \varphi_i(y_1 \ldots y_i) : A_{in}(y_1) \ldots A_{in}(y_i) : dy_1 \ldots dy_i. \tag{19}$$

Then

$$T(A_{in}(x)\ S) = \sum_{i=0} \int \varphi_i(y_1 \ldots y_i)\, T$$
$$\times (A_{in}(x) : A_{in}(y_1) \ldots A_{in}(y_i) :)\, dy_1 \ldots dy_i.$$

In order to convince ourselves that the field defined by Eq. (18) satisfies the asymptotic condition (5), let us substitute Eq. (18) in Eq. (5), for example for $t \to -\infty$:

$$\lim_{t\to-\infty} (\Phi, A^f(t)\Psi)$$
$$= \lim_{t\to-\infty} i \int_{x^0=t} d\mathbf{x}\, (\Phi, [S^+T(A_{in}(x)\,S)] \frac{\overleftrightarrow{\partial}}{\partial x^0} f(x)\, \Psi)$$
$$= i \lim_{t\to-\infty} \sum_{i=0} \int dy_1 \ldots dy_i \varphi_i(y_1 \ldots y_i)$$
$$\times \int_{x^0=t} d\mathbf{x}\, (\Phi, [S^+T(A_{in}(x) : A_{in}(y_1) \ldots A_{in}(y_i) :)]$$
$$\times \frac{\overleftrightarrow{\partial}}{\partial x^0} f(x)\, \Psi). \tag{20}$$

In accordance with the comment made in Sec. 1 we shall assume that the coefficient functions of the S matrix are such that in the right member of Eq. (20) we can take the process $\lim_{t\to-\infty}$ inside the sign of integration over y_k. Then

$$\lim_{t\to-\infty} (\Phi, A^f(t)\ \Psi) = i \sum_{i=0} \int dy_1 \ldots dy_i \varphi_i(y_1 \ldots y_i) \lim_{t\to-\infty}$$
$$\times \int_{x^0=t} d\mathbf{x}\, (\Phi, [S^+ : A_{in}(y_1) \ldots A_{in}(y_i) : A_{in}(x)]$$
$$\times \frac{\overleftrightarrow{\partial}}{\partial x^0} f(x)\, \Psi) = \lim_{t\to-\infty} i \left(\Phi, S^+S \int_{x^0=t} d\mathbf{x} A_{in}(x) \frac{\overleftrightarrow{\partial}}{\partial x^0} f(x)\, \Psi\right)$$
$$= (\Phi, A^f_{in}(t)\Psi). \tag{21}$$

We emphasize that these arguments do not make use of the specific properties of the T-product, which in principle can be just as general as in the definition (11). Of course, we have here a special case, with all the k_i equal to 1. Besides this, we are here using the previously indicated possibility of simplifying the T-product defined in Eq. (11), and shall introduce $M^{\cdots k_i \cdots}$ only for different types of products of chronological contractions in the T′-product. Since in the given case we encounter only one contraction, we have

$$T(\varphi(x):\varphi(y_1)\ldots\varphi(y_n):)$$
$$= T'(\varphi(x):\varphi(y_1)\ldots\varphi(y_n):)$$
$$+ P(y_1\mid y_2\ldots y_n)\,M_2(x, y_1):\varphi(y_2)\ldots\varphi(y_n):, \quad (22)$$

where we have introduced a new notation $M_2(x,y) = M^{11}(x,y)$.

Substituting Eq. (22) in Eq. (18) we get the following expression for the interpolating field:

$$A(x) = A_{in}(x) + \frac{1}{i}\int D^{ret}(x-y)\,S^{(1)}(y)$$
$$+ \int M_2(x,y)\,S^{(1)}(y). \quad (23)$$

Here and in what follows we use the abbreviated notation

$$S^{+}\delta S/\delta A_{in}(x) = S^{+}\delta_x S = S^{(1)}(x).$$

Obviously the field $A(x)$ is Hermitian if $M_2(x,y)$ is pure imaginary. The following assertion can be made about the field defined by the formula (23).

If the S matrix satisfies the causality condition[6]

$$\delta_y S^{(1)}(x) = 0 \text{ for } x \lesssim y, \quad (24)$$

then a field $A(x)$ of the form (23) satisfies the locality condition (8).

In fact,

$$[A(x), A(y)] = [A_{in}(x), A_{in}(y)]$$
$$+ \frac{1}{i}\int [D^{ret}(y-z) + iM_2(y,z)]\,[A_{in}(x), S^{(1)}(z)]\,dz$$
$$+ \frac{1}{i}\int [D^{ret}(x-u) + iM_2(x,u)]\,[S^{(1)}(u), A_{in}(y)]\,du$$
$$- \int [D^{ret}(x-u) + iM_2(x,u)]\,[D^{ret}(y-z)$$
$$+ iM_2(y,z)]\,[S^{(1)}(u), S^{(1)}(z)]\,du\,dz. \quad (25)$$

Let us use the obvious formulas

$$[S^{(1)}(u), S^{(1)}(z)] = \delta_z S^{(1)}(u) - \delta_u S^{(1)}(z), \quad (26)$$

$$[A_{in}(x), S^{(1)}(z)] = \frac{1}{i}\int D(x-u)\,\delta_u S^{(1)}(z)\,du. \quad (27)$$

Then we get

$$[A(x), A(y)] = [A_{in}(x), A_{in}(y)] + \int [D^{ret}(y-z)$$
$$+ iM_2(y,z)]\,[D^{adv}(x-u) + iM_2(x,u)]\,\delta_u S^{(1)}(z)\,du\,dz$$
$$- \int [D^{ret}(x-u) + iM_2(x,u)]\,[D^{adv}(y-z)$$
$$+ iM_2(y,z)]\,\delta_z S^{(1)}(u)\,du\,dz. \quad (28)$$

It is clear that in virtue of the locality of $A_{in}(x)$, the causality condition (24), and the properties of the functions $D^{ret}(x-y)$, $D^{adv}(x-y)$, and $M_2(x,y)$ each term in Eq. (28) vanishes for $x \sim y$.

We note that, starting from Eqs. (18) and (11), we can also get for the local interpolating field the more general expression

$$A(x) = A_{in}(x) + \frac{1}{i}\int D^{ret}(x-y)\,S^{(1)}(y)\,dy$$
$$+ \sum_{k=1} \frac{1}{k!}\int M_{k+1}(x, y_1, \ldots y_k)\,S^{(k)}(y_1\ldots y_k)\,dy_1\ldots dy_k, \quad (29)$$

if we assume that in Eq. (12)

$$\Lambda^{1\ldots1}(x_1, \ldots x_{\nu_1}) = M_{\nu_1}(x_1\ldots x_{\nu_1}).$$

The local character of this field is proved in an analogous way, but more cumbersome calculations are required.

Let us consider as an example the case in which $M_2(x,y)$ is a polynomial of finite degree in $\partial/\partial x$ applied to δ functions, with constant pure imaginary coefficients. Then when the requirements of relativistic invariance are taken into account $A(x)$ takes the form

$$A(x) = A_{in}(x) + \int D^{ret}(x-y)\,j(y)\,dy$$
$$+ \int Q(\Box_x)\,\delta(x-y)\,j(y)\,dy, \quad (30)$$

or

$$A(x) = \tilde{A}(x) + Q(\Box_x)\,j(x), \quad (31)$$

where

$$\tilde{A}(x) = A_{in}(x) + \int D^{ret}(x-y)\,j(y)\,dy, \quad (32)$$

and[6]

$$j(x) = -iS^{(1)}(x). \quad (33)$$

Equation (31) is identical with Eq. (9). Thus the ambiguity in the definition of the interpolating field pointed out in Borchers' paper[3] has been shown to be capable of being expressed in terms of the ambiguity in the definition of the T-product with a given S matrix.

4. THE INTERPOLATING FIELD AND THE AMBIGUITY IN THE DEFINITION OF THE S MATRIX

In this section we approach our problem from a somewhat more general standpoint and show that any ambiguity in the definition of the interpolating field can be interpreted as an ambiguity in the definition of the S matrix off the energy surface.

First let us give some attention to the problem of the definition of the S matrix off the energy surface. As is well known, the S matrix can be represented [1] in the form

$$S = \sum_{n=0} \frac{1}{n!} \int dk_1 \ldots dk_n \delta (k_1 + \ldots + k_n)$$
$$\times h_n (k_1 \ldots k_n) \delta (k_1^2 - m^2)$$
$$\ldots \delta (k_n^2 - m^2) : A_{in} (k_1) \ldots A_{in} (k_n) :. \quad (34)$$

It can be seen from the expression (34) that the S matrix depends only on the values of $h_n(k_1 \ldots k_n)$ on the energy surface, i.e., for $k_1^2 = \ldots = k_n^2 = m^2$.

This, however, is not sufficient for a complete formulation of the theory, and in particular for the formulation of the causality condition. We also need to know the Fourier transforms $h_n(x_1 \ldots x_n)$, and consequently it is necessary to define in some way $h_n(k_1 \ldots k_n)$ off the energy surface. We shall suppose that the S matrix is defined off the energy surface if the $h_n(k_1 \ldots k_n)$ are prescribed in the entire momentum space. In this case we can define the variational derivative of the S matrix in the form

$$\delta_x S = \sum_{n=1} \frac{1}{(n-1)!} \frac{1}{(2\pi)^{4/2}}$$
$$\times \int e^{ik_1 x} \delta (k_1 + \ldots + k_n) \delta (k_2^2 - m^2) \ldots \delta (k_n^2 - m^2)$$
$$\times h_n (k_1 \ldots k_n) : A_{in} (k_2) \ldots A_{in} (k_n) : dk_1 \ldots dk_n. \quad (35)$$

It can be seen from Eq. (35) that the value of $\delta_x S$ depends in an essential way on the definition of $h_n(k_1 \ldots k_n)$ off the energy surface. It is clear, however, that an expression of the form $\int f(x) \delta_x S \, dx$, where $f(x)$ is an arbitrary solution of the Klein-Gordon equation, depends only on the values of $h_n(k_1 \ldots k_n)$ on the energy surface. It follows from this that the values $(\delta_x S)_1$ and $(\delta_x S)_2$ which correspond to two different definitions of $h_n(k_1 \ldots k_n)$ off the energy surface can differ only by a function $Q(x)$ which satisfies the condition

$$\int f (x) \, Q (x) \, dx = 0. \quad (36)$$

It is also not hard to verify the converse, namely: if $(\delta_x S)_1$ is defined by Eq. (35) with certain fixed functions $h_n^{(1)}(k_1 \ldots k_n)$, and

$$(\delta_x S)_2 = (\delta_x S)_1 + Q (x), \quad (37)$$

where $Q(x)$ satisfies Eq. (36), then $(\delta_x S)_2$ can be represented in the form (35) with certain other functions $h_n^{(2)}(k_1 \ldots k_n)$ which differ from $h_n^{(1)}(k_1 \ldots k_n)$ only off the energy surface.

From these arguments we can conclude that the possibility of different definitions of $h_n(k_1 \ldots k_n)$ off the energy surface reduces to the possibility of an arbitrariness in the current defined by Eq. (33), of the form

$$j (x) = j_0 (x) + J (x), \quad (38)$$

where $j(x)$ corresponds to some definite choice of $h_n(k_1 \ldots k_n)$ off the energy surface and $J(x)$ is some operator satisfying the equation

$$\int f (x) \, J (x) \, dx = 0. \quad (39)$$

In order not to make additional complications we shall assume that $J(x)$ satisfies the requirements of Hermiticity and of translational and relativistic invariance, and also, if the theory is causal, the causality condition.

Let us now consider the question to what extent the asymptotic condition (5) fixes the definition of the interpolating field. For this purpose let us write down the difference between the expressions for the asymptotic condition for $t \to +\infty$ and $t \to -\infty$. We have

$$\left(\Phi, \int \frac{\partial}{\partial x^0} \left[A (x) \frac{\overleftrightarrow{\partial}}{\partial x^0} f (x)\right] dx \, \Psi\right)$$
$$= \left(\Phi, \int [A_{out} (x) - A_{in} (x)] \frac{\overleftrightarrow{\partial}}{\partial x^0} f (x) \, dx \, \Psi\right). \quad (40)$$

It follows from Eq. (4) that

$$A_{out} (x) - A_{in} (x) = \int D (x - y) \, j (y) \, dy. \quad (41)$$

Substituting Eq. (41) in Eq. (40) and making some transformations, we get

$$\int f (x) \, K_x A (x) \, dx = - \int f (x) \, j (x) \, dx. \quad (42)$$

It follows from this that

$$K_x A (x) = - j (x) - J_1 (x), \quad (43)$$

where

$$\int f (x) \, J_1 (x) \, dx = 0.$$

Using the fact that the definition of $j(x)$ itself contains an arbitrariness [cf. Eq. (38)], we can put Eq. (43) in the form

$$K_x A (x) = - j (x). \quad (44)$$

Thus the asymptotic condition (5), which is fundamental in this approach, [1] leads to a formula (44) for the determination of the interpolating field which shows that all of the ambiguity in the definition of this field is due to the ambiguity in $j(x)$, i.e., to the ambiguity in the definition of the S matrix off the energy surface.

5. DISCUSSION

We now draw some conclusions. It was shown in Sec. 4 that all of the ambiguity in the interpolating field can be regarded as an ambiguity in the definition of the S matrix off the energy surface. On the other hand, in Sec. 3 it was indicated that there is a possible ambiguity in the definition of the interpolating field associated with the ambiguity in the definition of the T-product with a prescribed S matrix. Therefore it is interesting to see whether we can establish a connection between these two approaches to the problem.

In Sec. 3 the ambiguity in the interpolating field was actually determined in terms of the ambiguity of the expression $T(A_{in}(x)S)$, which can be written out explicitly as follows:

$$T(A_{in}(x)S)=\sum_{i=0}\int dy_1\ldots dy_i P(y_1\,|\,y_2\ldots y_i)\,\varphi_i(y_1\ldots y_i)$$
$$\times\left(\frac{1}{i}D^c(x-y_1)+M_2(x,y_1)\right):A_{in}(y_2)\ldots A_{in}(y_i):. \qquad (45)$$

Since $M_2(x,y)$ can be represented in the form $i^{-1}Q(\Box_y)\,\delta(x-y)$, Eq. (45) can be written in the form

$$T(A_{in}(x)S)=\sum_{i=0}\int dy_1\ldots dy_i P(y_1\,|\,y_2\ldots y_i)\,\varphi_i(y_1\ldots y_i)$$
$$\times\frac{1}{i}\Big(D^c(x-y_1)-Q(\Box_{y_1})$$
$$\times K_{y_1}D^c(x-y_1)\Big):A_{in}(y_2)\ldots A_{in}(y_i):. \qquad (46)$$

Let us integrate Eq. (46) by parts, assuming that when the limits are substituted the expression in question vanishes, in accordance with the remark made in Sec. 1. Then we get

$$T(A_{in}(x)S)$$
$$=\sum_{i=0}\int dy_1\ldots dy_i P(y_1\,|\,y_2\ldots y_i)\,\frac{1}{i}D^c(x-y_1)$$
$$\times[\varphi_i(y_1\ldots y_i)-Q(\Box_{y_1})K_{y_1}\varphi_i(y_1\ldots y_i)]:$$
$$\times A_{in}(y_2)\ldots A_{in}(y_i):=T'(A_{in}(x)\widetilde{S}), \qquad (47)$$

where

$$\widetilde{S}=\sum_{i=0}\int dy_1\ldots dy_i\,[\varphi_i(y_1\ldots y_i)$$
$$-Q(\Box_{y_1})K_{y_1}\varphi_i(y_1\ldots y_i)]:A_{in}(y_1)\ldots A_{in}(y_i):. \qquad (48)$$

It is clear from the structure of the expression (48) that the matrix $\widetilde{S}$ differs from the S matrix only off the energy surface.

Thus we see that the ambiguity in the definition of the interpolating field associated with the ambiguity in the definition of the T-product with a given S matrix can be reformulated in such a way that it turns out to be connected with the ambiguity in the definition of the S matrix off the energy surface with a fixed definition of the T-product.

A very curious situation arises. It turns out that not only is there quite a variety of interpolating fields corresponding to an S matrix defined off the energy surface in a prescribed way, but also conversely there are several expressions for the S matrix off the energy surface corresponding to a single interpolating field. We now recall that the definition of the S matrix off the energy surface is closely connected with the definition of the Dyson matrix $S(\sigma_1, \sigma_2)$ which connects state vectors prescribed on two arbitrary spacelike surfaces. In this connection it becomes very interesting to find out how the ambiguity noted here affects the definition of the Dyson matrix, and we intend to continue with the study of this question.

In conclusion we express our deep gratitude to B. V. Medvedev for suggesting the topic and for his constant interest in the work. We also express our gratitude to M. K. Polivanov for a discussion.

[1] Lehmann, Symanzik, and Zimmermann, Nuovo cimento **1**, 205 (1955), **6**, 319 (1957). Glaser, Lehmann, and Zimmermann, Nuovo cimento **6**, 1122 (1957).

[2] C. N. Yang and D. Feldman, Phys. Rev. **79**, 972 (1950).

[3] H.-J. Borchers, Nuovo cimento **15**, 784 (1960).

[4] N. N. Bogolyubov and D. V. Shirkov, Vvedenie v teoriyu kvantovannykh polei (Introduction to the Theory of Quantized Fields), Gostekhizdat, 1957.

[5] A. D. Sukhanov, JETP **41**, 1915 (1961), this issue, p. 1361.

[6] Bogolyubov, Medvedev, and Polivanov, Voprosy teorii dispersionnykh sootnoshenii (Problems of the Theory of Dispersion Relations) Fizmatgiz, Moscow, 1958.

Translated by W. H. Furry
327