ISOTOPIC STRUCTURE OF WEAK INTERACTIONS AND PROCESSES RESULTING FROM THE ABSORPTION OF A NEUTRINO OR ANTINEUTRINO BY NUCLEONS

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Submitted to JETP editor July 18, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1953-1961 (December, 1961)

Certain processes of (anti) neutrino absorption by nucleons should give rise to systems of baryons and mesons with total strangeness zero. It is assumed that the weak interaction responsible for these processes is local, CP-invariant, and includes a baryon-meson current which transforms in isotopic spin space like a component of a vector. A relation is established between cross sections of similar reactions, resulting from the absorption of a neutrino or antineutrino respectively, and a method is indicated for qualitatively distinguishing between the V, A and S, P, T variants in the weak interaction. Isotopic equalities between amplitudes for various processes of neutrino or antineutrino absorption by nucleons are also written down.

1. INTRODUCTION

As is known, the processes that occur as a result of absorption by nucleons of high-energy neutrinos or antineutrinos may be divided into two categories. For one class of reactions the total strangeness of the strongly interacting particles in the final state is the same as that of a nucleon (i.e., zero), and for the other class it differs by unity. The following are examples of processes of the first type

$$\nu + n \to p + l^-,\quad \nu + p \to p + \pi^+ + l^-,\quad \nu + n \to \Lambda + K^+ + l^-,$$
$$\bar{\nu} + p \to n + l^+,\quad \bar{\nu} + n \to n + \pi^- + l^+,\quad \bar{\nu} + p \to \Lambda + K^0 + l^+$$

etc., whereas the following are examples of the second type

$$\bar{\nu} + p \to \Lambda + l^+,\qquad \bar{\nu} + p \to p + K^- + l^+,$$
$$\nu + p \to p + K^+ + l^-$$

and so forth ($l^{\pm}$ denotes a charged lepton, i.e., either electron or μ meson). Processes of the first type, proceeding with strangeness conservation, are caused by the same weak interaction that is responsible for nuclear β decay and μ capture, whereas the interaction leading to reactions in which strangeness is not conserved appears only in leptonic decays of K mesons and hyperons.

It is known about the first interaction that for small momentum transfers it consists of the V and A 4-fermion variants, and that the baryon-meson current in it transforms like a component of a vector in isotopic space.[1] This latter circumstance, by the way, cannot be considered as rigorously proven. The reason for this lies in the fact that the decay of $\pi^{\pm}$ mesons, as well as the β decay of the free neutron and a majority of mirror nuclei (it is precisely these processes that give rise to most of the information on the structure of weak interactions) is due to only the (+) component of an isotopic vector. Even if the weak interaction were to include a current transforming like a component of a second rank tensor in isotopic space (such a current can, for example, be constructed out of the $\bar{\Sigma}$ and Σ operators), it would not contribute to the decays mentioned. An argument against the existence of such a current comes from the observation that it would contribute to the decay of O^{14}, which occurs between different components of an isotopic vector, leading to a disagreement between the vector constants in the decays of $\mu^{\pm}$ and O^{14}. This last consideration, however, is of a quantitative nature, and furthermore the equality of the constants has lately become subject to doubt.[2] In composite models of elementary particles it turns out, as a rule, to be possible to construct only isovector currents,[3] however these models themselves are greatly in need of experimental verification.

Nothing definite can be said at this time about the interaction of strongly interacting particles in which strangeness is not conserved. According to the Feynman–Gell-Mann scheme[1] the baryon-meson currents are, generally speaking, sums of isospinors of rank $1/2$ and $3/2$. In a number of papers (see, e.g., the review article by Okun'[3]) the hypothesis has been advanced that the isospinor of rank $3/2$ is absent.

A study of reactions resulting from the absorption by nucleons of neutrinos or antineutrinos would give information about the properties of weak interactions for large energies and momentum transfers. A direct calculation of cross sections for specific processes will not, apparently, be very useful, because a large number of unknown form factors appears in the calculations. As a result the calculations, as well as their comparison with experimental data, become difficult or in practice impossible. A different approach seems more reasonable, in which experimental consequences of the general properties of the weak interactions are predicted. It is from this point of view precisely that the neutrino and antineutrino absorption by nucleons was discussed in the article of Lee and Yang.[4]

In this work, as in [4], the consequences of locality, CP invariance, and the proposed isotopic structure of the weak interactions are investigated. From the beginning we include in the considerations all five possible variants of the weak interaction.

In Sec. 2 the strangeness-conserving processes are investigated. The hypothesis that the baryon-meson current transforms like a component of a vector in isotopic space leads to certain relations between the processes of absorption of neutrinos and antineutrinos; these relations turn out to be different for different weak interaction variants. The difference between the V, A and S, P, T variants turns out to be most pronounced for the case when the energy of the neutrino or antineutrino in the laboratory frame is large is comparison with the transferred momenta. In Sec. 3 additional consequences of crossing symmetry for production processes of an arbitrary number of π and K mesons are described. Section 4 is devoted to the establishment of isotopic relations between amplitudes for various reactions.

Questions relating to the identity of electron and μ-meson neutrinos, or to the feasibility of obtaining sufficiently intense beams of these particles, are not discussed in this paper.

2. PROCESSES PROCEEDING WITH NO CHANGE IN STRANGENESS OF THE STRONGLY INTERACTING PARTICLES

Reactions of neutrino or antineutrino absorption by nucleons, resulting in the production of baryons and mesons with zero total strangeness, may be written in general form as

$$\text{a)}\ \nu + N \rightarrow B + l^-,$$
$$\text{b)}\ \bar{\nu} + N_1 \rightarrow B_1 + l^+, \qquad (1)$$

where N refers to the nucleon in the initial state, and B to the system of all strongly interacting particles at the end of the reaction. The states N_1 and B_1 may be obtained from N and B by applying the charge symmetry operator. If it is assumed that the latter coincides with rotations by 180° about the first axis in isotopic space, then this operation amounts to the substitutions

$$p \leftrightarrow n,\quad \Lambda \rightarrow \Lambda,\quad \Sigma^+ \leftrightarrow \Sigma^-,\quad \Sigma^0 \rightarrow -\Sigma^0,\quad \Xi^0 \leftrightarrow \Xi^-,$$
$$\pi^+ \leftrightarrow \pi^-,\quad \pi^0 \rightarrow -\pi^0,\quad K^+ \leftrightarrow K^0,\quad K^- \leftrightarrow \overline{K}^0. \qquad (2)$$

To each of processes a) there corresponds a definite reaction b), and vice versa. The amplitudes for processses a) and b) are simply related to each other, in the same manner as in ordinary β decay. If the weak interaction is local the matrix element for reaction a) may be written in first order of perturbation theory as

$$M = \sum_i \Gamma_i\ (\bar{u}_l O_i (C_i + C_i'\gamma_5)\, u_\nu), \qquad (3)$$

where the sum is over the five variants of the 4-fermion interaction, O_i are the spin matrices for these covariants, and Γ_i are the matrix elements of the corresponding baryon-meson currents taken between the states of N and B; $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$; C_i and C_i' are the ordinary β-decay constants.

If CP invariance holds, and if the baryon-meson current transforms like a component of an isovector, then accurate to within the possible appearance of an immaterial common factor of (-1) the matrix element for process b) may be written in the form

$$M_1 = \sum_i \Gamma_i\ (\bar{u}_{\bar{l}} O_i \eta_i (C_i - C_i'\gamma_5)\, u_{\bar{\nu}}), \qquad (4)$$

where (after replacing N and B by N_1 and B_1) Γ_i is the same as in Eq. (3) and η_i is determined by (C is the charge-conjugation matrix)

$$C^{-1} O_i C = \eta_i O_i^T,$$
$$\eta_i = \begin{cases} 1 & i = A, S, P, \\ -1 & i = V, T. \end{cases} \qquad (5)$$

It follows from a comparison of Eqs. (4) and (5) with (3) that the transition from reaction a) to b) is accomplished by having

$$C_V,\ C_A',\ C_S',\ C_P',\ C_T \ \text{change sign}$$
$$\text{and}\ C_V',\ C_A,\ C_S,\ C_P,\ C_T' \ \text{do not change sign.} \qquad (6)$$

In other words, terms involving interferences of constants appearing in the same line in Eq. (6) are the same in the cross sections for reactions a) and b), whereas interferences between lines contribute to the cross sections terms with opposite signs.

Let us restrict ourselves in this section to the cases where in processes (1) the masses of the leptons can be ignored. In practice this is permissible whenever the lepton l is an electron. If furthermore the energy of the incident neutrinos and the transferred momenta are much larger than the μ-meson mass, then our conclusions are valid also for the case $l = \mu$. In the case of massless leptons the free Lagrangian is invariant under the γ_5 transformation of the lepton operators, under which in the matrix element (3)

$$C_S, C_P, C_T, C'_S, C'_P, C'_T \text{ change sign, and } C_V, C_A, C'_V, C'_A \text{ do not change sign.} \quad (7)$$

Therefore after neglecting m_l the expressions for cross sections for processes (1) cannot contain interferences of constants from the two groups in Eq. (7), i.e., interferences of the V and A variants with the S, P and T variants.

Let us consider now the spinor transformation, used previously in application to leptonic decays:[5]

$$u_\nu(p_\nu) \to v_l^c(-p_l) = C\bar{u}_l(p_l), \quad u_l(p_l) \to v_\nu^c(-p_\nu) = C\bar{u}_\nu(p_\nu) \quad (8)$$

and amounting, in essence, to a crossing transformation of the leptons. The transformation (8) is, apparently, equivalent to the exchange

$$p_\nu \leftrightarrow -p_l \quad (9)$$

(one should also add $m_\nu \longleftrightarrow m_l$ except that we have assumed that the leptonic masses may be ignored) in the projection operator or in the density matrix resulting from squaring the matrix element (3). On the other hand, that matrix element remains unchanged if simultaneously with the transformation (8) we require that

$$C'_V, C_A, C_S, C'_S, C_P, C'_P \text{ change sign and } C_V, C'_A, C_T, C'_T \text{ do not change sign.} \quad (10)$$

It therefore follows that the expression for the cross section for processes (1) should be left unchanged under the simultaneous transformations (9) and (10). What has been said above is valid provided that the "vertex parts" Γ_i that appear in Eq. (3) are functions of only the four-vector $p_\nu - p_l$, which is invariant under (9), and not the four-vector $p_\nu + p_l$. This is the case for a local weak interaction, as well as in the case when a virtual charged boson exists; we will refer to this latter case as a quasilocal interaction.

A number of conclusions can be derived from the invariance of the cross section under transformation (9)—(10). To this end we observe that, as a consequence of locality or quasilocality of the weak interaction, the cross section for processes (1) may depend on the four-vector $s \equiv (p_\nu + p_l)$ not more than bilinearly. That is to say, the cross section divided by the statistical weight factor may be expressed in the form

$$a(sq_1)(sq_2) + b + d(sq_3), \quad (11)$$

where q_1, q_2 and q_3 are four-vectors made up out of the momenta and polarizations of the strongly interacting particles in the initial and final states, as well as of the four-momentum $p_\nu - p_l$, which is related to the others by the laws of momentum conservation. The coefficients a, b, d are functions of invariants formed out of the same four-vectors. We do not include in Eq. (11) the term proportional to s^2 because

$$s^2 = -(p_\nu - p_l)^2 + 2(m_l^2 + m_\nu^2) \approx -(p_\nu - p_l)^2.$$

Under the transformation (9) the vectors q_i do not, and the vector s does, change sign. It then follows from the invariance under (9), (10) and the circumstance that the coefficients a, b and d depend on C_i and C'_i bilinearly, that the coefficients a and b in Eq. (11) may contain only such interferences of the constants that are even under (10), i.e.,

$$a, b \sim C_V^2 + C_V'^2,\ C_A^2 + C_A'^2,\ C_V C'_A + C'_V C_A,$$
$$C_S^2 + C_S'^2,\ C_P^2 + C_P'^2,\ C_S C_P + C'_S C'_P,\ C_T^2 + C_T'^2,$$
$$C_S C'_S,\ C_P C'_P,\ C_S C'_P + C'_S C_P,\ C_T C'_T, \quad (12)$$

whereas the coefficient d should be odd:

$$d \sim C_V C_A + C'_V C'_A,\ C_V C'_V,\ C_A C'_A,$$
$$C_S C_T + C'_S C'_T,\ C_P C_T + C'_P C'_T,$$
$$C_S C'_T + C'_S C_T,\ C_P C'_T + C'_P C_T. \quad (13)$$

Comparing now Eqs. (12) and (13) with (6) we find that in the case of a V, A interaction the coefficients a and b are the same for processes a) and b) in Eq. (1), whereas d differs in sign. The sum of the cross sections a) and b) will therefore not contain d, and consequently the interference of the V and A variants may enter only through the combination $C_V C'_A + C'_V C_A$, i.e., only through a pseudoscalar quantity. The scalar part of the sum of the cross sections on the other hand, can contain the V and A variants only in the form of a sum of squares, which is obviously larger than the contribution from a pure V interaction. If the Feynman-Gell-Mann hypothesis relating the β-decay current to the isovector part of the electromagnetic current is valid, then the V contribution may be determined from experiments on electroproduction of the system B or B_1 from nucleons. Such a calculation was carried out by Azimov[6] for the processes $\nu + N \to N + \pi + l$, $\nu + N \to \Lambda + K + l$.

In the presence of the S, P, T variants the situation for the scalar part of the cross section is the same as in the "pure" V, A case, i.e., a and b are the same for processes a) and b) and d differs in sign. Consequently the experimental comparison of this part of the cross section for processes a) and b) permits one to test independently of what variants might be involved in the fundamental assumptions of the theory, namely locality, CP invariance, and—which is most problematic—the isovector nature of the baryon-meson current. For the pseudoscalar quantities, such as the longitudinal polarization of the produced fermions, the situation is different. Namely for the case of S, P, T, d is the same and a, b differ in sign for processes a) and b), whereas for V, A the opposite is the case. Consequently a comparison of quantities of this type in processes a) and b) would allow, in principle, to establish whether the S, P, T variants contribute at high energies.

Experimentally the coefficients a, b and d can be distinguished by varying the momenta of $\nu(\bar{\nu})$ and l in such a way that only the four-vector $s = p_\nu + p_l$ changes, while $p_\nu - p_l$, as well as the momenta of all the other particles, remain fixed. One can, in particular, in studying any of the processes (1) select only those events for which the neutrino energy in the laboratory system E_ν is large and is almost entirely transferred to the electron (or μ meson). In that case the only large invariants in Eq. (11) are (sq) (with $q \neq p_\nu - p_l$). If, for example, q coincides with the four-momentum of the initial nucleon, then $(sq) = m(E_\nu + E_l) \approx 2mE_\nu$, whereas the invariants not involving the vector s are of the order of either $Q^2 \equiv (p_\nu - p_l)^2$ or $\sim m\sqrt{|Q^2|}$. For $E_\nu \gg \sqrt{|Q^2|}$ only the first term in Eq. (11), i.e., a, is important. Therefore under these conditions the scalar parts of the cross sections for a) and b) should coincide independently of what variants are involved in the weak interaction. (In the case of the V, A interaction the cross section for each of the processes a) and b) —and not only their sum—appears now in the form of a sum of positive contributions from the V and A variants, the first of which may be related to electroprocesses.) The pseudoscalar quantities should have the same signs in the case of the V, A interaction, and opposite signs in the case of S, P, T. Thus again they provide a possibility for determining what variants are involved.

If we consider the cross section integrated over all internal variables of the system B in Eq. (1), then there remain only three independent vectors in the problem: $s = p_\nu + p_l$, $p_\nu - p_l$ and p_N, so that $q = p_N$ and $(sq) = m(E_\nu + E_l)$. At that Eq. (11) means that the cross section depends quadratically on $E_\nu + E_l$ for fixed $E_\nu - E_l$ and the angle of emission of l with, according to what has been said, the terms in the scalar part of the cross section linear in $E_\nu + E_l$ being of opposite sign for processes a) and b), the other terms being of the same sign. This fact has been pointed out previously by Lee and Yang [4] in somewhat different terms.

In conclusion we note that the cross sections for the processes of (anti) neutrino absorption resulting in a change in the strangeness of the strongly interacting particles also have the structure of Eq. (11), with, for $(sq) \gg m\sqrt{|Q^2|}$, the first term dominating. In that case we again have invariance under the transformations (9)—(10), however no analogy of the type (6) between various processes exists.

3. CROSSING SYMMETRY

From among the processes (1) we may select those in which the system B or B_1 consists of one nucleon and an arbitrary number of mesons. In that case besides the relations between the various reactions, that occur as a result of neutrino or antineutrino absorption, described in the previous section it is possible to obtain additional relations due to crossing symmetry. Let us consider the processes

$$\begin{aligned} &\alpha)\quad \nu + N_1 \to N_2 + A + l^-,\\ &\beta)\quad \bar{\nu} + N_1 \to N_2 + \bar{A} + l^+, \end{aligned} \tag{14}$$

where N_1 and N_2 denote nucleons in the initial and final states, and A stands for a system consisting of an arbitrary number of π and K mesons. $\bar{A}$ differs from A in that all particles are replaced by their antiparticles. If the initial nucleon in the reaction α is the same as the final nucleon in the reaction β, in the sense that they are either both protons or both neutrons, and if the same is true of the other pair of nucleons, i.e., if

$$N_{1\alpha} = N_{2\beta}, \qquad N_{2\alpha} = N_{1\beta}, \tag{14a}$$

then the matrix elements of reactions α and β are related by crossing symmetry. Let us denote the momenta of the particles in reaction α by p_ν, p_1, p_2, p_A, p_l, and those in reaction β by q_ν, q_1, q_2, q_A, q_l (p_A and q_A stand for the totality of the momenta of all particles in A or $\bar{A}$) and let us assume in what follows that Eq. (14a) holds. Then the expression for the cross section of process β in Eq. (14) coincides with the expression for the reaction α, provided that we make the following substitutions in the matrix element for the latter, i.e., in Eq. (3):

$$p_1 \to q_2, \quad p_2 \to q_1, \quad p_A \to -q_A,$$
$$p_\nu \to q_l, \quad p_l \to q_\nu, \quad m_\nu \leftrightarrow m_l. \qquad (15)$$

If the nucleons are polarized then we must add to Eq. (15): $\zeta_1 \longleftrightarrow \eta_2$, $\zeta_2 \longleftrightarrow \eta_1$, where ζ_i and η_i are the polarization vectors of the i-th nucleon (i = 1, 2) in the reaction α and β. The last relation in Eq. (15) may be omitted if the neglect of the mass of the lepton l is justified. If such is not the case then it becomes convenient to add to Eq. (15) the transformation (9), (10), after which the transition from α to β consists of the replacement (10) together with

$$p_1 \to q_2, \quad p_2 \to q_1, \quad p_A \to -q_A, \quad p_\nu \to -q_\nu,$$
$$p_l \to -q_l \qquad (\zeta_1 \to \eta_2, \quad \zeta_2 \to \eta_1). \qquad (16)$$

Let us consider in more detail the specific example of "weak" production of a single π meson, i.e., the reactions

$$\alpha_1)\ \nu + p \to p + \pi^+ + l^-, \qquad \beta_1)\ \bar{\nu} + p \to p + \pi^- + l^+,$$
$$\alpha_2)\ \nu + n \to n + \pi^+ + l^-, \qquad \beta_2)\ \bar{\nu} + n \to n + \pi^- + l^+,$$
$$\alpha_3)\ \nu + n \to p + \pi^0 + l^-, \qquad \beta_3)\ \bar{\nu} + p \to n + \pi^0 + l^+. \qquad (17)$$

These reactions are related by the transformation (15) or (10), (16). On the other hand, the processes β_1, β_2, β_3 stand in the same relation to α_1, α_2, α_3 as did processes b) to a) in Eq. (1), so that the transition between them is accomplished according to the rule (6), provided that the conditions listed in Sec. 2 are fulfilled. It is easy to see on comparing Eq. (6) with Eqs. (10) and (16) that in the case of the V, A interaction the expression for the cross sections α_3 and β_3 should be even under the exchange $p_1 \longleftrightarrow p_2$ and a change of sign of p_ν, p_l and p_π, whereas the cross sections α_1 and α_2, or β_1 and β_2, should under the same transformation go into each other. In the case of S, P, T what has been said above remains true for the scalar part of the cross section, whereas the pseudoscalar part changes sign.

An experimental test of crossing symmetry would require obtaining the dependence of the cross section on the invariants that can be constructed out of the four-vectors entering into Eqs. (15) or (16) which is, apparently, difficult. It may be that selecting events for which N_2 is approximately at rest will somewhat facilitate this task. At that in Eq. (17), as well as in the general case (14), there remain after integration over the internal variables of the system A or $\bar{A}$ only three independent vectors: p_ν, p_l and $p_1 \approx p_2$, out of which one can construct three invariants: $Q^2 \equiv (p_\nu - p_l)^2$, $(p_\nu - p_l) \cdot p_1 = m(E_\nu - E_l)$ and $(sp_1) \equiv (p_\nu + p_l) \cdot p_1 = m(E_\nu + E_l)$. Under the transformation (16) the last two change sign. At large energies E_ν and for $|Q^2| \ll m(E_\nu + E_l)$ the dependence on the last invariant is quadratic so that the transformation (16) amounts to a change in the sign of the one invariant $(p_\nu - p_l) \cdot p_1 = m(E_\nu - E_l)$ only.

4. ISOTOPIC RELATIONS

If the baryon-meson current has definite isotopic structure, i.e., transforms as a component of an isovector or isospinor of a given rank, then the amplitudes for the various processes resulting from absorption of (anti)neutrinos by nucleons are related to each other as a consequence of isotopic invariance. Such relations are given below for reactions with no more than three strongly interacting particles in the final state. In the usual manner there follow from equalities among amplitudes inequalities among cross sections.

Let us consider first processes in which the total strangeness does not change, and let us suppose that the baryon-meson current transforms like a component of an isotopic vector.

1) If the amplitudes for the reactions

$$a)\ \nu + p \to p + \pi^+ + l^-,$$
$$b)\ \nu + n \to n + \pi^+ + l^-,$$
$$c)\ \nu + n \to p + \pi^0 + l^- \qquad (18)$$

are denoted respectively by T_a, T_b, T_c, then the relation between them may be written as

$$T_a - T_b - \sqrt{2}\, T_c = 0. \qquad (19)$$

The same relation holds also for the processes

$$a)\ \nu + p \to \Sigma^+ + K^+ + l^-,$$
$$b)\ \nu + n \to \Sigma^+ + K^0 + l^-,$$
$$c)\ \nu + n \to \Sigma^0 + K^+ + l^-,$$

or

$$a)\ \nu + p \to \Lambda + K^+ + \pi^+ + l^-,$$
$$b)\ \nu + n \to \Lambda + K^0 + \pi^+ + l^-,$$
$$c)\ \nu + n \to \Lambda + K^+ + \pi^0 + l^-. \qquad (20)$$

The inequalities for the cross section arising from Eq. (19) are obvious.

2) In the reactions

$$a)\ \nu + p \to p + \pi^+ + \pi^0 + l^-,$$
$$b)\ \nu + p \to n + \pi^+ + \pi^+ + l^-,$$
$$c)\ \nu + n \to p + \pi^+ + \pi^- + l^-,$$
$$d)\ \nu + n \to p + \pi^0 + \pi^0 + l^-,$$
$$e)\ \nu + n \to n + \pi^+ + \pi^0 + l^- \qquad (21)$$

the two pions may be produced in an even or odd orbital state. If the corresponding amplitudes are denoted by T^+ and T^-, and if the Bose nature of the pions is taken into account, then

$$T_a^+ = T_b^+/\sqrt{8} = -T_e^+ = (T_c^+ - T_d^+)/\sqrt{2},$$
$$T_b^- = T_d^- = 0, \qquad T_c^- = (T_a^- - T_e^-)/\sqrt{2}. \qquad (22)$$

In the total cross section the amplitudes T^+ and T^- do not interfere. Consequently one obtains from Eq. (22) inequalities for the total cross sections: $\sigma_a \geq \sigma_b/4$, $\sigma_e \geq \sigma_l/4$, etc. If reaction (21) takes place near threshold then T^- is clearly small in comparison with T^+, so that Eq. (22) gives for the differential cross sections

$$d\sigma_a \approx d\sigma_e \approx d\sigma_b/8 \leqslant (\sqrt{d\sigma_c} + \sqrt{d\sigma_d})^2/2,$$
$$\sqrt{2d\sigma_a} + \sqrt{d\sigma_c} \geqslant \sqrt{d\sigma_d}, \qquad \sqrt{2d\sigma_a} + \sqrt{d\sigma_d} \geqslant \sqrt{d\sigma_c}. \qquad (23)$$

For the total cross sections the relations (23) remain valid provided that

$$d\sigma_a \to \sigma_a, \quad d\sigma_c \to \sigma_c, \quad d\sigma_e \to \sigma_e, \quad d\sigma_b/2 \to \sigma_b, \quad d\sigma_d/2 \to \sigma_d.$$

Since pions obey Bose statistics the amplitudes T^+ and T^- may be interpreted as corresponding to the production of the pion pair in a state with even or odd total isotopic spin. In this sense Eq. (22) is also valid for the processes

$$a)\ \nu + p \to \Sigma^+ + \pi^0 + K^+ + l^-,$$
$$b)\ \nu + p \to \Sigma^+ + \pi^+ + K^0 + l^-,$$
$$c)\ \nu + n \to \Sigma^+ + \pi^- + K^+ + l^-,$$
$$a')\ \nu + p \to \Sigma^0 + \pi^+ + K^+ + l^-,$$
$$c')\ \nu + n \to \Sigma^- + \pi^+ + K^+ + l^-,$$
$$e')\ \nu + n \to \Sigma^0 + \pi^+ + K^0 + l^-,$$
$$d)\ \nu + n \to \Sigma^0 + \pi^0 + K^+ + l^-,$$
$$e)\ \nu + n \to \Sigma^+ + \pi^0 + K^0 + l^-, \qquad (24)$$

with $T_{a'}^\pm = \pm T_a^\pm$, $T_{c'}^\pm = \pm T_c^\pm$, $T_{e'}^\pm = \pm T_e^\pm$. The latter equalities are a consequence of isotopic invariance alone. In Eq. (22) the superscripts ± refer now only to the isotopic parity of the $\Sigma\pi$ system and have no relation to the orbital angular momentum. Consequently T^+ and T^- can interfere also in the total cross section.

3) The amplitudes of the reactions

$$a)\ \nu + p \to \Xi^0 + K^+ + K^+ + l^-,$$
$$b)\ \nu + n \to \Xi^0 + K^+ + K^0 + l^-,$$
$$c)\ \nu + n \to \Xi^- + K^+ + K^+ + l^- \qquad (25)$$

are connected by the equalities (T^+ and T^- now again refer to the amplitudes for production of pions in even or odd states of orbital angular momentum)

$$T_a^+ = T_c^+ = 0, \qquad T_a^- - T_c^- - 2T_b^- = 0. \qquad (26)$$

Since T^+ and T^- do not interfere in the total cross section we get from Eq. (26) the inequalities

$$\sqrt{2\sigma_b} + \sqrt{\sigma_c} \geqslant \sqrt{\sigma_a}, \qquad \sqrt{2\sigma_b} + \sqrt{\sigma_a} \geqslant \sqrt{\sigma_c}. \qquad (27)$$

So far all the relations in this Section have been written for processes involving neutrinos, i.e., reactions of type a) in Eq. (1). The same results are naturally valid for the corresponding reactions of type b) involving antineutrinos.

In conclusion we consider processes in which strangeness changes. As was already remarked, the baryon-meson current is in this case in general a sum of terms of isospin ½ and 3/2. In the table are given relations between the amplitudes for various processes in the two extreme cases when the current has isospin ½ or 3/2. Only processes satisfying the $\Delta S = \Delta Q$ rule [1] are considered and only if they result in the production of no more than two strongly interacting particles. The two types of reactions in the last group in the table refer to processes of the type β and α, Eq. (14), and are connected by crossing relations. Relations between cross sections for processes

Reactions	Relations between the amplitudes when the current has isospin	
	½	3/2
$a)\ \bar\nu + p \to \Sigma^0 + l^+$ or $a)\ \bar\nu + p \to \Lambda + \pi^0 + l^+$ $b)\ \bar\nu + n \to \Sigma^- + l^+$ $\quad b)\ \bar\nu + n \to \Lambda + \pi^- + l^+$	$T_a = T_b/\sqrt{2}$	$T_a = -T_b\sqrt{2}$
$a)\ \bar\nu + p \to \Xi^0 + K^0 + l^+$ $b)\ \bar\nu + p \to \Xi^- + K^+ + l^+$ $c)\ \bar\nu + n \to \Xi^- + K^0 + l^+$	$T_a + T_b - T_c = 0$	$T_a = T_b = -T_c$
$a)\ \bar\nu + p \to \Sigma^+ + \pi^- + l^+$ $b)\ \bar\nu + p \to \Sigma^0 + \pi^0 + l^+$ $c)\ \bar\nu + n \to \Sigma^0 + \pi^- + l^+$ $d)\ \bar\nu + p \to \Sigma^- + \pi^+ + l^+$ $e)\ \bar\nu + n \to \Sigma^- + \pi^0 + l^+$	$T_c = -T_e$ $= \sqrt{2}(T_b - T_a)$ $= \sqrt{2}\ (T_d - T_b)$	$T_b = \frac{1}{2}(\sqrt{2}T_c - T_a)$ $= -(T_a + T_d)$ $= T_a + \sqrt{2}T_e$
$a)\ \bar\nu + p \to p + K^- + l^+$ or $a)\ \nu + p \to p + K^+ + l^-$ $b)\ \bar\nu + p \to n + \bar K^0 + l^+$ $\quad b)\ \nu + n \to p + K^0 + l^-$ $c)\ \bar\nu + n \to n + K^- + l^+$ $\quad c)\ \nu + n \to n + K^+ + l^-$	$T_a - T_b - T_c = 0$	$-T_a = T_b = T_c$

$\bar{\nu} + N \rightarrow \Sigma + l^+$ were obtained previously by Behrends and Sirlin.[7]

The author is indebted to Ya. I. Azimov for discussions.

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Translated by A. M. Bincer
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