EFFECT OF MUTUAL ENTRAINMENT OF ELECTRONS AND PHONONS ON THE TRANS-VERSE ELECTRICAL CONDUCTIVITY IN A STRONG MAGNETIC FIELD

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It is shown that the entrainment of phonons by electrons significantly changes the transverse electrical conductivity in a strong magnetic field at low temperatures ($T \ll \Theta$). The order of magnitude and the temperature dependence change in metals and semimetals. In semiconductors, the dependence on the magnetic field may also change.

1. INTRODUCTION

LET the magnetic field H in a crystal be directed along the z axis. We shall regard this field as strong in the sense that $\omega\tau \gg 1$, where $\omega = eH/mc$ is the Larmor frequency (m is the effective mass of the electron) and τ is the relaxation time of the electrons. We shall be interested in the electric current along the x axis, due to the electric field acting in the same direction. The corresponding component of the electrical conductivity σ_{xx} will therefore be denoted simply σ.

In the displacement of the electron along the x axis, the location of the center of the Landau oscillator x^0 changes. Consequently, the y component of the associated electron momentum also changes ($x^0 = cp_y/eH$). If the electrons are scattered by the phonons, then their displacement along the x axis is associated with the transfer of the y component of the momentum to the phonon, which leads to the formation of a phonon flux along the y axis. If the phonons interacted only with the electrons, then the momentum in the stationary state, obtained by the phonons from the electrons, would be equal to the momentum given up by the electrons in the opposite process. This would produce an electron current along the x axis. In the opposite limiting case, when the phonons give up momentum to defects or to boundaries, or lose momentum as the result of transport processes more rapidly than they obtain it from the electrons, the phonons are virtually in an equilibrium state, and the usual relaxation of electrons takes place relative to the phonons. Here we are interested in the case in which the relaxation time of the phonons relative to electrons, τ_{pe}, is smaller than their nonelectronic relaxation time τ_p.

It is important to bear in mind that the current in the direction of the electric field can be caused by the transfer of the y component of the electronic momentum not only to the phonons, but also to different types of defects in the crystal. Of course, the effect studied by us is of importance only if that part of the electrical conductivity which we tentatively call the defect part (σ_d) is much smaller than the phonon part ($\sigma_d \ll \sigma_p$).

The entrainment effect significantly decreases the transverse electrical conductivity and changes its dependence on the temperature and on the magnetic field. In particular, at sufficiently low temperatures there is a dependence of the electrical conductivity on the dimensions of the specimen in the direction of the Hall current for semiconductors and semimetals. In semiconductors, the electrical conductivity in this temperature region is shown to be inversely proportional to the magnetic field intensity. For somewhat higher temperatures, the electrical conductivity can be shown to be exponentially dependent on the temperature. In Secs. 3 and 4 we consider the effect of entrainment in semimetals (and metals with a closed Fermi surface) and in semiconductors for different phonon relaxation mechanisms.

2. GENERAL THEORY OF THE EFFECT

The quantitative expressions for the case of an arbitrary spectrum of the electrons can be obtained from the equation for the density with the aid of a diagram technique [1]; however, inasmuch as the discussion is of a new physical effect, we shall, for the sake of a better understanding, limit ourselves in the present work to a very simple derivation for the case of an isotropic quadratic spectrum for the electrons.

The kinetic equation for the phonons expresses

the fact that a change in the phonon distribution function per unit time (as a consequence of the emission and absorption of phonons by electrons), which brings about an electric current, is compensated by a change in the distribution function as a result of the relaxation of the phonons relative to the electrons and other scatterers. We shall estimate the relaxation of the phonons in the "relaxation time approximation." As regards the term expressing their entrainment, which is associated with the electrons, it can be easily obtained by starting out from the known formula for the electric current:[1,2]

$$j_1 = \frac{2\pi e}{V\hbar^2 T} \sum_{\alpha\beta q} |J_{\beta\alpha}|^2 |C_q|^2 N_q n_\alpha (1 - n_\beta) \, \delta \, (\omega_{\alpha\beta} + \omega_q) \, X_{\beta\alpha}^2 eE. \tag{1}$$

Here α and β are the aggregates of quantum numbers of the electron in the homogeneous magnetic field (with account of spin), n_α is the equilibrium Fermi function, N_q is the Planck function, ω_q is the phonon function, $J_{\beta\alpha}$ is the matrix element of the operator $e^{iq \cdot r/\hbar}$ (q is the momentum of the phonon), $X_{\beta\alpha} = X_\beta^0 - X_\alpha^0$ is the shift in the center of the oscillator upon transition from the state α to the state β, T is the temperature in energy units, and V is the normalized volume; the quantity C_q characterizes the interaction of the electrons with the phonons; for acoustic phonons, it has the form

$$C_q = E_0 \sqrt{qa^3/MsV},$$

where E_0 is the deformation potential, M is the mass of the elementary cell, s is the sound velocity, and a is the lattice constant.

Let g_q be the deviation of the phonon distribution function from its equilibrium value. Then the change in the distribution function as the result of emission and absorption of phonons by electrons is

$$(\partial g_q/\partial t)_e = g_q/\tau_{pe} + g_q/\tau_p. \tag{2}$$

The contribution to the current from the interaction of the electrons with phonons of momentum q can be treated as the difference between the number of absorbed and emitted phonons per unit time and per unit volume, multiplied by $eX_{\beta\alpha}$ (the latter is independent of the indices β and α, as will be shown below). Then, from (1),

$$\left(\frac{\partial g_q}{\partial t}\right)_e = -\frac{2\pi e}{\hbar^2 T} \sum_{\alpha\beta} |J_{\beta\alpha}|^2 |C_q|^2 N_q n_\alpha \, (1 - n_\beta) \, \delta$$

$$\times \, (\omega_{\alpha\beta} + \omega_q) \, X_{\beta\alpha}E. \tag{3}$$

The relaxation time of the phonons relative to

the equilibrium electrons is determined by the formula

$$\tau_{pe}^{-1} = -\frac{2\pi}{\hbar} \sum_{\alpha\beta} |C_q|^2 |J_{\alpha\beta}|^2 (n_\alpha - n_\beta) \, \delta \, (\varepsilon_\alpha - \varepsilon_\beta - \hbar\omega_q)$$

$$= \frac{2\pi}{\hbar^2} \sum_{\alpha\beta} |C_q|^2 |J_{\beta\alpha}|^2 n_\alpha \, (1 - n_\beta) \, \frac{\delta \, (\omega_{\alpha\beta} + \omega_q)}{N_q + 1}. \tag{4}$$

We now note that $|J_{\beta\alpha}|^2$ preserves the law of conservation of momentum for electron-phonon interactions:

$$|J_{\beta\alpha}|^2 = |J_{nn'}|^2 \, \delta_{p_{y\beta}, \, p_{y\alpha}+q_y} \delta_{p_{z\beta}, \, p_{z\alpha}+q_z} \tag{5}$$

$$J_{nn'} = \int \varphi_n \, (x - x_\beta^0) \, \varphi_{n'} \, (x - x_\alpha^0) \, e^{iq_x x/\hbar} \, dx, \tag{6}$$

where $\varphi_n(x - x^0)$ are the normalized wave functions of the oscillator. Then $X_{\beta\alpha}$ does not depend on the indices α and β:

$$X_{\beta\alpha} = cq_y/eH. \tag{7}$$

Substituting (7) and (3) in (2), we get

$$N_q (N_q + 1) \frac{E}{H} \frac{cq_y}{T\tau_{pe}} + \frac{g_q}{\tau_{pe}} + \frac{g_q}{\tau_p} = 0, \tag{8}$$

whence

$$g_q = -\frac{\tau_p}{\tau_p + \tau_{pe}} N_q \, (N_q + 1) \frac{E}{H} \frac{cq_y}{T}. \tag{9}$$

We now proceed to the calculation of the transverse electric current. It consists of the component (1), which is not connected with the entrainment, and a second component brought about by the absorption and emission of phonons, which forms (in its non-equilibrium part) a current along the y axis, as has already been pointed out. The change in the momentum p_y of the electron in such an interaction also creates an additional current.

The number of electron transitions from the state α to the state β, due to interaction with phonons of momentum q, is equal to

$$-\frac{2\pi}{\hbar} |C_q|^2 |J_{\alpha\beta}|^2 \, [n_\alpha \, (1 - n_\beta) \, (N_q + g_q + 1)$$

$$- n_\beta \, (1 - n_\alpha) \, (N_q + g_q)] \, \delta \, (\varepsilon_\alpha - \varepsilon_\beta - \hbar\omega_q). \tag{10}$$

Since the number vanishes for equilibrium of the phonons, we have only terms with g_q left, so that the number of transitions is:

$$-\frac{2\pi}{\hbar} |C_q|^2 |J_{\alpha\beta}|^2 \, (n_\alpha - n_\beta) \, \delta \, (\varepsilon_\alpha - \varepsilon_\beta - \hbar\omega_q) g_q.$$

Multiplying by $eX_{\beta\alpha}/V$ and summing over all α, β, and q, we obtain the record part of the current:

$$j_2 = \frac{2\pi e}{\hbar^2 V} \sum_{\alpha\beta q} |C_q|^2 |J_{\alpha\beta}|^2 n_\alpha \, (1 - n_\beta) \, \delta \, (\omega_{\beta\alpha} - \omega_q) \, X_{\beta\alpha} \frac{g_q}{N_q + 1}. \tag{11}$$

Combining (11) and (1), we get the total current

$$j = j_1 + j_2 = \frac{2\pi e}{V\hbar^2 T} \sum_{\alpha\beta q} |J_{\beta\alpha}|^2 |C_q|^2 N_q n_\alpha (1 - n_\beta)$$

$$\times \delta(\omega_{\alpha\beta} + \omega_q) X_{\beta\alpha} \left(eE X_{\beta\alpha} + \frac{T}{N_q(N_q + 1)} g_q \right), \qquad (12)$$

which is identical with the result obtained in[1].

By using (3) and (9), one easily gets

$$\sigma_p = \frac{c^2}{H^2 T V} \sum_q q_y^2 N_q (N_q + 1)/(\tau_p + \tau_{pe}). \qquad (13)$$

If the phonons interact only with the electrons, i.e., if $\tau_p \rightarrow \infty$, then the transverse current vanishes, in accord with the qualitative considerations given earlier. In the opposite limiting case, when the phonons are in equilibrium ($\tau_p \ll \tau_{pe}$), (13) transforms into (1).

In the classical case ($\hbar\omega \ll \varphi$, where φ is the characteristic energy), Eq. (13) can be obtained by solution of the set of kinetic equations for phonons and electrons, when $\omega\tau \gg 1$.

3. THE ENTRAINMENT EFFECT FOR THE CASE OF DEGENERATE ELECTRONS

We limit ourselves to the case in which $\hbar\omega \ll \zeta$, where ζ is the chemical potential. In this case, the collision integral of electrons with phonons does not depend on the magnetic field, and the classical expression can be used for τ_{pe}:

$$\tau_{pe}^{-1} = -\frac{4\pi |C_q|^2 V}{\hbar (2\pi\hbar)^3} \int d^3 p \, \delta(\varepsilon_p - \varepsilon_{p-q} - \hbar\omega_q)(n_p - n_{p-q}). \qquad (14)$$

Here, since usually $\omega > \omega_0$, where ω_0 is the frequency of the transitions between states with opposite spin orientations, the sum over the spin indices is replaced by the factor 2. Carrying out the integration over the angles by means of the δ function, we get

$$\tau_{pe}^{-1} = -\frac{1}{\pi} \frac{|C_q|^2 Vm}{\hbar^4 q} \int_{p_{min}}^{\infty} p \, [n(\varepsilon_p) - n(\varepsilon_p - \hbar\omega_q)] \, dp,$$

where $p_{min} = q/2 + ms$. Proceeding to integration over the energy, and replacing $n(\varepsilon_p) - n(\varepsilon_p - \hbar\omega_q)$ by $(dn/d\varepsilon)\hbar\omega_q$, we easily obtain

$$\tau_{pe}^{-1} = \begin{cases} (E_0 m a^2/\hbar^2)^2 \, q/\pi M a & q < 2\sqrt{2m\zeta} \\ 0 & q > 2\sqrt{2m\zeta}. \end{cases} \qquad (15)$$

We now consider the case in which the Fermi momentum of the electron is larger than the thermal momentum of the phonon. ($2\sqrt{2m\zeta} > T/s$). Transforming in (13) from a sum over q to an integration, we get

$$\sigma_p = \frac{c^2}{H^2 T (2\pi\hbar)^3} \int d^3q \cdot q_y^2 N_q \frac{N_q + 1}{\tau_p + \tau_{pe}}. \qquad (16)$$

Here the principal role in the integral is played by

values of q on the order of the thermal momentum of the phonon ($q_T \sim T/s$).

There are three fundamental mechanisms of non-electronic relaxation of the thermal phonons: 1) relaxation on boundaries with the characteristic time $\tau_L \sim L/s$, where L is the dimension of the specimen in the direction of the y axis, 2) relaxation on defects [3]

$$\tau_d^{-1} = \frac{\omega_D}{10} \left(\frac{\hbar\omega_q}{\Theta} \right)^4 \left[x + x_i \left(\frac{\Delta M}{M} \right)^2 \right], \qquad (17)$$

where $\Theta = \hbar\omega_D$ is the Debye temperature and x and x_i are the relative concentrations of the impurity atoms and isotopes, respectively, and 3) relaxation with the help of umklapp processes, with a relaxation time [3]

$$\tau_t \approx \frac{1}{30} \frac{1}{\omega_D} \frac{Ms^2}{\Theta} \frac{T}{\hbar\omega_q} e^{\Theta/\alpha T}, \quad T < \Theta, \quad \alpha \gg 1. \qquad (18)$$

These mechanisms were shown in the order of their importance for an increase in temperature.

In the low temperature region, where the first mechanism dominates the phonon relaxation, the conditions $\tau_p \sim \tau_L \gg \tau_{pe}$ leads to the inequality

$$30 \frac{m}{M} \frac{L}{a} \left(\frac{E_0}{\Theta} \right)^2 \frac{ms^2}{\Theta} \frac{T}{\Theta} \gg 1,$$

which is practically always satisfied. Substituting $\tau_p = \tau_L$ in Eq. (16) and neglecting τ_{pe}, we get

$$\sigma_p \approx \left(\frac{T}{ms^2} \right)^2 \left(\frac{T}{\hbar\omega} \right)^2 \frac{e^2}{a\Theta} \frac{s}{L}. \qquad (19)$$

Thus the electrical conductivity is shown to be dependent on the dimensions of the specimen in the y direction, perpendicular to the electric and magnetic fields. On the other hand, since

$$\sigma_d \approx \sigma_d^0 (\omega\tau)^{-2} = ne^2 / m\omega^2 \tau \qquad (20)$$

(n is the electron concentration, σ_d^0 is the electrical conductivity produced by defects in the absence of a magnetic field, while

$$\tau^{-1} = xa^{-3}v\Delta \sim xv/a, \qquad (21)$$

where $\Delta \sim a^2$ is the scattering cross section, v is the Fermi velocity of the electrons), we have the relation

$$\frac{\sigma_p}{\sigma_d} \sim 10 \left(\frac{T}{\Theta} \right)^4 \frac{1}{(na^3)^{4/3}} \frac{a}{Lx}. \qquad (22)$$

In very pure semimetals at the low temperatures which are necessary for the appearance of the effect of phonon scattering on boundaries, the relation (22) can be shown to be larger than unity, so that the entrainment effect can be measured.

A still more important effect is seen in the case in which the phonons relax through the agency of umklapp processes at temperatures below the

Debye temperature. The ratio of the relaxation times is, according to (15) and (18), equal to

$$\frac{\tau_p}{\tau_{pe}} \sim 10^{-2} \frac{T}{\Theta} \left(E_0 \frac{ma^2}{\hbar^2} \right)^2 e^{\Theta/2T},$$

which can be larger than unity. Here, we get for σ_p:

$$\sigma_p \sim 10^2 \frac{e^2}{a\hbar} \frac{\Theta}{Ms^2} \left(\frac{T}{\hbar\omega} \right)^2 \left(\frac{T}{ms^2} \right)^2 e^{-\Theta/2T}, \tag{23}$$

while, by (20)—(21),

$$\frac{\sigma_p}{\sigma_d} \sim 10^3 \frac{T}{Ms^2} \left(\frac{T}{\Theta} \right)^3 e^{-\Theta/2T} \frac{1}{(na^3)^4 sx}.$$

In this case of not very contaminated materials, the latter ratio is also much larger than unity. It is interesting to note that Eq. (23) gives the exponential temperature dependence of the electrical conductivity.

A similar analysis can also be carried out for the case in which the phonons relax on defects of an atomic scale. The ratio of the relaxation times shows that even in this case, the entrainment of the phonons is quite considerable.

However, if the phonons relax on impurities, then the phonon electrical conductivity is shown to be smaller than the impurity conductivity (or commensurate with it). In this case, one falls off along with the other upon decrease in the concentration of impurities. If the phonons relax on isotopes, then, the phonon electrical conductivity can predominate over the impurity conductivity in low contamination materials with a large quantity of isotopes. Here $\sigma_p \sim T^8$, while, without account of entrainment, $\sigma_p \sim T^5$.

Now let us consider the case in which $\sqrt{2m\zeta} < T/s$. In this case, all the phonons that interact with the electrons are subthermal. The most effective relaxation mechanism of the subthermal phonons is the relaxation on normal phonons. In this case, the electrons entrain both the transverse and longitudinal phonons.

For semiconductors and semimetals, the energy of interaction of the electrons with phonons can be written in the form $\Sigma E_{ik} u_{ik}$, where u_{ik} is the tensor of the interaction constant. In the general case of such anisotropic interaction, the coupling of the electrons with transverse and longitudinal phonons is identical in order of magnitude. This is precisely the situation in such crystals as bismuth, germanium, silicon and, perhaps, in most real conductors with which experimenters deal.

A different situation holds for crystals of the type InSb; here the electron spectrum is isotropic and, as a result of the cubic symmetry, the tensor E_{ik} reduces to a scalar. In these crystals, the electrons interact with longitudinal phonons more

strongly than with the transverse. However, keeping in mind the very special situation and the estimated character of our entire investigation, we shall regard the coupling of the electrons with transverse and longitudinal phonons as comparable in magnitude, even though we took the isotropic character of the electron spectra into account. For order-of-magnitude estimates, this cannot lead to a contradiction.

Thus, for not too low temperatures, one need take only the transverse phonons into account, inasmuch as they relax on thermal phonons more rapidly and, consequently, they make a larger contribution to the electrical conductivity. The relaxation time of the subthermal transverse phonons is, according to Landau and Rumer,[3,4]

$$\tau_\perp \approx \frac{1}{10} (aM / q)(\Theta / T)^4. \tag{24}$$

According to (15) and (24),

$$\tau_\perp / \tau_{pe} \approx \frac{1}{30} (\Theta / T)^4 (E_0 ma^2 / \hbar^2)^2.$$

For $T < \Theta$, this ratio can be large.

Substituting $\tau_\perp$ from (24) in (16), in place of τ_p, and neglecting τ_{pe}, it is easy to calculate the electrical conductivity. In this case, one must replace N_q and $N_q + 1$ by T/sq, and integrate over q up to $q_{max} = 2\sqrt{2m\zeta}$. As a result, we get

$$\sigma_p \approx \frac{e^2}{\hbar a} \left(\frac{\zeta}{\hbar\omega} \right)^2 \frac{T}{Ms^2} \left(\frac{T}{\Theta} \right)^4.$$

Thus $\sigma_p \sim T^5$ while, without calculation of the entrainment, $\sigma_p \sim T$. For crystals of a high degree of purity, $\sigma_d < \sigma_p$.

4. THE ENTRAINMENT EFFECT FOR NONDEGENERATE ELECTRONS

We limit ourselves to the quantum case ($\hbar\omega \gg T$) (in the classical case, the effect is clearly absent).

We begin by calculating τ_{pe}. In this case, (3) can be put in the form

$$\tau_{pe}^{-1} = \frac{4\pi}{\hbar^2} \frac{|C_q|^2}{N_q + 1}$$

$$\times \sum_{\alpha\beta} n_\alpha \delta (\omega_{\beta\alpha} - \omega_q) \delta_{p_{y\beta}, p_{y\alpha} + q_y} \delta_{p_{z\beta}, p_{z\alpha} - q_z} |J_{nn'}|^2.$$

Here it is taken into account that $\hbar\omega_0 \ll T$. For $\hbar\omega_0 \gg T$, the factor 4 is replaced by 2.

We transform from summation over p_y and p_z to integration, and set $n = n' = 0$:

$$\tau_{pe}^{-1} = \frac{4\pi}{\hbar^2} \frac{|C_q|^2}{N_q + 1} \int dp_y dp_z n_\alpha \delta (\omega_{\beta\alpha} - \omega_q) |J_{00}|^2 \frac{L_y L_z}{(2\pi\hbar)^2}.$$

Integration over p_y gives the factor $L_x m\omega$ (L_x, L_y, L_z are the linear dimensions of the normalized volume V):

$$\tau_{pe}^{-1} = \frac{4\pi}{\hbar} \frac{eH}{c} \frac{V |C_q|^2}{(N_q + 1)(2\pi\hbar)^2} \int dp_z \exp\left\{\frac{1}{T}\left(\zeta' - \frac{p_z^2}{2m}\right)\right\} |J_{00}|^2$$

$$\times \delta(\varepsilon_{p_z+q_z} - \varepsilon_{p_z} - \hbar\omega_q),$$

where

$$\zeta' = \zeta - \hbar\omega/2 = T \ln [\pi \sqrt{2\pi}n\hbar^2 / m^{3/2}T^{1/2}\omega],$$

$$|J_{00}|^2 = \exp(-q_\perp^2 / q_H^2), \quad q_\perp^2 = q_x^2 + q_y^2, \quad q_H^2 = 2eH\hbar/c.$$

We shall now assume that

$$q_H^2 / q_T^2 = 2 (ms^2 / T)(\hbar\omega/T) \ll 1,$$

i.e., that the magnetic field is not "superpowerful."[5] Then $N_q + 1 \approx T/sq$ and, after integration over p_z, we get

$$\tau_{pe}^{-1} = \frac{V\sqrt{2\pi}}{\hbar} \frac{E_0^2 q^2 m^{1/2} n a^3}{M q_z T^{3/2}} \exp\left(-\frac{q_\perp^2}{q_H^2} - \frac{q_z^2}{8mT}\right). \quad (25)$$

We set $\tau_p = Aq^{-t}$ and substitute this quantity, along with (25), in (16). We introduce cylindrical coordinates with the axis along q_z and integrate over the angles. Moreover, since $q_H > \sqrt{8mT}$, we can assume $q \approx q_\perp$. By introducing the dimensionless variables $\xi = q_z/\sqrt{8mT}$ and $\eta = q_\perp/q_H$, we obtain

$$\sigma_p = \frac{c^2}{s^2} \frac{T q_H^{t+2} \sqrt{8mT}}{H^2(2\pi\hbar)^3 A} \int \frac{\eta^{t+1}d\xi d\eta}{1 + C\xi\eta^{t-2}\exp(\xi^2 + \eta^2)}, \quad (26)$$

where C is the ratio of the coefficient of the exponential term in τ_{pe} to τ_p for $\xi = \eta = 1$. If C is large ($C \gg 1$), then the entrainment effect is absent [the unity term in the denominator of Eq. (26) can be neglected]. If $C \ll 1$, on the other hand, then the factor before the integral in (26) gives an order-of-magnitude estimate of the transverse electrical conductivity, while the integral depends logarithmically on C.

For low temperatures, where the scattering of phonons on the boundary ($A = L/s$ and $t = 0$) plays an important role, we have

$$\sigma_p \approx \frac{1}{100} \frac{e^2}{L\hbar} \frac{T}{\hbar\omega}\left(\frac{T}{ms^2}\right)^{1/2}. \quad (27)$$

For the quantity C,

$$C = \left(\frac{T}{E_0}\right)^2 \frac{s}{\omega L} \frac{M}{m} \frac{1}{na^3}. \quad (28)$$

It is seen from (28) that the entrainment effect increases with increasing electron concentration and with decreasing temperature. In semiconductors, however, the scattering by ionized impurities increases in this case. For estimates of the latter, one can make use of the usual formula[1]

$$\sigma_i = \frac{4\pi eN}{\hbar^2 T} \sum_{\alpha\beta q} |V_{ed}|^2 n_\beta(1 - n_\alpha) \delta(\omega_{\alpha\beta}) |J_{\beta\alpha}|^2 eX_{\beta\alpha}^2, \quad (29)$$

where

$$|V_{ed}|^2 = 16\pi^2 e^4 \hbar^4 / V^2 \varepsilon^2 (q^2 + \varkappa^2)^2,$$

N is the concentration of the impurities, $\varkappa$ is the momentum corresponding to the Debye screening radius, and ϵ is the dielectric constant.

Assuming that $q_H \gg \varkappa$, and summing (29) in the same fashion as was done above, one can obtain the following result by neglecting the logarithmic divergence that occurs in the integral:

$$\sigma_i \approx nNe^6/(mT)^{3/2}\omega^2\varepsilon^2. \quad (30)$$

Then the ratio of (30) to (27) takes the form

$$\frac{\sigma_i}{\sigma_p} \approx \frac{100}{\varepsilon^2}(nNa^6)\frac{e^4}{a^2T^2}\frac{L}{a}\left(\frac{q_H a}{\hbar}\right)^2\frac{\theta}{T}. \quad (31)$$

Equations (28) and (31) show that the entrainment effect can take place in magnetic fields of the order of tens of kilo-oersteds, at temperatures $\sim 10°K$, and in concentrations of electrons and impurities $N \sim n \sim 10^{14}$ cm^{-3}. Here, in accord with (27), $\sigma_p \sim 1/H$, while the transverse magnetoresistance $\rho_{xx} \approx \sigma/\sigma_{xy}^2 \sim H$. The fact that the concurrent mechanism—scattering by ionized impurities—gives a weak (logarithmic) dependence $\rho_{xx}(H)$ should make possible the experimental observation of this effect.

It is an important conclusion of our theory that the ratio of the electrical conductivity, brought about by the ionized impurities, to the phonon electrical conductivity (31) is proportional to the square of the concentration, while in the theory which does not take entrainment into account, the ratio is proportional to the first power of the concentration. This means that for high concentrations (when $C \ll 1$), the transition from the phonon electrical conductivity to the impurity conductivity occurs at much higher temperatures than follows from the theory which does not take the entrainment into account.

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