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INTERACTION OF γ QUANTA WITH ORIENTED NONSPHERICAL NUCLEI

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The results of an analysis of the possibilities for observing the effects of optical anisotropy by available experimental techniques are presented. The influence of the resolution in the measurement of the energy of the scattered photons and of the degree of the nuclear orientation on these effects is studied. Some consequences of the possible nonaxiality of nuclei are briefly discussed.

1. INTRODUCTION

A number of papers[1-7] have been recently devoted to the discussion of the different effects connected with the optical anisotropy of atomic nuclei. The term "optical anisotropy" signifies that the interaction of photons with the nucleus depends on the orientation of the nuclear spin relative to the wave vector of the photon and that the electric dipole polarizability thus has tensor character.[1] The term "optical anisotropy of nuclei" has been introduced to emphasize the far reaching analogy between the phenomena of the interaction of photons with nuclei and of molecular optics.[5] The investigation of molecular Raman spectra yields considerable amounts of information concerning their structure. One can hope that the investigation of the phenomena associated with the optical anisotropy of nuclei will add to our knowledge of the structure and characteristics of nuclei.

The parameters of the optical anisotropy are nuclear characteristics as basic as, say, the quadrupole or the magnetic moments. These parameters depend very sensitively on the characteristics of the nuclear models. The presently available experimental data on the optical anisotropy of nuclei are very poor. An analysis of the parameters on the basis of a model is thus rather difficult. Besides the experiments by Fuller and Hayward on Ho^{67} and Er^{68} (private communication by A. M. Baldin), where effects associated with the optical anisotropy were indirectly observed, there is no strong experimental proof for the existence of this important property in nuclei. However, the mentioned experiments and the existing theoretical investigations[3] present such strong arguments in favor of the existence of the optical anisotropy in nuclei that we shall be concerned only with the question of the quantitative determination of the relevant parameters.

As has been already pointed out by Baldin[4] a definite experimental proof could be obtained by studying the interaction of γ rays with oriented nuclei. An investigation of the absorption and of elastic and Raman scattering of photons on oriented nuclei can yield information sufficient to determine completely the parameters of the optical anisotropy. However, up to now this question has not been investigated experimentally at all and theoretically only in general terms (see e.g. [3,4], where the elastic scattering on oriented nuclei is treated). Unfortunately, the presently available experimental techniques do not allow the investigation of pure elastic scattering. The energy resolution is such that the elastically scattered photons cannot be separated from those inelastically scattered photons which excite the low lying states. The experiments of Fuller and Hayward are of this kind. However, the questions associated with the influence of the accuracy of the present day experiments and of the degree of the nuclear orientation on the effects of the optical anisotropy have not yet been investigated.

The aim of this paper is the investigation of the capabilities of the present day experiments concerning the observation of the effects of the optical anisotropy. The summary effect of elastic and inelastic photon scattering on oriented nuclei will be studied. From the effective cross section of this process one can deduce the influence of the energy resolution employed in the measurement of the scattered photons and of the degree of orientation on the effects of the optical anisotropy. On the basis of the model of the nonaxial nucleus developed by A. S. Davydov and co-workers, we consider briefly certain effects associated with the possible existence of nonaxial deformation of nuclei.

2. SCATTERING OF γ RAYS ON ORIENTED NUCLEI

It was mentioned above that in the present day experiments on elastic photon scattering on nuclei only an effective cross section can be obtained, with contributions both from purely elastically scattered photons and from those inelastic scattering events that excitate the low lying levels. It is well established experimentally that in highly deformed nuclei the lowest levels are connected with collective excitations which correspond, in particular, to rotations of the nucleus as a whole. It has been shown earlier[6] that the cross section of the inelastic scattering of photons associated with the excitation of rotational levels (the nuclear Raman scattering) can be described in terms of the parameters of the optical anisotropy, namely, the tensor and the vector polarizabilities. The combined effect of elastic and Raman scattering is thus determined by the same parameters (the optical anisotropy parameters) as the pure elastic scattering.

We consider now the combined effect of scattering on oriented axially symmetric nuclei. The scattering matrix for this process can be written in the form[4,6]

$$\langle mJ\boldsymbol{\lambda} | R | m'J'\boldsymbol{\lambda}'\rangle = \Big\{ c^0 (-1)^{\nu} \delta_{\mu,-\nu} \delta_{mm'} \delta_{JJ'}$$
$$+ \sqrt{\frac{2J'+1}{2J+1}}\, i(1J'0K | JK)$$
$$\times (1J'\mu + \nu m' | Jm) \frac{(11\,\mu\nu | 1\,\mu+\nu)}{(11-11 | 10)} c^v$$
$$+ \sqrt{\frac{2J'+1}{2J+1}} (2J'0K | JK)(2J'\mu + \nu m' | Jm)$$
$$\times \frac{(11\mu\nu | 2\mu+\nu)}{(1100 | 20)} c^t \Big\} (-1)^{\mu+\nu} \lambda'_{-\mu} \lambda_{-\nu} \frac{1}{2\pi} \left(\frac{\omega}{c}\right)^3;$$
$$c^0 = c^s - e^2Z^2/AM\omega^2. \qquad (1)$$

(It is assumed here that the wave function of an axially symmetric nucleus can be written in the form[8]

$$|JmK\rangle = \sqrt{\frac{2J+1}{16\pi^2}} \left(D^J_{mK}(\theta)\, \chi_K^{\tau}(q) + (-1)^{J-j} D^J_{m-K}(\theta)\, \chi^{\tau}_{-K}(q)\right).$$

Furthermore, the condition $\Delta\omega/\omega \ll 1$ was used, where $\Delta\omega$ is the frequency change of the photon on scattering.) In Eq. (1) J and J′ are the nuclear spins and m and m′ their projections on the z axis; $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'$ are the photon polarization vectors. The unprimed and primed quantities refer to the state before and after the scattering respectively. (1J′ 0K|JK) etc are Clebsch-Gordan coefficients. The quantity

$$\frac{1}{2\pi}\left(\frac{\omega}{c}\right)^3 \frac{e^2Z^2}{AM\omega^2}$$

is the amplitude of Thomson scattering on a nucleus of charge Z. c^s, c^v, c^t are the scalar, vector and tensor polarizabilities respectively.

The effective cross section for the combined scattering is obtained from (1) by means of the relation

$$d\sigma/d\Omega = (2\pi c/\omega)^2 \,\mathrm{Sp}\, R\rho R^+. \qquad (2)$$

Here ρ is the density matrix describing the state of the target nucleus.

Inserting the expression for the R-matrix into (2), averaging over the polarizations of the incoming photons, summing over the polarizations of the scattered photons and over the final-state spins and their projections, we obtain an expression for the effective cross section of the summary process of the scattering of unpolarized photons on oriented nuclei:

$$\left(\frac{d\sigma}{d\Omega} - \frac{d\sigma^{\mathbf{u}}}{d\Omega}\right) 2 \left(\frac{\omega}{c}\right)^{-4}$$
$$= \frac{2}{J+1} \mathrm{Re}\left[\left(\frac{1}{5} c^{t*} - c^{0*}\right) c^v\right] (\mathbf{k}'\mathbf{k})\, \overline{(\mathbf{J}[\mathbf{k}'\mathbf{k}])}$$
$$+ \frac{1}{(J+1)(2J+3)} \Big\{\Big[\mathrm{Re}\,(c^{0*}c^t) + \frac{2}{3}|c^v|^2$$
$$- \frac{1}{14}|c^t|^2\Big] [3\, \overline{(\mathbf{J}[\mathbf{k}'\mathbf{k}])^2} - J(J+1)[\mathbf{k}'\mathbf{k}]^2]$$
$$+ \Big[-\mathrm{Re}\,(c^{0*}c^t) + \frac{2}{3}|c^v|^2 - \frac{5}{14}|c^t|^2\Big][3\,\overline{(\mathbf{J}\mathbf{k}')^2}$$
$$+ 3\,\overline{(\mathbf{J}\mathbf{k})^2} - 2J(J+1)]\Big\} - \frac{6}{5} \frac{\mathrm{Re}\,(c^{t*}c^v)}{(J+1)(J+2)(2J+3)}$$
$$\times \Big\{5S\,\overline{(\mathbf{J}\mathbf{k}')(\mathbf{J}\mathbf{k})(\mathbf{J}[\mathbf{k}'\mathbf{k}])} - \Big[J(J+1) - \frac{1}{3}\Big](\mathbf{k}'\mathbf{k})\overline{(\mathbf{J}[\mathbf{k}'\mathbf{k}])}\Big\}$$
$$+ \frac{18}{35}|c^t|^2 \frac{1}{2(J+1)(2J+3)(J+2)(2J+5)}$$
$$\times \Big\{35\, \overline{S(\mathbf{J}\mathbf{k}')(\mathbf{J}\mathbf{k}')(\mathbf{J}\mathbf{k})(\mathbf{J}\mathbf{k})} -$$
$$- 5\left(J^2 + J - \frac{5}{6}\right) [\overline{(\mathbf{J}\mathbf{k}')^2 + (\mathbf{J}\mathbf{k})^2 + 2(\mathbf{J}\mathbf{k}')(\mathbf{J}\mathbf{k})(\mathbf{k}\mathbf{k}')} +$$
$$+ 2\,\overline{(\mathbf{J}\mathbf{k})(\mathbf{J}\mathbf{k}')(\mathbf{k}'\mathbf{k})}]$$
$$+ J(J-1)(J+1)(J+2)[1 + 2(\mathbf{k}'\mathbf{k})^2]\Big\}. \qquad (3)^*$$

Here **k** and **k′** are unit wave vectors of the photons before and after scattering, respectively, and S means a symmetrized sum [the quantity $S(a_1, a_2, \ldots a_n)$ is the sum of all permutations of the product $a_1, a_2, \ldots a_n$ divided by n!]. Mean values of the type $\overline{(\mathbf{J}\cdot\mathbf{k}\times\mathbf{k}')^2}$ are calculated with the density matrix in the usual way

$$\overline{(\mathbf{J}[\mathbf{k}'\mathbf{k}])^2} = \mathrm{Sp}(\mathbf{J}[\mathbf{k}'\mathbf{k}])^2\rho.$$

The quantity

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*$(\mathbf{k}'\mathbf{k}) = (\mathbf{k}\cdot\mathbf{k})$, $[\mathbf{k}'\mathbf{k}] = [\mathbf{k}'\times\mathbf{k}]$, $(\mathbf{J}[\mathbf{k}'\mathbf{k}]) \rightarrow (\mathbf{J}\cdot\mathbf{k}'\times\mathbf{k})$.

$$\frac{d\sigma^{u}}{d\Omega} = \frac{1}{2}\left(\frac{\omega}{c}\right)^4 \left\{|c^0|^2 (1 + (\mathbf{k}'\mathbf{k})^2) + \frac{1}{3}|c^v|^2 (3 - (\mathbf{k}'\mathbf{k})^2) + \frac{1}{20}|c^t|^2 (13 - (\mathbf{k}'\mathbf{k})^2)\right\}$$

is the summary cross section for the scattering of unpolarized photons on unoriented nuclei, the quantity which has been measured by Fuller and Hayward.

In order to estimate the effects connected with the optical anisotropy of nuclei we evaluate the quantity α in analogy with [4]:

$$\alpha \frac{d\sigma^{u}}{d\Omega} \equiv \frac{d\sigma}{d\Omega}(\mathbf{k}' = \mathbf{e}_x, \mathbf{k} = \mathbf{e}_y, [\mathbf{k}'\mathbf{k}] = \mathbf{e}_z) - \frac{d\sigma}{d\Omega}(\mathbf{k}' = \mathbf{e}_z, \mathbf{k} = \mathbf{e}_y), [\mathbf{k}'\mathbf{k}] = -\mathbf{e}_x). \quad (4)$$

Inserting the values for the cross section from (3) and putting $c^v = 0$ (as has been shown[4] the value of c^v is much smaller than c^0 and c^t) we find for fully polarized nuclei ($\rho_{mm'} = \delta_{mJ}\delta_{m'J}$) the following expression for α:

$$\alpha = \left\{3\left[\mathrm{Re}(c^{0*}c^t) + \frac{1}{7}|c^t|^2\right]\frac{J(2J-1)}{(J+1)(2J+3)} + \frac{9}{28}|c^t|^2 \times \frac{J(2J-1)(J-1)(2J-3)}{(J+1)(2J+3)(J+2)(2J+5)}\right\} \Big/ \left[|c^0|^2 + \frac{13}{20}|c^t|^2\right]. \quad (5)$$

In order to obtain a numerical estimate for α one has to know the optical anisotropy parameters c^0 and c^t. We use the expressions for the tensor and scalar polarizabilities given in [3] and find for the case where the nuclear spin equals $^7/_2$:

$$\alpha(\omega = \omega_1, c^t \approx 2c^0) = 1.04,$$
$$\alpha(\omega = \omega_2, \quad c^t \approx -c^0) = -0.7.$$

On the other hand, for the case of pure elastic scattering, α has the values

$$\alpha_{el} \ (\omega = \omega_1, \ c^t \approx 2c^0) = 1.5,$$
$$\alpha_{el} \ (\omega = \omega_2, \ c^t \approx -c^0) = -0.9.$$

One thus can conclude that the observable effects associated with the nuclear optical anisotropy are somewhat smaller if one measures the combined elastic and nuclear Raman scattering, as compared with the measurement of pure elastic scattering. However, experiments of this kind have the advantage that they yield direct evidence on the internal parameters of the optical anisotropy as can be seen from (3) and (5).

The above analysis of the consequences of the impossibility of observing purely elastic scattering from oriented nuclei with present-day techniques was based on (3) under the condition that the nuclei are fully oriented. It is experimentally impossible, however, to achieve full nuclear orientation. This must obviously lead in itself to a decrease in the magnitude of the observed effect. The influence of the degree of nuclear orientation can also be evaluated by (3), which is valid for any kind and degree of nuclear orientation. One just has to specify the density matrix appropriate to the particular case. We shall consider only the frequently applicable case when the density matrix is of the form $\rho_{mm'} = f(m)\delta_{mm'}$ (this case corresponds to a system having z as an axis of symmetry.

With this density matrix one can write (3) for a sufficiently high degree of nuclear orientation in a very simple and convenient way:

$$\left(\frac{d\sigma}{d\Omega} - \frac{d\sigma^{u}}{d\Omega}\right) 2\left(\frac{\omega}{c}\right)^{-4} \approx [-2\mathrm{Re}(c^{0*}c^v) + \frac{2}{5}\mathrm{Re}(c^{t*}c^v)](\mathbf{k}'\mathbf{k})[\mathbf{k}'\mathbf{k}]_z \frac{\overline{m}}{J+1} + [\mathrm{Re}(c^{0*}c^t) + \frac{2}{3}|c^v|^2 - \frac{1}{14}|c^t|^2]\left(\frac{3}{2}[\mathbf{k}'\mathbf{k}]_z^2 - \frac{[\mathbf{k}'\mathbf{k}]^2}{2}\right)\frac{3\overline{m^2} - J(J+1)}{(J+1)(2J+3)} + [-\mathrm{Re}(c^{0*}c^t) + \frac{2}{3}|c^v|^2 - \frac{5}{14}|c^t|^2](\frac{3}{2}k_z'^2 + \frac{3}{2}k_z^2 - 1) \times \frac{3\overline{m^2} - J(J+1)}{(J+1)(2J+3)}. \quad (6)$$

From (6) one sees immediately that for such a system the effects connected with the vector polarizability are determined by the orientation parameter $f_1 = \overline{m}/(J+1)$, while effects of the tensor polarizability are similarly associated with the orientation parameter $f_2 = [3\overline{m^2} - J(J+1)]/J(2J-1)$.

The highest value for f_2 which can be achieved experimentally is 0.4 to 0.5. We thus obtain for the observable effects associated with the tensor polarizability at incomplete nuclear orientations ($f_2 = 0.5$) the values

$$\alpha(\omega = \omega_1, c^t \approx 2c^0) = 0.5, \qquad \alpha(\omega = \omega_2, c^t \approx -c^0) = -0.3.$$

The following important circumstances have to be pointed out. As can be seen from (6) one can investigate the effects of the nuclear vector polarizability only with polarized nuclei while the effects of the tensor polarizability can be observed both with polarized and with aligned nuclei. We also note that the effects of the optical anisotropy are largest at a scattering angle of 90°.

It was above assumed that K is a good quantum number with K = J. This is only approximately true.[8,9] Admixture of states with $K \neq J_0$ can, generally speaking, decrease the magnitude of the effects of the optical anisotropy in the described experiment. However, this decrease will be insignificant since direct calculations, analogous to those of Bohr,[8] show that in the ground state K differs from J by at most a few percent. Thus the most important factors that tend to decrease the magnitude of the observable effects of the optical anisotropy in scattering experiments on oriented nuclei are the incomplete orientation of the nuclei and the poor energy resolution of the photons.

3. INFLUENCE OF THE POSSIBLE NONAXIALITY ON SOME CHARACTERISTICS OF NUCLEAR OPTICAL ANISOTROPY

In conclusion we shall say a few words on the interaction of photons with nonaxial nuclei. Such a model has been developed by Davydov and coworkers.[10] Generally speaking the operator of the tensor polarizability must be characterized by two independent parameters, since it is a symmetric tensor of second rank with zero trace. We consider all the quantities in a system of coordinates fixed with respect to the nucleus (we recall that the low lying nuclear excited states are supposed to be rotational states). We can then choose as the parameters, for example, the quantities $\langle\alpha^t_{zz}\rangle \equiv c^t$ and $\langle\alpha^t_{xx} - \alpha^t_{yy}\rangle$ (α^t_{ik} is the tensor part of the electric dipole polarizability operator). For the above-considered case of a strongly deformed axially symmetric nucleus, $\langle\alpha^t_{xx} - \alpha^t_{yy}\rangle$ equals zero and all effects of the optical anisotropy are given by only one parameter, the tensor polarizability c^t.

This is not true for a nonaxial nucleus, for which the parameter $\langle\alpha^t_{xx} - \alpha^t_{yy}\rangle$ evaluated in the nuclear coordinate system differs from zero. Clearly this parameter will lead to changes in the results obtained above. We consider, for example, the interaction of photons with even-even nonaxial nuclei for which there exist exact wave functions for the rotational states, as given by Davydov and Filippov.[10] In such nuclei the tensor polarizability does not manifest itself in elastic scattering processes. However one can attempt to observe it in the nuclear Raman scattering in which rotational levels are excited.[3 6] Utilizing the wave function of the rotational states of even-even nonaxial nuclei[10] and employing a procedure similar to one used previously,[6] one can show that the effective cross section for inelastic photon scattering under excitation of the states 2_1^+ and 2_2^+ is given by the expression

$$\left(\frac{d\sigma}{d\Omega}\right)_{0^+\to 2_i^+} = \frac{1}{40}\left(\frac{\omega}{c}\right)^4 \left| a_i(\gamma) + \frac{b_i(\gamma)}{\sqrt{3}} \frac{\langle \alpha^t_{xx} - \alpha^t_{yy}\rangle}{c^t}\right|^2 |c^t|^2 (13 + \cos^2\theta). \quad (7)$$

Here i = 1, 2; the quantities c^t and $\langle\alpha^t_{xx} - \alpha^t_{yy}\rangle$ can be calculated by means of the wave functions that describe the internal nuclear state.

We use for estimating $\langle\alpha^t_{xx} - \alpha^t_{yy}\rangle/c^t$ the model of an anisotropic oscillator (according to this model the nucleus is considered to consist of three oscillators with frequencies $\omega_i \sim 1/R$). It can be then shown that up to the order β^2, where β is the axial deformation parameter, we have

$$\langle \alpha^t_{xx} - \alpha^t_{yy}\rangle / c^t = \sqrt{3}\,\mathrm{tg}\,\gamma. \quad (8)^*$$

[In the derivation of (8) it was assumed that the damping constants of all three oscillators are equal. This assumption is obviously true for not too large β.] Inserting (8) into (7) we find for the ratio of the cross sections

$$\left(\frac{d\sigma}{d\Omega}\right)_{0^+\to 2_2^+} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{0^+\to 2_1^+} = \left(\frac{a_2 + b_2\,\mathrm{tg}\,\gamma}{a_1 + b_1\,\mathrm{tg}\,\gamma}\right)^2. \quad (9)$$

In the region of $\gamma \approx 30°$ (condition for $\Delta\omega/\omega \ll 1$ to be true) this ratio is considerably less than unity. Thus one can conclude that in nonaxial even-even nuclei the cross section for excitation of level 2_2^+ in a scattering process is much smaller than that for the level 2_1^+.

Since the wave functions of nonaxial odd-A nuclei are unknown, it is at present impossible to make any predictions on the influence of the nonaxiality on interactions of photons with oriented nuclei.

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*tg = tan.