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AN ANALYSIS OF 9 BeV *PROTON-NUCLEON INTERACTIONS IN NUCLEAR EMULSION*

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The dependence of the angular and energy characteristics of proton-nucleon interactions on the observed multiplicity is investigated. An approximate method has been developed for this purpose, based on the assumption that the transverse-momentum distribution is constant.

1. ANGULAR DISTRIBUTION OF THE SECONDARY PARTICLES IN THE LABORATORY SYSTEM OF COORDINATES

NUCLEON-NUCLEON showers were studied in NIKFI (type R) emulsion irradiated by the 9-BeV proton beam from the proton synchrotron of the Joint Institute for Nuclear Research. As a selection criterion, we used the conditions ($n_b + n_g \le 1$, the absence of a recoil nucleus, etc.) already mentioned in the literature.[1] All events observed were grouped according to multiplicity. The number of interactions N in each group, the half-angle $\theta_{1/2}$, and the mean geometrical angle θ_g are shown in the table. The angular distribution of the secondary particles in the laboratory system of coordinates (l.s.) is shown in Fig. 1. It follows from the data that the angular distribution of shower particles becomes somewhat wider with increasing multiplicity, which is seen most clearly by comparing the 3- and 8-prong stars.

2. ANGULAR DISTRIBUTION IN THE C.M.S.

A. It was of interest to compare the different assumptions on which the approximate transformation of the shower-particle angles from the l.s. into the c.m.s. are based. For this we have used

Type of star	N	$\theta_{1/2}$	θ_g	A ($\beta_c/\beta' = 1$)	A	$\frac{K^{\pm}}{n_s - 1.25}$	$\frac{m_2}{\mu_\pi}$
3-prong	110	$11°07'^{+2°02'}_{-1°}$	13°16′	+0.36±0.08	+0.04±0.08	0.21	⩾1.6
4-prong	53	$15°30'^{+3°}_{-2°30'}$	16°29′	+0.26±0.08	−0.08±0.10	0.16	⩾1.9
5-prong	19	$16°^{+3°12'}_{-1°}$	17°02′	+0.24±0.14	−0.04±0.14	0.13	⩾3.0
6-prong	23	$18°36'^{+2°}_{-2°}$	17°07′	+0.24±0.12	−0.18±0.12	0.17	⩾6.0
7-prong	6	$18°24'^{+3°}_{-5°}$	18°15′	−0.04±0.22	−0.20±0.22	0.16	⩾5.8
8-prong	7	$27°24'^{+3°36'}_{-3°24'}$	25°	−0.20±0.17	−0.36±0.17	0.16	⩾6.2
8-prong*	13	$26°^{+5°30'}_{-4°00'}$	26°27′	−0.12±0.07	−0.30±0.07	0.17	⩾5.6

*The last row contains the complete data on 8-prong stars found in the High-Energy Laboratory of the Nuclear Physics Institute of the Academy of Sciences, Kazakh S.S.R. (7 cases) and in the High-Energy Laboratory of the Joint Institute for Nuclear Research (6 cases).

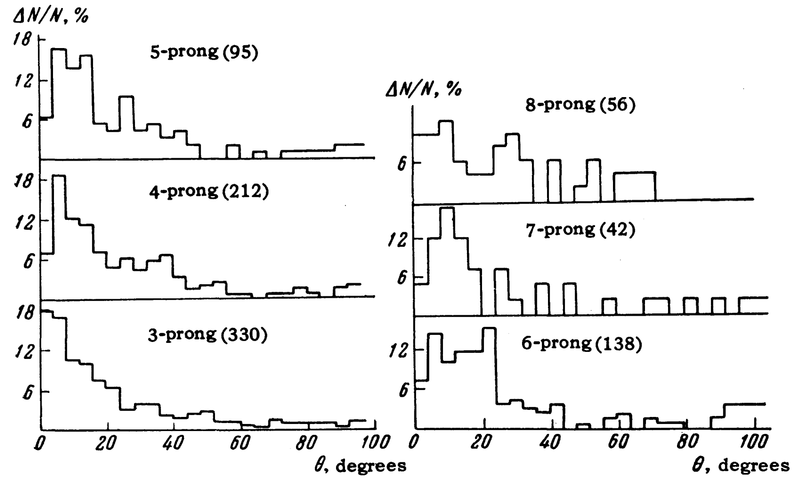

FIG. 1. Angular distribution in the l.s. of secondary particles in proton-nucleon collisions.

a group of identified particles from 3-prong stars (86 protons and 99 π mesons) [2] whose angular distribution in the c.m.s. has been obtained directly by measuring the angles and the energies in the l.s. (Fig. 2, histogram 1). For this group of particles, we have constructed also a c.m.s. distribution, assuming a constant transverse momentum [3] and assuming the c.m.s. particle velocity β' equal to the velocity β_c of the c.m.s. with respect to the l.s. (Fig. 2, histograms 2—5).

The observed transverse momentum distribution of secondary particles is found to be constant over a wide range of primary-particle energy, of the observed multiplicity, and of the target mass (Fig. 3). We therefore consider a method of angle transformation in which we postulate that the form of the shower particle transverse-momentum distribution remains constant. As a satisfactory approximation of the experimental distribution of $p_\perp$ we consider the function

$$\Delta N/N\Delta p_\perp = cp_\perp \exp(-p_\perp^2/b^2), \qquad (1)$$

which is shown in Fig. 3 for different $p_\perp$. In order to use this distribution in going over from the l.s. to the c.m.s., it is first necessary to find the correlation between the particle emission angles and the transverse momentum. The calculated coefficients of this correlation for protons and π mesons were found to be small ($r_p = r_\pi \leq 0.3$).

The variation of the mean value of $p_\perp$ with θ is shown in Fig. 2. The width of the angle intervals over which $p_\perp$ is averaged is denoted by horizontal segments. It can be seen from Fig. 4 that at small angles the mean value of $p_\perp$ tends to increase with increasing angle θ. This is explained by the influence of the momentum conservation law. (For fixed secondary-particle emission angles there is a limiting transverse momentum, which is determined by the energy of the primary nucleon.) The limiting value of $p_\perp$ increases with the angle θ,

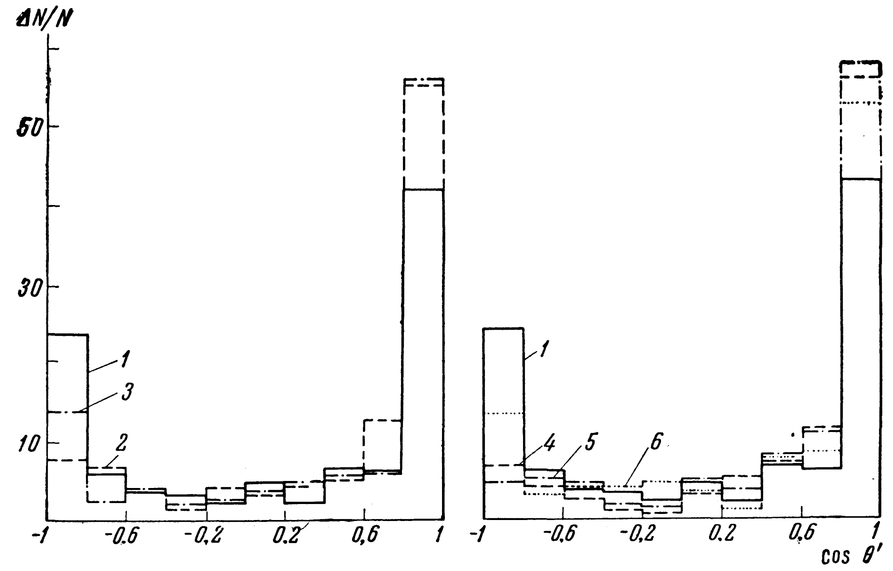

FIG. 2. Total angular distribution of protons and π mesons in the c.m.s. Histograms: 1 — experimental distribution constructed assuming that the transverse momentum is constant[3] respectively for $p_\perp = 1.6\ \mu_\pi$ (mean value), $p_\perp = 0.8\ \mu_\pi$ (most probable value), and $p_\perp \gg \mu_\pi$; histogram 5 — for β_c/β'; and 6 — taking into account the fact that the tranverse momentum distribution is constant.

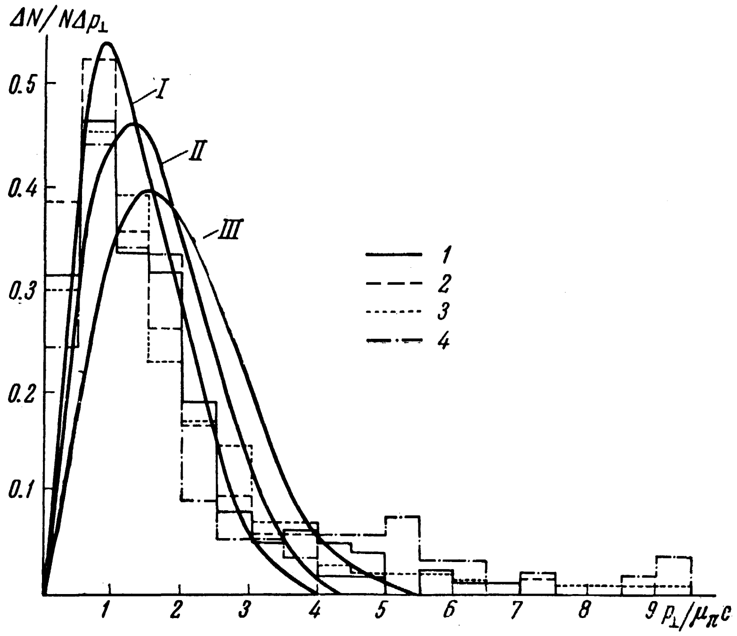

FIG. 3. Transverse momentum distribution of secondary particles: histograms 1, 3 – nucleon-nucleon interactions at $E = 10^{10}$ eV[2] and $E = 10^{11}$ eV[5] respectively; 2, 4 – nucleon-nucleus interactions at $E = 10^{10}$ eV[4] and $E = 10^{11}$ eV[6] respectively. Curves I, II, and III correspond to distributions of the type for $b = 0.630\ (\mu_\pi)^{-1}$ ($\bar{p}_\perp = 1.6\ \mu_\pi$), and $0.480\ (\theta_\pi)^{-1}$ ($\bar{p}_\perp = 1.825\ \mu_\pi$) (velocity of light is everywhere assumed to be unity).

which evidently causes a small decrease in the mean value of $p_\perp$ with decreasing θ.

With increasing multiplicity, the relative number of particles in the range of small angles decreases (see Fig. 1). Therefore, the correlation between $p_\perp$ and θ is small, and, as an approximation, we can assume that $p_\perp$ is independent of θ.* Using this assumption and averaging the formula for the Lorentz transformation of angles over the distribution (1), we obtain the expression†

$$\operatorname{ctg}\theta_i' = \gamma_c \operatorname{ctg}\theta_i - \sqrt{\gamma_c^2 - 1}\, F(\theta_i, b),$$

$$F(\theta_i, b) = \frac{t_i}{\sin\theta_i} \exp t_i\,[K_0(t_i) + K_1(t_i)],$$

$$t_i = \frac{b^2 \sin^2\theta_i}{2}, \qquad (2)‡$$

where θ_i and θ_i' are the angles of emission of particles in the l.s. and c.m.s. respectively, and K_0 and K_1 are Bessel functions of imaginary argument of zero and first orders, respectively.

Using Eq. (2), we obtained the angular distribution of particles in the c.m.s. for $b = 0.556$ (histogram 6 in Fig. 2). From a direct comparison it follows that the proposed method leads to a better agreement with the observed c.m.s. angular distribution than the approximate method used earlier. It should be noted that the c.m.s. angular distribution of the particles obtained with account of the distribution of transverse momentum leads nevertheless to a certain overestimate of the fraction of particles propagating forwards, because a mass μ_π is ascribed to the protons. However, for large multiplicities this is not essential (an analysis of showers with $n_s \geq 10$ for $E \geq 10^{11}$ eV showed that no such systematic error occurs in this case[5]). The method given above can be fully recommended for the analysis of showers detected in a cloud chamber.

B. Let us now consider the angular distribution of shower particles in the c.m.s. ($\gamma_c = 2.4$) for all proton-nucleon interactions. This is shown in Fig. 5 as a function of the observed multiplicity. The histograms 1 and 2 were obtained assuming $\beta_c/\beta' = 1$ and using a method in which the $p_\perp$ distribution is assumed constant (the curve II of Fig. 3 was used). It follows from the graph that the angular distribution of shower particles becomes more isotropic with increasing multiplicity.

A tendency of the asymmetry of particles to change, on the average, with increasing multiplic-

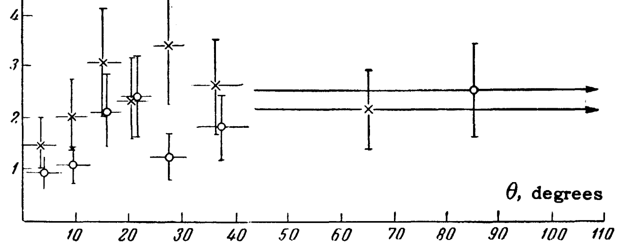

FIG. 4. Transverse momentum of shower particles as a function of the angle of emission in the l.s. at $E = 10^{10}$ eV: × – protons (86); ○ – mesons (99) from 3-prong stars.

*An analysis of the variation of $p_\perp$ with θ for cases with $n_s \geqslant 4$ according to [4] leads to analogous results.

†It is assumed here that the mass of all particles is that of π mesons.

‡ctg = cot.

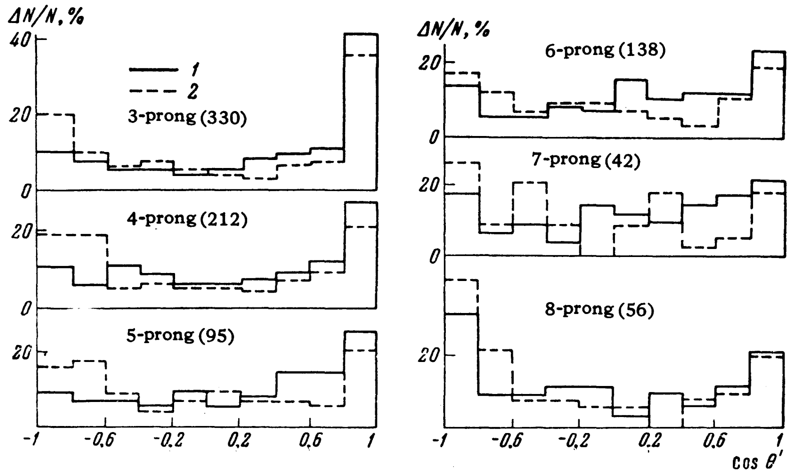

FIG. 5. Angular distribution of secondary particles in proton-nucleon interactions.

ity is also observed. The degree of asymmetry was estimated according to the formula $A = (N_+ - N_-)/(N_+ + N_-)$ (see table) where N_+ and N_- are the numbers of particles propagating forwards and backwards in the c.m.s. of the colliding nucleons, respectively. The values given in column 6 are closer to reality, as follows from Fig. 2. The observed asymmetry in the angular distribution of shower particles in 8-prong stars may apparently be ascribed to π mesons, for within the framework of the approximate method used the protons propagate predominantly forwards.

3. KINEMATIC ESTIMATE OF THE LORENTZ FACTOR

In various kinematic methods used to estimate the energy of primary particles [3,7-10] it is necessary to assume symmetry of the particle emission in the c.m.s. The methods differ mainly in the assumptions concerning the symmetry [the so-called detailed symmetry (I), or the equality of the total number of particles in both hemispheres in the c.m.s. (II)] and concerning the energy of the secondary particles. Within the framework of the method developed earlier, [3] it is possible to take the transverse-momentum distribution into account. If we use Eq. (1), then it is easy to obtain for the two types of symmetry of the angular distribution of shower particles the following formulas for the estimate of the Lorentz factor: [11]

$$\text{I)}\ \gamma_c = \varphi_1/[\varphi_1^2 - \varphi_2^2]^{1/2},$$
$$\text{II)}\ \gamma_c = F(\theta_\mu, b)/[F^2(\theta_\mu, b) - \operatorname{ctg}^2\theta_\mu]^{1/2}, \qquad (3)$$

where

$$\varphi_1 = \sum_{i=1}^{N_s} F(\theta_i, b), \qquad \varphi_2 = \sum_{i=1}^{N_s} \operatorname{ctg}\theta_i.$$

and θ_μ is the median angle.

It is interesting to compare the methods listed above for showers produced by protons with a given energy ($E = 10^{10}$ eV). The corresponding values of γ_c are shown in Fig. 6 as a function of the observed multiplicity (the stars of one group were considered as one composite shower). It follows from the figure that in the mean multiplicity range ($3 < n_s < 8$) the values of γ_c estimated by various methods differ little and the best agreement with the expected value $\gamma_c = 2.4$ is obtained with the method that takes the transverse-momentum distribution into account [Eq. (3,II)].* The methods that take the steep energy spectrum $d\epsilon/\epsilon^2$ of produced mesons into account [9,10,12] also lead to good agreement. In contrast, the use of the condition $\beta_c/\beta' = 1$ leads to a systematic overestimate of the energy by an average factor of two.

A relatively rapid change of γ_c is observed between 3- and 4- and also between 7- and 8-prong stars. Moreover, independently of the method used, the estimated values in 3-prong stars were found to be greater and in 8-prong stars lower than the expected value $\gamma_c = 2.4$ (curve 1 in Fig. 6). An agreement with the observed character of the dependence of γ_c on n_s can be obtained (curve 2) by assuming that the 3- and 8-prong stars are produced as a result of a single-meson exchange between the colliding nucleons. In the first case the incident proton captures a virtual π meson of the target nucleons and becomes excited, and in the second case the picture is reversed.† A serious objection to such an interpretation, however, is that the maximum energy released in such collisions is insufficient to produce 8-prong stars.

*The indicated arrows in γ_c were calculated from the condition of loss of symmetry.

†From the kinematical point of view, such a system is close to that which follows from the theory of peripheral collisions in the Weizsäcker-Williams approximation. [13]

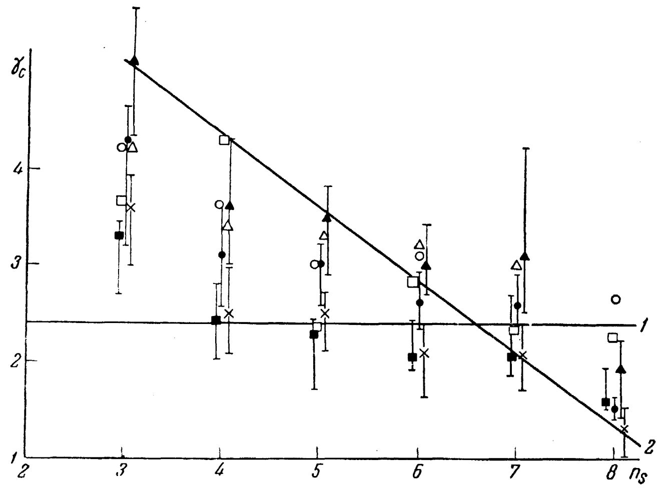

FIG. 6. Lorentz factor of the c.m.s. as a function of multiplicity. The points in the figure refer to different assumptions: ○, ● – $p_\perp$ constant and assuming respectively I and II; [3] □ – according to (3, I); ■ – from formula (3, II); △, ▲ – $\beta_c/\beta' = 1$ and assuming respectively I[7] and II[8]; × – energy spectrum of secondary particles of the form $d\varepsilon/\varepsilon^2$[9,10,12] and the assumption II.

It should be noted that neglect of the difference in the nature of secondary particles leads to an overestimate of γ_c, which clearly manifests itself more in low-multiplicity events. It is possible that this explains the overestimated value of γ_c for 3-prong stars, where the relative fraction of nucleons is most considerable. The observed decrease of γ_c for 8-prong stars can be explained if we do not accept the selection criterion for nucleon-nucleon events and assume that the contribution of collisions between the proton and several nucleons of the nucleus increases with increasing multiplicity. Such an explanation, however, is in itself based on a hypothesis that needs experimental confirmation (say by analysis of the data obtained in hydrogen bubble chambers), and even from this point of view it is impossible to explain the considerable difference between the values of γ_c for 7- and 8-prong stars.

4. INELASTICITY IN NUCLEON-NUCLEON COLLISIONS

The energy fraction $K^\pm$ carried away by charged mesons in the shower can be found from the formula

$$K^\pm \leqslant \alpha^\pm (1 - \bar{n}_p/n_s),$$

where $\bar{n}_p$ is the mean number of protons per star (for these stars we have assumed $\bar{n}_p = 1.25$, which follows from the direct measurement in 3-prong stars [2] and agrees, within the limits of experimental error, with the values given by other authors for large multiplicities [4,14]); $\alpha^\pm$ is the energy fraction carried away by all charged particles, assuming that they are π mesons. Assuming a constant transverse-momentum distribution (Sec. 2), it is easy to obtain for $\alpha^\pm$:

$$\alpha^\pm = \frac{\mu_\pi}{MN(\gamma_0 - 1)}\left[N_s + \frac{\sqrt{\pi}}{2}\sum_{i=1}^{N_s} f(\theta_i, b)\right] \text{ in the l.s.} \quad (4)$$

$$\alpha_1^\pm = \frac{\mu_\pi}{MN}\left\{u_1\left[N_s + \frac{\sqrt{\pi}}{2}\sum_{i=1}^{N_s} f(\theta_i, b)\right] - u_2\frac{\sqrt{\pi}}{2b}\sum_{i=1}^{N_s}\operatorname{ctg}\theta_i\right\} \text{ in the c.m.s.} \quad (5)$$

where γ_0 is the dimensionless energy of the primary nucleon in the l.s.,

$$f(\theta_i, b) = \exp(b^2\sin^2\theta_i)\,[1 - \Phi(\sqrt{2}\,b\sin\theta_i)]/b\sin\theta_i,$$
$$u_1 = (\gamma_0 + 1)^{1/2}/2\,[(\gamma_0 + 1)^{1/2} - \sqrt{2}],$$
$$u_2 = (\gamma_0 - 1)^{1/2}/2\,[(\gamma_0 + 1)^{1/2} - \sqrt{2}],$$

Φ is the Gaussian probability integral, and $N_s = Nn_s$.

Using Eqs. (4) and (5), we have calculated the values $K^\pm$ averaged over all showers of each group (the notation $K_1^\pm$ refers to the c.m.s.). The results are shown in Fig. 7 as functions of the observed multiplicity.* It follows from the figure that $K^\pm$ increases both in the l.s. and in the c.m.s. with increasing multiplicity. For comparison, the

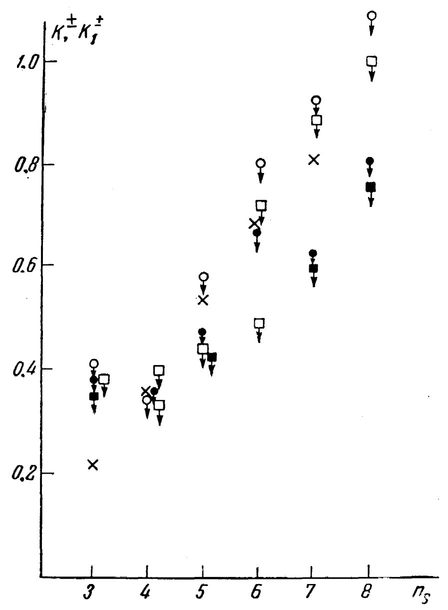

FIG. 7. Variation of the inelasticity coefficient $K^\pm$ and $K_1^\pm$ with multiplicity. In the l.s., $K^\pm$: ● – for $p_\perp$ = const,[3] ■ – according to Eq. (3); in the c.m.s. ($K_1^\pm$): ○ – for $p_\perp$ = const,[3] □ – according to formula (4), × – according to the data of[14].

*A detailed derivation of formulae (2)–(5) is given in[11], where the value of $K^\pm$ is found to agree well with measurements for 3-prong stars.

values of $K_1^{\mp}$ obtained in [14] are shown in Fig. 7. These values are in good agreement with ours. The energy fraction per produced meson (column 7 of the table) depends little on the observed multiplicity and on the average amounts to 17%. Therefore, if the energy spectrum of the π^0 and $\pi^{\pm}$ mesons is the same, then for 7- and 8-prong stars the ratio of the π^0 mesons to the number of $\pi^{\pm}$ mesons is less than 0.5.

According to Birger and Smorodin,[15] we can estimate the lower limit of the target mass from the inequality

$$m_2 \geqslant \frac{\mu_\pi}{M}\left\{N_s + \frac{\sqrt{\pi}}{2}\sum_{i=1}^{N_s}\left[f(\theta_i, b) - \frac{1}{b}\operatorname{ctg}\theta_i\right]\right\}. \qquad (6)$$

The corresponding values of m_2 are also given in the table. Since m_2 is estimated from charged particles, the increase in this value with increasing multiplicity may be due both to the relative redistribution of the number of charged and neutral π mesons (to a decrease in the fraction of π^0 mesons) and to a possible increase in the contribution of central collisions of the nucleons. From energy considerations one would think that the relative fraction of π^0 mesons in 8-prong stars is small since the target mass m_2 is estimated for them most reliably and is less than the nucleon mass. This indicates that in 8-prong stars the contribution of the collisions between the primary protons and the group of nucleons in the nucleus is small.

CONCLUSIONS

1. A new approximate method has been developed for the estimate of the c.m.s. angular and energy characteristics of nuclear disintegrations, based on the assumption of a constant transverse-momentum distribution.

2. The application of this method to all proton-nucleon interactions enables us to obtain the following results:

a) The degree of anisotropy of the c.m.s. angular distribution of shower particles ($\gamma_c = 2.4$) decreases with increasing multiplicity, and a marked deviation from symmetry in the particle emission forwards and backwards is observed for 3- and 8-prong stars.

b) In the range of medium multiplicity ($3 < n_s < 8$) the best fit to the expected value ($\gamma_c = 2.4$) results from the kinematic method, in which a constant transverse-momentum distribution of shower particles is assumed. The assumption $\beta_c/\beta' = 1$ leads to a systematic overestimate of the energy by a factor of two; regardless of the method of estimate, the value of γ_c is found to be overestimated for 3-prong stars and underestimated for 8-prong stars. Taking this fact into account, there is a general tendency for the Lorentz factor of the system to decrease with increasing multiplicity, assuming angular symmetry of the secondary particles.

c) With increasing multiplicity, an increase in the energy fraction carried away by charged mesons is observed both in the l.s. and in the c.m.s. The fraction of the energy per meson (17%) is almost constant. Hence, it follows that $n(\pi^0)/n(\pi^{\pm}) < 0.5$ for 7- or 8-prong stars if the energy spectra of the π^0 and $\pi^{\pm}$ mesons are identical. The estimated mass of the target increases with increasing multiplicity. The mass, however, is not greater than the nucleon mass, which is an indirect indication of the correctness of the selection criteria for nucleon-nucleon interactions.

In conclusion, the authors express their gratitude to the workers of the Joint Institute for Nuclear Research for taking part in the discussion of the results of the present work. They would like to thank I. M. Gramenitskii and M. I. Podgoretskii for supplying the data on the angular distribution of particles in 8-prong stars prior to publication.

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