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FERMI SURFACE OF SILVER

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The topology, shape and some details of the Fermi surface in silver are discussed on the basis of galvanomagnetic measurements. The main dimensions of the Fermi surface of silver (and also of gold and copper) are estimated; a comparison is made with other methods.

IN an earlier investigation[1] we obtained stereographic projections of the special directions of the magnetic field for an open Fermi surface in silver. This projection was constructed on the basis of galvanomagnetic measurements on a relatively small number of specimens, which did not let us determine the quantitative characteristics of the two-dimensional regions. The problem was also left unsolved as to whether there exists a special 001 line in the stereographic projection. (The special line is a trace of a plane. We shall denote it by the indices of the plane without the parentheses.) This same question also exists for the stereographic projection of gold. For copper, as follows from the measurements of Pippard,[2] there exists a total of three special lines: 001, 110, and 111.

Priestley[3] considered the experimental results on the de Haas—van Alphen effect obtained by Shoenberg,[4] and showed that this special line should exist for gold and silver, as well as for copper. It was also of importance to carry out measurements of the Hall effect, from which one could obtain a number of parameters of the Fermi surface.

The principal object of the measurements was silver.* Single crystals of silver, prepared by the Obreimov-Shubnikov method, had a length of ~ 30 mm and a diameter of ~ 2 mm. The value of the ratio of the resistance at room temperature to the resistance at $T = 4.2°K$ was in most cases $\rho_{300}/\rho_{4.2} \approx 1000$.

Measurement of the emf was carried out on a potentiometer with a photoelectric multiplier. The accuracy of the measurements was to within 1×10^{-9} V. The measurements were made at a temperature of 4.2°K in fields up to 24 kOe. The magnetic field was rotated in a plane perpendicular to the axis of the specimen. The dependence of the resistance on the angle between the field and the crystallographic axes was determined for ten single crystal specimens of silver in a constant magnetic field $H = 23.5$ kOe. Examples of the rotation diagrams are given in Fig. 1.

Two pairs of mutually perpendicular contacts were used for the measurement of the angular dependence of the Hall emf. The emf's obtained on them were geometrically additive.

Like the resistance, the Hall emf has a strongly anisotropic character (Fig. 2). Dependence of the Hall emf on the magnetic field is linear at the maxima, which take place for the [001], [110], and [111] directions. The linear dependence of the Hall emf is also observed for the three field directions which are located in the regions II of the stereographic projection (Fig. 3). In the region I, the Hall emf has a field dependence that is somewhat greater than linear. In the [112] direction, the Hall emf approaches saturation in the magnetic field.

As is well known,[1] the resistance of silver single crystals exhibits saturation in a magnetic field at the minima of the angular dependence. At the maxima, it increases approximately with the square of the field. In the case of silver, as well as for gold and copper, the Fermi surface has an open cross section for those directions of the magnetic field in which a quadratic increase in the resistance is observed.[5] Such directions of the magnetic field, which were obtained for all measured specimens, have been plotted on a stereographic projection (Fig. 3). On the basis of this projection, it can be concluded that the stereographic projection of the special directions of the magnetic field, also pictured in Fig. 3, corresponds to an open Fermi surface for silver.

This projection is confirmed by data on measurements of the Hall emf: a linear dependence on

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*The authors take this opportunity to thank V. A. Gromakovskii for help in the measurements.

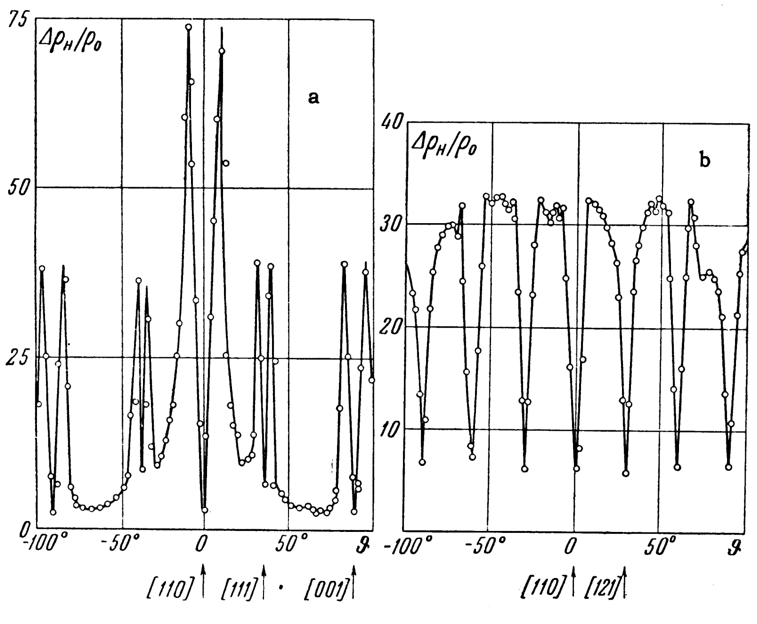

FIG. 1. Rotational diagram of the resistance of a single crystal of silver in a constant magnetic field H = 23.5 kOe, T = 4.2°K: a – axis of specimen parallel to [110]; b – axis of specimen parallel to [111].

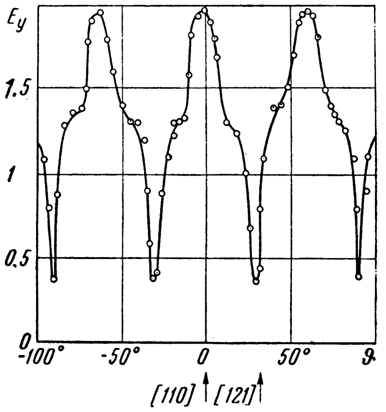

FIG. 2. Rotational diagram of the Hall emf (arbitrary units) in the constant magnetic field H = 2.35 kOe. The axis of the specimen is parallel to [111]; T = 4.2° K.

the magnetic field for the rational directions [001], [110], and [111] and the regions II of the projection, a nonlinear dependence for the regions I and saturation for the [112] direction, which is characteristic for two crossed open directions of the Fermi surface.[5,6]

We note that in the measured series of single crystals it was possible to observe very clearly the maxima of the resistance in the case when the magnetic field was located in the (001) plane. This confirmed the existence of the special line 001 in the stereographic projection.

It was concluded earlier[7] that the special line 001 does not exist for gold, since maxima in the resistance do not appear in the rotational diagrams of the resistance in the direction of the line of intersection of the plane of the magnetic field with the (001) plane. It then follows that there is no "corrugated cylinder" on the Fermi surface of gold in the [001] direction. Carrying out a comparison of the results from the de Haas–van Alphen

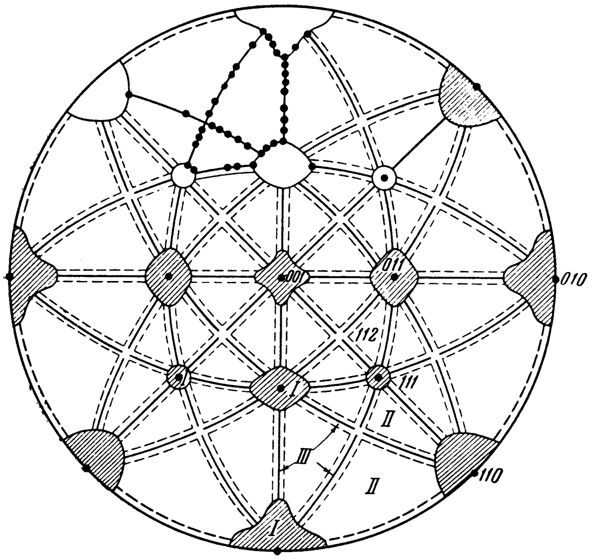

FIG. 3. Stereographic projection of the special directions of the magnetic field for the Fermi surface in silver, gold, and copper. (In the upper part the points correspond to the directions in which maxima of the resistance are observed in the rotational diagrams for silver in a constant magnetic field.)

Regions I – two-dimensional regions of directions of the magnetic field for which there exist open electron trajectories. (The dimensions of the regions are given for silver.) In this case, the closed trajectories possess two signs ("electrons" and "holes"). Along the boundary and in the center of the region I, the thickness of the layer of open trajectories vanishes.

For directions of the magnetic field located in the regions II, there exist closed trajectories of a single sign (electrons).

Regions III – regions of directions of the magnetic field for which there are elongated trajectories of the electrons. The open trajectories exist only for those directions which lie on the axial lines.

Region	Measurement planes	Radii of the two dimensional regions (deg)		Diameter of the "neck"[111]	
		Measurements of the authors	Calculations of Priestley from data of Schoenberg	Calculations from the size of the two-dimensional regions	from the data of Morse [8,9]
			Silver		
001	(100)	10	10,3		
	(110)	6	7,3		
110	(001)	7	9	0,10 b	0.11b
	$(1\bar{1}0)$	10	12,5	(0,11 b from data on Hall effect measurements)	
	$(1\bar{1}1)$	7	—		
111	$(0\bar{1}1)$	3,5	4		
	$(\bar{1}\bar{1}2)$				
			Gold		
001	(100)	13	12,4		
	(110)	7,3	9.2		
110	(001)	7.5	11,2		
	$(1\bar{1}0)$	13	15.7	0.12 b	0,13b
	$(1\bar{1}1)$	8	—		
111	$(0\bar{1}1)$	5	4,6		
	$(\bar{1}\bar{1}2)$	6	5,3		
			Copper		
001	(100)	12	15		
	(110)	8	10,8		
110	(001)	9	12.5		
	$(1\bar{1}0)$	12	18,5	0.13 b	0,13 b
	$(1\bar{1}1)$	9	—		
111	$(0\bar{1}1)$	6	5,3		
	$(\bar{1}\bar{1}2)$	—	6,2		

effect, Priestley[3] noted that the "corrugated cylinder" could escape observation because the angle α between the current and the [001] axis was close to 90° in the experiments. And since $\rho \sim H^2 \cos^2 \alpha$, the magnitude of the maximum of the resistance in the rotational diagram could be insignificant.

Because the 001 line was discovered in silver, we carried out additional measurements on two single crystals of gold, the orientation of which made it possible to check on the existence of the special line 001 in gold. The measurements have shown that the line 001 exists on the stereographic projections of the special directions of the magnetic field for the Fermi surface of gold. Figure 4 shows the rotational diagram of the resistance in a magnetic field for one of the specimens. It is seen from the drawing that sharp maxima appear in the resistance along the line of intersection of the plane of the magnetic field with the planes (010) and (100).

Thus the only difference of the stereographic projections of the special directions of the magnetic field of the Fermi surfaces for silver, gold [7] and copper [1] consists in the size of the two-dimensional regions I. In the table, the basic dimensions of the (two-dimensional) regions I, obtained in our measurements, are shown for these three metals.

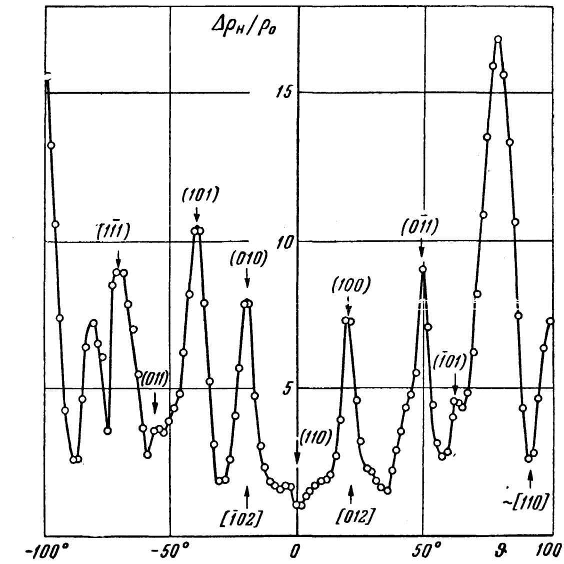

FIG. 4. The rotational diagram of the resistance of a single crystal of gold. The axis of the specimen departs from the [001] axis by 70° and is located in the plane which makes an angle of 41° with the (100) plane. The value of $\rho_{300}/\rho_{4.2}$ = 750, H = 23.5 kOe, T = 4.2° K.

According to I. Lifshitz and Peschanskii,[6] the stereographic projection shown in Fig. 3 corresponds to an open Fermi surface of the type of a three-dimensional net of "corrugated cylinders." The axes of these cylinders are parallel to the principal rational directions [001], [110], and [111].* It is most natural to assume that the open Fermi surface in silver, gold, and copper is a surface formed by "spheres," joined in the [111] direction by "necks." [2] Depending on the dimensions of the "necks" and "spheres," the Fermi surface can be open only along the [111] direction,† or additional "openness" can appear along the [110] and [001] directions.‡

In the case of silver, gold, and copper, all three directions should be open (Fig. 5). The diameters of the "necks" can be estimated from the dimensions of the two-dimensional areas. Estimates accurate to 10 per cent are given in the table.

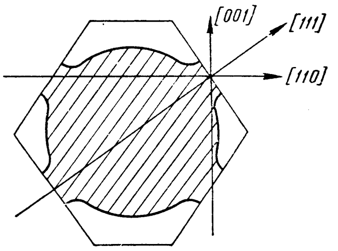

FIG. 5. Section of the fundamental element of the Fermi surface of silver, gold and copper by the $(1\bar{1}0)$ plane.

These same diameters can be determined from a knowledge of the Hall constant in the rational directions and the directions of the regions II of stereographic projection. [5] For silver, the measurements give the following values:

$$R_{[001]} = R_{[110]} = 1.4\cdot 10^{-3}, \quad R_{[111]} = 1.8\cdot 10^{-3}$$

and the Hall constant for the regions II:

$$R_{II} = 1\cdot 10^{-3} \text{ cgs emu.}$$

(The sign of the Hall constant is electronic. The accuracy of the measurements was ± 10 per cent.) Thus the diameter of the "neck" is equal to 0.11 b [$b = 2(2\pi a)$, a is the lattice constant].*

The Hall constant R_{II} is related to the value of the total volume of the Fermi surface of silver (gold, copper). Calculations for silver yield $V = 0.26\,b^3$. This value agrees with the volume of the Fermi sphere in silver, $V = 0.25\,b^3$ in the model of a free electron gas with one electron per atom. This coincidence is due to the small value of the region of intersection of the Fermi surface with the (111) boundaries of the Brillouin zone.

There is no necessity here of carrying out a comparison of these results with those obtained by Shoenberg in the study of the de Haas–van Alphen effect, since a detailed comparison was given by Priestley. [3] We only note that the results of these two methods are in excellent agreement. This can be established by a comparison of the dimensions of the two-dimensional regions of the projection, obtained experimentally by us, and calculated by Priestley from the data of Shoenberg.

Excellent quantitative agreement exists with the results of researches on magnetoacoustic resonance, [8,9] which is seen from a comparison of the diameters of the "necks." (see the table).

In conclusion, it should be emphasized that the results of all the methods applied in the investigation of silver, gold, and copper (the anomalous skin effect, galvanomagnetic measurements, the de Haas–van Alphen effect, magnetoacoustic and cyclotron resonance [14]) are in excellent qualitative and quantitative agreement, and give a complete representation of the Fermi surface of these metals.

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*On the basis of stereographic projection, it is impossible to draw a unique conclusion on the shape of the Fermi surface. Such a projection allows us only to draw a general conclusion on the topological feature of the Fermi surface. Such topological peculiarities are the concepts "corrugated cylinder," "corrugated plane," "three-dimensional and plane nets of corrugated cylinders," etc. Therefore, there is no sense in identifying the shape of the Fermi surface in the general case with its topology, as was done by Klauder and Kunzler.[10]

†This type of surface is realized in lead.[11] It should be noted that the Fermi surface of lead suggested by Gold,[12] in contrast with the opinions expressed by us in [11], can have an open direction [111]. In this case, the Fermi surfaces of lead assumed by Gold and by us will be topologically equivalent.

Our attention was called to the possibility of the existence of an open direction$[11\bar{1}]$ by R. S. Young (private communication), to whom we express our grateful appreciation.

‡Naturally, open directions with large indices appear with further increase in the diameter of the "necks." This problem is considered in detail in [13].

*The "neck" diameter d is computed from the formula $\tan\vartheta' = d/0.82\,b$, where ϑ' is the radius of the two-dimensional region 110 in the (111) plane, or by the formula: $V_{II} - V_{[110]} = 0.7b^2d$, where V_{II} and $V_{[110]}$ are the volumes of the Fermi surfaces, obtained from the Hall constants R_{II} and $R_{[110]}$.

[1] N. E. Alekseevskii and Yu. P. Gaĭdukov, JETP **37**, 672 (1959), Soviet Phys. JETP **10**, 481 (1960).

[2] A. B. Pippard, Phil. Trans. Roy. Soc. **250**, 325 (1957).

[3] M. G. Priestley, Phil. Mag. **5**, 111 (1960).

[4] D. Shoenberg, Phil. Mag. **5**, 105 (1960).

[5] I. M. Lifshitz and V. G. Peschanskii, JETP **35**, 1251 (1958), Soviet Phys. JETP **8**, 875 (1959).

[6] I. M. Lifshitz and V. G. Peschanskii, JETP **38**, 188 (1960), Soviet Phys. JETP **11**, 137 (1960).

[7] Yu. P. Gaĭdukov, JETP **37**, 1281 (1959), Soviet Phys. JETP **10**, 913 (1960).

[8] Morse, Myers, and Walker, Phys. Rev. Lett. **4**, 605 (1960).

[9] R. W. Morse, Proc. Intern. Conference on Fermi Surfaces, Cooperstown, New York, August, 1960 (Wiley, London, 1960), p. 214.

[10] J. R. Klauder and J. E. Kunzler, Phys. Chem. Solids **18**, 256 (1961).

[11] N. E. Alekseevskii and Yu. P. Gaĭdukov, JETP **41**, 354 (1961), Soviet Phys. **14**, 256 (1962).

[12] A. V. Gold, Phil. Trans. Roy. Soc. **251**, 85 (1958).

[13] Alekseevskii, Gaĭdukov, Lifshitz, and Peschanskii, JETP **39**, 1201 (1960), Soviet Phys. JETP **12**, 837 (1961).

[14] A. F. Kip, Proc. Intern. Conference on Fermi Surfaces, Cooperstown, New York, August, 1960 (Wiley, London, 1960), p. 146.

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