POLARIZATION AND CHARGE EXCHANGE IN HIGH-ENERGY πp *SCATTERING*

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If the charge-exchange cross section in πp-scattering approaches $\sigma_{ex}(\Delta^2, \infty) \neq 0$ as $E \to \infty$ then the limiting value of the polarization will be $|P_\infty| < [\sigma_{ex}(\Delta^2, \infty)/\sigma(\Delta^2, \infty)]^{1/2}$; where $\sigma(\Delta^2, \infty)$ is the limiting cross section for elastic π^+p scattering into a unit interval of the squared transferred momentum. The limiting polarization should be equal but opposite in sign in the cases of π^+p and π^-p scattering. If $\sigma_{ex}(\Delta^2, \infty) = 0$, then $P_\infty = 0$.

THE description of elastic scattering at very high energies is usually based on the so-called diffraction picture, which is characterized by the fact that the total cross section and the cross section for elastic scattering into a unit interval of the square of the momentum transfer $\sigma(\Delta^2)$ tend to a constant limiting value with increasing energy; here $\Delta^2 = k^2(1 - \cos\theta)/2$, where θ is the c.m.s. momentum. Along with the dispersion relations (d.r.) for the final momentum transfer, the diffraction picture leads to a certain asymptotic relation between the charge-exchange cross section and the possible value of the polarization in elastic π^+p scattering. If the charge-exchange cross section for large energies tends to a limit $\sigma_{ex}(\Delta^2, \infty) \neq 0$, then the possible value of the limiting polarization is

$$|P_\infty| < [\sigma_{ex}(\Delta^2, \infty)/\sigma(\Delta^2, \infty)]^{1/2}.$$

The limiting polarization should have the same modulus but opposite sign for π^+p and π^-p scattering. If $\sigma_{ex}(\Delta^2, \infty) = 0$, then $P_\infty = 0$.

These relations arise since the limiting value of charge exchange in the scattering of pions on nucleons is determined only by the real part of the amplitudes $k^{-1}g_{\pi\pm}$ and $k^{-1}h_{\pi\pm}$ in the π^+p elastic scattering matrix $M_{\pi\pm} = g_{\pi\pm} + ih_{\pi\pm}\sigma_{ex}$ where σ_{ex} is the component of the spin operator in the direction of the normal to the scattering plane.* On the other hand

$$\sigma(\Delta^2)P = \frac{8\pi}{k^2}(\mathrm{Im}\, g_{\pi\pm}\, \mathrm{Re}\, h_{\pi\pm} - \mathrm{Im}\, h_{\pi\pm}\, \mathrm{Re}\, g_{\pi\pm}), \quad (1)$$

and therefore the polarization effect vanishes if $g_{\pi\pm}$ and $h_{\pi\pm}$ are purely imaginary.

*The cross section for scattering into a unit interval of the square of the momentum transfer is connected with the scattering into a unit interval of solid angle and with elements of the matrix M by the relation $\sigma(\Delta^2) = 4\pi k^{-2}\sigma(\theta) = 4\pi k^{-2}|M|^2$.

Owing to isotopic invariance, the cross sections of each of the charge-exchange reactions $\pi^-p \to \pi^0 n$, $\pi^0 p \to \pi^+ n$ are related to the matrix elements $M_{\pi\pm}$ in the following way:

$$\sigma_{ex}(\Delta^2) = 8\pi k^{-2}(|g^-|^2 + |h^-|^2). \quad (2)$$

The superscript $\mp$ denotes the half difference (half-sum) of the corresponding quantities for π^-p and π^+p scattering.

In order to explain the relation between $k^{-1}g^-$, $k^{-1}h^-$ and $k^{-1}g_{\pi\pm}$, $k^{-1}h_{\pi\pm}$ at high energies, we use the dispersion relations in the form given by Chew, Goldberger, Low, and Nambu.[1] These dispersion relations were established for the invariant amplitudes $A^\pm(\nu, \Delta^2)$ and $B^\pm(\nu, \Delta^2)$ (where $\nu = E_l - \Delta^2 M^{-1}$, and E_l is the meson energy in the laboratory system), which are related to the physical amplitudes $g^\pm$, and $h^\pm$ as follows:

$$g^\pm = \frac{(W+M)^2 - \mu^2}{16\pi W^2}[A^\pm + (W-M)B^\pm] + \left(1 + \frac{2\Delta^2}{k^2}\right)\frac{(W-M)^2-\mu^2}{16\pi W^2}[-A^\pm + (W+M)B^\pm], \quad (3)$$

$$-h^\pm = \frac{2\Delta}{k}\sqrt{1 + \frac{\Delta^2}{k^2}}\,\frac{(W-M)^2-\mu^2}{16\pi W^2} \times [-A^\pm + (W+M)B^\pm]. \quad (4)$$

Here, M and μ are the nucleon and pion masses and $W^2 = M^2 + \mu^2 + k^2 + 2\sqrt{(\mu^2 + k^2)(M^2 + k^2)}$ is the square of the total c.m.s. energy ($\hbar = c = 1$).

The limited range of interaction and the requirement of unitarity restrict the behavior of $A^\pm$ and $B^\pm$ in a definite way as the energy tends to infinity (see Pomeranchuk[2] and Finn[3]):

$$A^\pm(\infty, \Delta^2) = O(k^2), \qquad B^\pm(\infty, \Delta^2) = O(1). \quad (5)$$

Condition (5) simplifies the relation between the physical and invariant amplitudes.* For $W \gg M$ and $\Delta^2 \ll W^2$

$$g^{\pm} \rightarrow \frac{k}{4\pi} B^{\pm}, \qquad h^{\pm} \rightarrow \frac{1}{8\pi} \frac{\Delta}{k} A^{\pm}. \tag{6}$$

A second important consequence of conditions (5) is the limiting of the necessary number of subtractions in the d.r. for $A^{\pm}$ and $B^{\pm}$ to one subtraction;[3] here†

$$\nu \operatorname{Re} Z^{+}(\nu, \Delta^2) - \nu_0 \operatorname{Re} Z^{+}(\nu_0, \Delta^2) = \frac{2}{\pi}(\nu^2 - \nu_0^2) \mathrm{P} \int_0^{\infty} \frac{\operatorname{Im} Z^{+}(\nu', \Delta^2)\, \nu'^2 d\nu'}{(\nu'^2 - \nu^2)(\nu'^2 - \nu_0^2)}, \tag{7}$$

$$\operatorname{Re} Z^{-}(\nu, \Delta^2) - \operatorname{Re} Z^{-}(\nu_0, \Delta^2) = \frac{2}{\pi}(\nu^2 - \nu_0^2) \mathrm{P} \int_0^{\infty} \frac{\operatorname{Im} Z^{-}(\nu', \Delta^2)\, \nu' d\nu'}{(\nu'^2 - \nu^2)(\nu'^2 - \nu_0^2)}, \tag{8}$$

where

$$Z^{\pm}(\nu, \Delta^2) \equiv B^{\pm}(\nu, \Delta^2), \qquad \nu^{-1} A^{\pm}(\nu, \Delta^2).$$

For large k we have $\nu \approx k^2/M$ and therefore conditions (5) can be replaced by

$$Z^{\pm}(\infty, \Delta^2) = O(1). \tag{9}$$

The investigation of the asymptotic behavior of $\operatorname{Re} Z^{\pm}$ and $\operatorname{Im} Z^{\pm}$ will be carried out by the method used earlier in [4]. We split the interval of integration in (7) and (8) into two parts, $0 - \epsilon$ and $\epsilon - \infty$, and we set $\nu^2 \ll \epsilon^2 \ll \nu_0^2$. If we expand the integrals with the limits $0 - \epsilon$ in powers of ν^2/ν_0^2 and the integrals with the limits $\epsilon - \infty$ in powers of ν^2/ν'^2 and consider only the first terms (we can do this if the integrals converge), then the variables ν and ν_0 separate.‡ As a result, we obtain from (7) and (8) the relations

$$\operatorname{Re} Z^{-}(\nu, \Delta^2) - \frac{2}{\pi} \mathrm{P} \int_0^{\varepsilon} \frac{\operatorname{Im} Z^{-}(\nu', \Delta^2)\, \nu' d\nu'}{\nu'^2 - \nu^2} = \operatorname{Re} Z^{-}(\nu_0, \Delta^2) - \frac{2}{\pi} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{\nu_0^2 \operatorname{Im} Z^{-}(\nu', \Delta^2)\, d\nu'}{(\nu'^2 - \nu_0^2)\, \nu'} = p(\varepsilon, \Delta^2), \tag{10}$$

$$\nu \operatorname{Re} Z^{+}(\nu, \Delta^2) - \frac{2}{\pi} \mathrm{P} \int_0^{\varepsilon} \frac{\operatorname{Im} Z^{+}(\nu', \Delta^2)\, \nu'^2 d\nu'}{\nu'^2 - \nu^2} = \nu_0 \operatorname{Re} Z^{+}(\nu_0, \Delta^2) - \frac{2}{\pi} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{\nu_0^2 \operatorname{Im} Z^{+}(\nu', \Delta^2)\, d\nu'}{\nu'^2 - \nu_0^2} = q(\varepsilon, \Delta^2). \tag{11}$$

Analysis of the behavior of the right-hand part of (10) as $\nu_0 \rightarrow \infty$ leads to the following conclusions. If $\operatorname{Im} Z_{\pi\pm}(\nu', \Delta^2)$ tends to a limit as $\nu' \rightarrow \infty$, then the fact that $\operatorname{Re} Z^{-}(\nu_0, \Delta^2)$ is bounded as $\nu_0 \rightarrow \infty$ means that $\operatorname{Im} Z^{-}(\infty, \Delta^2) = 0$, and, moreover,

$$\int^{\infty} (\nu')^{-1} \operatorname{Im} Z^{-}(\nu', \Delta^2)\, d\nu' < \infty.$$

The first result is a generalization of Pomeranchuk's theorem[2] and the second is a generalization of the theorem of Amati, Fierz, and Glaser.[5]

Investigation of the right-hand part of (11) shows that the condition that $\operatorname{Im} Z^{+}(\infty, \Delta^2)$ goes to a limit* is equivalent to requiring that $\nu_0 \operatorname{Re} Z^{+}(\nu_0, \Delta^2)$ increase more slowly than ν_0 as $\nu_0 \rightarrow \infty$, i.e., it means that $\operatorname{Re} Z^{+}(\infty, \Delta^2) = 0$.

The asymptotic properties of the amplitudes obtained in this way give, for $k \rightarrow \infty$ and $\Delta^2 < \infty$:

$$k^{-1} \operatorname{Re} g^{+}(\infty, \Delta^2) = 0, \tag{12}$$

$$k^{-1} \operatorname{Im} g^{-}(\infty, \Delta^2) = 0, \tag{13}$$

$$\int^{\infty} k^{-2} \operatorname{Im} g^{-} dk < \infty, \tag{14}$$

and similar relations for $k^{-1}h$.

We now consider two possible cases.

1) The charge-exchange cross section per unit interval of the square of the momentum transfer tends to zero for high energies. Then, by (2),

$$k^{-1} \operatorname{Re} g^{-}(\infty, \Delta^2), \qquad k^{-1} \operatorname{Re} h^{-}(\infty, \Delta^2) = 0. \tag{15}$$

This case, because of (12), corresponds to vanishing real parts of the $\pi^{\pm}$ scattering amplitude, and, owing to (1),

$$P_{\pi^{\pm}\infty} = 0.$$

2) The charge-exchange cross section $\sigma_{ex}(\infty, \Delta^2)$ is not equal to zero. Then, owing to (13),

$$k^{-1} \operatorname{Re} g^{-}(\infty, \Delta^2), \quad k^{-1} \operatorname{Re} h^{-}(\infty, \Delta^2) \neq 0$$

and, consequently, [see (12)]

$$k^{-1} \operatorname{Re} g^{-}(\infty, \Delta^2) = k^{-1} \operatorname{Re} g_{\pi^{+}}(\infty, \Delta^2) = -k^{-1} \operatorname{Re} g_{\pi^{-}}(\infty, \Delta^2) \neq 0. \tag{16}$$

*Conditions (5) exclude the case $\sigma_{tot} \rightarrow 0$ ($W \rightarrow \infty$), which corresponds to $A^{\pm}(\infty, \Delta^2) < O(k^2)$, $B^{\pm}(\infty, \Delta^2) = 0$. Also excluded is the case $A^{\pm}(\infty, \Delta^2) < O(k^2)$, $B^{\pm}(\infty, \Delta^2) = O(1)$, which corresponds to the vanishing of the polarization for a trivial reason, since here, owing to (3), (4), we have $h(g)^{-1} \rightarrow 0$ as $W \rightarrow \infty$.

†The d.r. for $B^{\pm}$ contain pole terms which were not written in explicit form, since they are not important for the investigation of the asymptotic behavior of $B^{\pm}(\Delta^2, \nu)$ ($\nu \rightarrow \infty$, Δ^2 = const).

‡The possibility of separating the variables in equations (10) and (11) was pointed out by V. P. Kanavets.

*If $\operatorname{Im} Z^{+}(\infty, \Delta^2)$ is nonmonotonic or oscillates in going to the limit, then $\nu_0 \operatorname{Re} Z^{+}(\nu_0, \Delta^2)$ may have an infinite peak, whose area, however, tends to zero as $\nu \rightarrow \infty$, since, on the average, $\operatorname{Re} Z^{+}(\nu_0, \Delta^2) \rightarrow 0$. This has been shown by E. M. Landis.

We obtain similar relations for $k^{-1}\mathrm{Re}\, h_{\pi\pm}(\infty, \Delta^2)$.

Our theorem that the moduli of the polarization in π^+p and π^-p scattering are equal and of opposite sign now follows from (16), (13), and (1). Moreover, we obtain from (16) and (13) the following relations:

$$\sigma_{ex}(\Delta^2, \infty) = 8\pi k^{-2} (\mathrm{Re}\, g^2_{\pi\pm} + \mathrm{Re}\, h^2_{\pi\pm}). \qquad (17)$$

We now consider an identity resulting from (1) and (17):

$$P^2_{\pi\pm\infty} \equiv \frac{\sigma_{ex}(\infty, \Delta^2)}{\sigma(\infty, \Delta^2)} \left[\frac{(\mathrm{Im}\, g\, \mathrm{Re}\, h - \mathrm{Im}\, h\, \mathrm{Re}\, g)^2}{(\mathrm{Re}\, g^2 + \mathrm{Re}\, h^2)(\mathrm{Re}\, g^2 + \mathrm{Re}\, h^2 + \mathrm{Im}\, g^2 + \mathrm{Im}\, h^2)} \right]. \qquad (18)$$

[In the right-hand part of (18), we have dropped the subscripts $\pi^\pm$ and ∞ of g and h.] It is readily shown that the expression in the brackets is less than unity. Hence

$$|P_\infty| < \left(\frac{\sigma_{ex}(\infty, \Delta^2)}{\sigma(\infty, \Delta^2)} \right)^{1/2} = \left(\frac{8\pi (\mathrm{Re}\, g^2 + \mathrm{Re}\, h^2)}{k^2 \sigma(\infty, \Delta^2)} \right)^{1/2}. \qquad (19)$$

Integrating (19) over the momentum transfer, we obtain

$$\overline{P^2_\infty} < \frac{(\sigma_{ex})_{tot}}{\sigma_{el}} = \frac{(\sigma_{ex})_g + (\sigma_{ex})_h}{\sigma_{el}}, \qquad (20)$$

where $\overline{P^2_\infty}$ is the mean value of the square of the limiting polarization weighted for the effective scattering cross section and $(\sigma_{ex})_{g,\, h}$ are the fractions of the total charge-exchange cross section associated with $(\mathrm{Re}\, g)^2$ and $(\mathrm{Re}\, h)^2$ at very high energies. At present, we do not have any reliable data on the total charge-exchange cross sections at energies of several BeV.

According to measurements at the Joint Institute for Nuclear Research,* the value of σ_{el} is $\sim 6 \times 10^{-27}$ cm^2. The d.r. for zero-angle scattering make it possible to estimate the quantity (see [4])

$$p(\varepsilon) = \lim_{k\to\infty} k^{-1}\, \mathrm{Re}\, g(0) + \frac{1}{4\pi} \int_\varepsilon^\infty (\sigma^+_{tot} - \sigma^-_{tot})\, k^{-1} dk;$$

ϵ is the energy to which $\sigma^\pm_{tot}$ is known. According to [4], the value of p(5.2 BeV) is -1.4×10^{-29} cm^2. We can expect that the order of magnitude of $\lim k^{-1}\mathrm{Re}\, g(0)$ $(k \to \infty)$ is not greater than this value. With the aid of the optical model (refracting sphere with $R \lesssim 10^{-13}$ cm), we can then estimate $(\sigma_{ex})_g$:

$$(\sigma_{ex})_g = 8\pi \int k^{-2}\, \mathrm{Re}\, g\, d(\Delta^2) \lesssim 2 \cdot 10^{-29} \text{ cm}^2.$$

*Private communication from N. G. Birger.

We assume that at high energies $(\sigma_{ex})_h$, too, does not exceed this value. In this case (20) gives for the limiting value of the polarization

$$\sqrt{\overline{P^2_\infty}} \lesssim 6\%.$$

In conclusion, we note the following.

1) The converse theorem (if the polarization vanishes, then charge-exchange vanishes) is not valid, since the polarization vanishes when the difference between the real phases $(\delta_{l+1/2}) - (\delta_{l-1/2}) \to 0$, i.e., $h(g)^{-1} \to 0$, where the phases themselves need not tend to zero ($k^{-1}\mathrm{Re}\, g_{\pi\pm} \neq 0$ as $k \to \infty$).

Our conclusion also does not apply to Wolfenstein's parameter $\sin\beta \sim \mathrm{Im}(g^* ih)$,[6] i.e., the change in the spin direction as a result of the $\pi^\pm$ scattering on polarized protons does not necessarily vanish at very high energies if $\sigma_{ex}(\infty, \Delta^2) = 0$.

2) Equalities (13) and (14) and the relations which follow from them are also valid for any final momentum transfer as the energy tends to infinity. Since $\sin(\theta/2) = \Delta/k$, then (13) and (14), formally speaking, are valid only for the zero angle. Nevertheless, the relations obtained are useful, since, according to the diffraction picture, the overwhelming majority of the scattered particles are already in the region $\Delta \lesssim 3\mu$.

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