INELASTIC PION-NUCLEON INTERACTIONS AT HIGH ENERGIES

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We consider peripheral pion-nucleon interactions in the single-meson approximation. A comparison of the calculated values of the multiplicity of the particles created, of the angular distribution and energy distribution of the recoil nucleon, and of its transverse momentum with experimental data shows that at high energies the π-N collisions caused by π-π interactions are the dominant ones.

1. INTRODUCTION

In the present state of the theory quantitative characteristics of strong interaction processes can be obtained only by using some well-defined assumptions about the mechanism of the phenomenon. Fermi's statistical theory was for a long time such a model theory. One of the authors, however, criticized this theory already in 1956 in a contribution to a CERN symposium and suggested to divide inelastic interactions into "central" and "peripheral" ones.[1,2] Since then many new experimental data have been obtained on N-N and π-N collisions at high energies and several calculations have been performed (see [3-8] where a detailed bibliography is given). It seems at the present moment that this more detailed picture is in satisfactory agreement with experiment.

We describe in the following calculations of inelastic π-N interactions at energies $E > 1$ BeV which were recently performed in Dubna using the physical picture of two kinds of collisions.

2. π-N INTERACTIONS IN THE SINGLE-MESON APPROXIMATION

We performed our calculations assuming that the momentum exchange mechanism between the pion and the nucleon was a single-meson one. Typical diagrams are given in Fig. 1. It is clear that we need distinguish processes with an even and with an odd number of created pions. In the first case the basic process is the creation of pions in peripheral π-π collisions (diagram A). In the case of an odd number of pions this process is accompanied by the creation of one pion during the scattering of the virtual meson by the nucleon (diagram B).

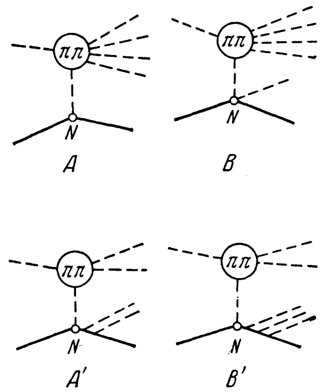

FIG. 1. The most important typical diagrams A and B for inelastic π-N scattering and the less essential diagram A′ and B′.

The process given in diagram A′ of Fig. 1 may compete with the process of diagram A. Similarly, process B may be imitated by the process given by diagram B′.

Rodberg[9] has shown that diagram a, corresponding to an interaction with the nucleon core (see Fig. 2a), gives an appreciably smaller contribution than diagram b (Fig. 2b) which is a π-N interaction. These diagrams occur as components in the diagrams of Fig. 1. For this reason it turns out to be advantageous, in particular, to replace two pions created by a nucleon core (diagram A′) by two pions created in a π-π collision (diagram A). A similar argument applies also to diagrams B and B′.

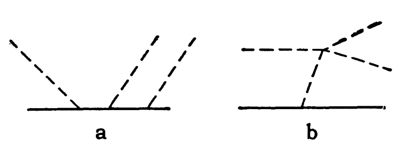

FIG. 2. Diagram a has an appreciably smaller amplitude than diagram b.

It is thus sufficient to consider only processes A and B. The cross sections corresponding to them are of the form

$$\sigma_{2n}(E) = g^2 \frac{1}{8\pi^2 v} \int_0^{q_{max\,n}} \frac{q^2 dq}{q_0 p_0 \omega} \sigma_{\pi\pi}^{(2n)}(Q)\, K(Q) \left\{ \frac{1}{4pq} \ln\left(1 + \frac{4pq}{2p_0 q_0 - 2pq - 2M^2 + \mu^2}\right) - \frac{\mu^2}{(2p_0 q_0 - 2M^2 + \mu^2)^2 - 4p^2 q^2} \right\}, \tag{1}$$

$$\sigma_{2n+1}(E) = \frac{1}{4\pi^3 v} \int_0^{P_{max\,n}} \frac{P^2 dP}{p_0 \omega} \int_{\sqrt{P^2 + (M+\mu)^2}}^{E - \sqrt{P^2 + m_n^2}} K(S)\, \sigma_{\pi\pi}^{(2n)}(S)\, dP_0 \, \frac{\sigma_{\pi N}(p) \sqrt{(R^2 - M^2 - \mu^2)^2 - 4M^2\mu^2}}{(2P_0 p_0 - M^2 - R^2 + \mu^2)^2 - 4P^2 p^2}. \tag{2}$$

Here

$$q_{max\,n} = \frac{1}{2E} \sqrt{(E + M + m_n)(E + M - m_n)(E - M + m_n)(E - M - m_n)},$$
$$K(Q) = Q^2 \sqrt{1 - (2\mu/Q)^2},$$
$$S^2 = (E - P_0)^2 - P^2, \quad Q^2 = (E - q_0)^2 - q^2, \; R^2 = P_0^2 - P^2,$$
$$p_0^2 = p^2 + M^2; \qquad q_0^2 = q^2 + M^2,$$
$$P_{max\,n} = q_{max\,n} \big|_{m_n \to m_n + \mu},$$

E is the total energy of the primary particles in the c.m.s. (center of mass system), p the momentum of the primary nucleon in the c.m.s., M the nucleon mass, m the total mass of the particles created in the π-π collision, v the relative velocity of the pion and the nucleon in the c.m.s., $\sigma_{\pi\pi}^{(n)}$ the π-π interaction cross section, g^2 the π-N coupling constant, and ω and μ the pion energy and mass in the c.m.s. Equation (2) has a meaning if only pions are created during the π-π collision; if nucleon-antinucleon pairs are created Eq. (1) is valid both for an even and for an odd number of pions.*

In the present calculation where we assume that we can apply the single-meson approximation for arbitrarily large momentum transfers (we shall see in the following that the contribution from very large momentum transfers turns out to be, indeed, small) all π-π interactions are connected at the vertices of the diagrams A and B. Diagram B can then be replaced by diagram D (Fig. 3) which is equivalent.

Calculations have shown that diagrams such as D contribute appreciably less to the π-N interaction than diagrams such as A. This is clear from Table I where we give the ratios of the cross sections for even and odd numbers of mesons for different energies E_0 of the primary pion.

For each value of n the ratio of cross-sections $\sigma_{2n+1}/\sigma_{2n}$ increases with increasing energy E_0, but with increasing energy the relative contribution of a channel with given n decreases fast. One can say that channels giving the main contribution to the interaction lie around the diagonal of the table; in that case $\sigma_{2n} \gg \sigma_{2n+1}$.

*We note that one obtains also numerically close results by using the von Weizsäcker-Williams method.[8]

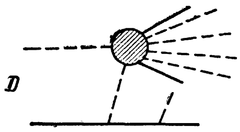

FIG. 3.

Table I

E_0, BeV	σ_3/σ_2	σ_5/σ_4
2	0.1	
7	0.15	
100	1	0.25
1000	2	0.5

The data from Table I show that a study of the creation of an odd number of mesons at high energies of the primary pion may serve as a means of studying the nucleon core. One must, however, bear in mind that these channels are secondary ones; the creation of a large, even number of pions will be the dominant process at these high energies.

3. THE π-π INTERACTION

If we wish to evaluate the relative multiplicity of pions, the momentum transfer, and the angular distributions we need know the partial π-π interaction cross sections. The partial cross sections for the inelastic interactions were evaluated assuming that there were no correlations between the particles created in the π-π collision. Such an assumption leads to a factorization of the matrix element for the creation of particles during π-π collisions, of the form $\sim\Omega^{n-1}(E)\rho_n(E)$, where E is the energy of the primary particles in the c.m.s., $\rho_n(E)$ the momentum-space volume for n particles, and $\Omega(E)$ a three-dimensional volume. The calculations were performed making the assumption of the Fermi theory*

$$\Omega(E) = (\hbar/\mu c)^3 Mc^2/E.$$

In our statistical calculations of the π-π interactions we took account of the resonance interaction of the pions at $\omega = M^*_{\pi\pi} = 0.6$ M in the state with isotopic spin T = 1 and spin S = 1. Channels with n up to 8 were taken into consideration in the calculations, as well as channels involving the creation of nucleon-antinucleon pairs.

Along with inelastic π-π interactions we must also take into account the elastic π-π scattering. There are at the present moment no experimental data whatever about the cross section for such a scattering. One might expect from an analogy with the π-N and the N-N interaction[11] that the cross section $\sigma_{el}(\pi\pi)$ for the elastic π-π scattering would be about a third of the cross section for the inelastic π-π scattering. Estimates showed that within the limits of accuracy of the present-day experimental data the results of the calculations are not very sensitive to the assumptions made about the magnitude of $\sigma_{el}(\pi\pi)$.

If we take the inelastic cross section for a π-N collision to be equal to 23 mb and the π-N interaction constant $g^2 = 14.5$ we can estimate from Eqs. (1) and (2) the effective cross section for the π-π interaction: $\sigma_{\pi\pi} \sim 40$ mb.

4. RESULTS OF THE CALCULATIONS

We give in Fig. 4 the calculated multiplicity of the particles created (counting the recoil nucleon as one):†

$$\bar{n}(E_0) = \sum_n (2n+1)\,\sigma_{2n}(E_0) \Big/ \sum_n \sigma_{2n}(E_0). \tag{3}$$

One can see from the figure the increase in multiplicity with energy which agrees well with experimental data. (See [4-6] for references to experimental papers.)

In Fig. 5 we show the momentum spectrum of the recoil protons in the c.m.s. for a primary π^--meson energy $E_0 = 7$ BeV (l.s.)

$$W(q) = \frac{q^2}{2q_0} K(Q) \left\{ \frac{1}{4pq} \ln\left(1 + \frac{4pq}{2p_0q_0 - 2pq - 2M^2 + \mu^2}\right) - \frac{\mu^2}{(2p_0q_0 - 2M^2 + \mu^2)^2 - 4p^2q^2} \right\} \sum_i n^i_p \sigma^i_{\pi\pi}(S), \tag{4}$$

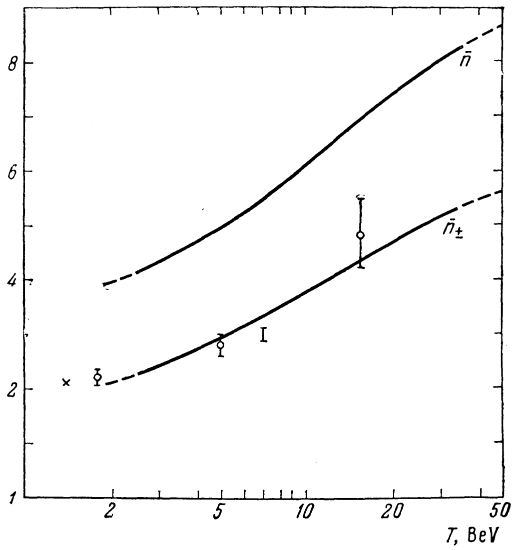

FIG. 4. Average number of particles created in π-p collisions at different energies of the primary pion (in the l.s.). $\bar{n}$ is the total number, $\bar{n}_\pm$ the number of charged particles. The recoil nucleon is also included in these values.

where n^i_p is the number of recoil protons; $\sigma^i_{\pi\pi} = w_i\sigma_{\pi\pi}$ is the cross section of the i-th π-π interaction channel and w_i the statistical weight of this channel. In the calculations we assumed that the cross section $\sigma_{\pi\pi}$ does not depend strongly on energy (this does not refer to the partial cross sections $\sigma^i_{\pi\pi}$ which may show resonance features at low energies; cf. Sec. 3). The rest of the notation is as in Eqs. (1) and (2).

It is clear that the calculated values of the recoil momenta are close to the experimental ones; however, as a whole, the theoretical spectrum W(q) is softer than the experimental one.

In Fig. 6 we have shown for a π^--meson energy $E_0 = 7$ BeV the angular distribution of the recoil protons in the c.m.s.:

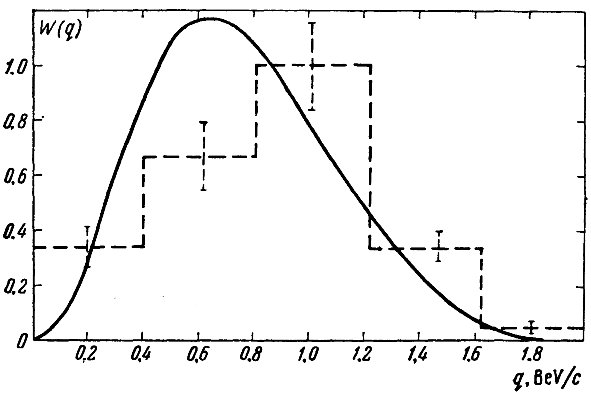

FIG. 5. Momentum spectrum of the recoil protons (c.m.s.) at an energy of the incident pion of $E_0 = 7$ BeV. The dotted line indicates the experimental histogram from [5]. The values are given in units of BeV/c.

*Assuming also that there is no correlation in the initial state leads to $\Omega(E)$ = const (Sudarshan's theory); in practice both assumptions give closely similar results (see also[10]).

†The numerical calculations were performed on the M-20 computer in the calculating center of the Joint Institute for Nuclear Research.

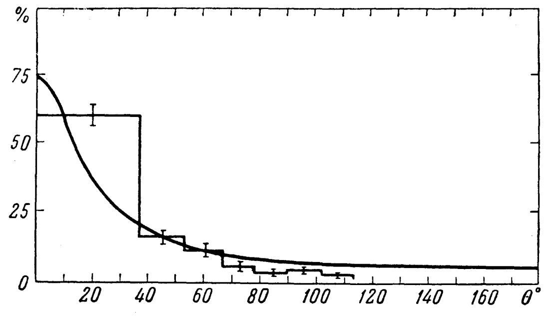

FIG. 6 Angular distribution of the recoil protons (c.m.s.) at an incident pion energy of $E_0 = 7$ BeV. The histogram shows the experimental data from[4].

$$W(\theta) = \sum_i n_p^i \int_0^{q_{max\,i}} \frac{q^2}{2q_0} K(Q)\,\sigma^i_{\pi\pi}(S)$$

$$\times \frac{(p_0q_0 - pq\cos\theta - M^2)\,dq}{(2p_0q_0 - 2pq\cos\theta - 2M^2 + \mu^2)^2}. \qquad (5)$$

The agreement between experiment and theory is satisfactory.

In Fig. 7 we give the angular distribution of the recoil protons, $W(\theta)$, for $E_0 = 3$ and 16 BeV. These distributions are normalized such that

$$2\pi \int_0^\pi W(\theta)\sin\theta\, d\theta = 1.$$

In Table II we show for different values of the π^--meson kinetic energy E_0 (in the l.s.) the average values of the momentum of the recoil proton (in the c.m.s.)

$$\bar{q} = \int qW(q)\,dq \Big/ \int W(q)\,dq \qquad (6)$$

and the average values of its transverse momentum

$$\bar{q}_\perp = \int q_\perp W_\perp(q_\perp)\,dq_\perp \Big/ \int W_\perp(q_\perp)\,dq_\perp, \qquad (7)$$

where

$$W_\perp(q_\perp) = q_\perp \sum_i n_p^i \int_{-\sqrt{q^2_{max\,i} - q^2_\perp}}^{\sqrt{q^2_{max\,i} - q^2_\perp}} \frac{K(Q)}{2q_{0\perp}} \sigma^i_{\pi\pi}(S')$$

$$\times \frac{p_0q_{0\perp} - pt - M^2}{(2p_0q_{0\perp} - 2pt - 2M^2 + \mu^2)^2}\,dt,$$

$$S' = \sqrt{(E - q_{0\perp})^2 - q_\perp^2 - t^2}, \quad q^2_{0\perp} = M^2 + q^2_\perp + t^2. \qquad (8)$$

It is clear from Table II that the theoretical values $\bar{q}_\perp$ change very slowly in accordance with the experimental data obtained in accelerators and in cosmic-ray experiments.

One can from the calculations performed reach the conclusion that the peripheral π-π interaction mechanism involving a single-meson exchange is the dominant one.

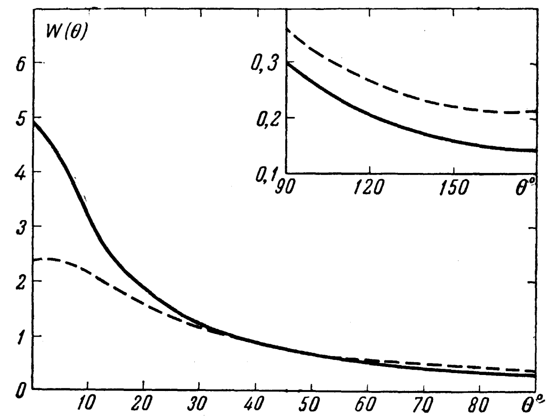

FIG. 7. Angular distribution of the recoil protons (c.m.s.) for an incident π^--meson energy of $E_0 = 3$ BeV (dotted line) and of 16 BeV (full-drawn curve). The values are in units of BeV/c.

Table II

E_0, BeV	$\bar{q}$, BeV/c theory	$\bar{q}$, BeV/c experiment	$\bar{q}_\perp$, BeV/c theory	$\bar{q}_\perp$, BeV/c experiment
3	0,5	—	0,3	—
7	0.75	0.89±0.04	0.4	0.37±0.04
10	0.8	—	0.45	—
16	1.05	—	0.5	0.4

One should, however, bear in mind that the reliability of the known experimental data is not yet very high and the question of the contribution of the many-meson processes which may essentially influence the transfer of large momenta remains so far open.

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