DEVIATIONS OF THE P^{32} β SPECTRUM FROM THE ALLOWED SHAPE

CH'ING CH'ENG-JUI and L. S. NOVIKOV

Institute of Nuclear Physics, Moscow State University

Submitted to JETP editor July 26, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 42, 364-369 (February, 1962)

It is shown that deviations of the β spectrum from the allowed shape are well described by a form factor of the form of Eq. (4), which is equivalent to the Gell-Mann correction for the weak magnetism effect.[1,3] The sign of the deviation differs from that given by Gell-Mann for the case of electron decay.

THE study of β spectra has acquired new interest in connection with possible deviations from the allowed shape which stem from Gell-Mann's hypotheses regarding the nonrenormalizability of the vector interaction constant in β decay, as well as from radiative corrections.[1-3]

A β transition of the type $\Delta I = \pm 1$ (no) is in general dependent on the axial vector variant of the interaction (through the principal matrix element $\int\sigma$ and the second-forbidden matrix element $\int\gamma_5 \mathbf{r}$). The vector variant in this transition only gives rise to a correction in the form of the second-forbidden matrix element $\int \boldsymbol{\alpha} \times \mathbf{r}$. According to Gell-Mann, the matrix element of the vector interaction equals, apart from a numerical factor, the matrix element of the magnetic dipole γ transition from a state of the same isotopic spin. This matrix element differs from the corresponding matrix element in the old theory by $\mu/\sqrt{2}$, where μ is the magnetic moment of the corresponding electromagnetic transition. Gell-Mann designated by the term "weak magnetism effect" those corrections to the β-spectrum shape which are due to interference between the principal matrix element of the axial vector interaction and the second-forbidden matrix element of the vector interaction.

According to Gell-Mann's calculations, the correction factor for an allowed β spectrum of the type $\Delta I = \pm 1$ (no) which takes into account the three matrix elements ($\int\sigma$, $\int\gamma_5\mathbf{r}$, and $\int \boldsymbol{\alpha} \times \mathbf{r}$) will have the form

$$1 + \tfrac{8}{3} AE - \tfrac{2}{3} B/E, \qquad (1)$$

where E is the electron energy and A and B are constants proportional to the ft value of the given transition. The AE term gives the correction for the weak magnetism effect.

The coefficient A is related to the radiative width of the level with the same isotopic spin by the expression

$$A = \frac{1}{E_\gamma}\left[\frac{3}{4}\left(\frac{137\,\Gamma_\gamma}{E_\gamma}\right)\frac{ft}{(ft)_{O^{14}}}\right]^{1/2}, \qquad (2)$$

where Γ_γ is the width of the level with the same isotopic spin and E_γ is the energy of the γ transition. Thus, a simultaneous measurement of the shape of the β spectrum and the γ width of the level of the corresponding M1 transition constitutes a complete test of Gell-Mann's hypothesis regarding weak magnetism.

It is distinctly interesting to examine the shape of the allowed β spectrum of P^{32} ($\Delta I = \pm 1$, no). This spectrum has an end-point energy $E_0 = 1710$ keV and an ft value of 7.9, which is large for an allowed spectrum. For this reason we can expect that the AE term in the energy region near E_0 will give appreciable corrections to the spectrum shape.

Several works have recently been devoted to the investigation of deviations of the P^{32} β spectrum from the allowed shape.

Deviation from the Kurie plot in the β spectra of P^{32}, K^{42}, and As^{76} was studied by Pohm, Waddell, and Jansen.[4] The investigations were made on a thin-lens spectrometer, the resolving power of the apparatus being 4%. The form factor obtained in the experiment for the P^{32} spectrum was a straight line parallel to the energy axis.

Porter, Wagner, and Freedman[5] investigated the β spectra of Na^{24} and P^{32}. The investigations were performed on a double-lens spectrometer with 2% resolution. No deviations from the allowed shape were observed in the spectrum of Na^{24}. The form factor for the P^{32} β spectrum sloped towards the energy axis. A function of the form $1 + B/E$ was chosen to describe the form factor. For B a value was found lying within the limits $0.05 < B < 0.093$.

Coefficients* of the curves describing the P^{32} form factor, calculated by the method of least squares

Source no.	Density, mg/cm²		$y = a(1 + AE + B'E)$ (260 — 1500 keV)	$y = a'(1 + A'E)$ (800 — 1500 keV)	$y = a''(1 + B''/E)$ (260 — 1500 keV)
	Active layer	Backing			
1	0.2	1.5	$A = -5.45 \cdot 10^{-5}$, $B = 61.82$	$A' = -7.71 \cdot 10^{-5}$	$B'' = 95.18$
2	0.1	0.3	$A = -4.81 \cdot 10^{-5}$, $B = 77.16$	$A' = -9.49 \cdot 10^{-5}$	$B'' = 111.4$
3	0.1	0.3	$A = -4.38 \cdot 10^{-5}$, $B = 48.17$	$A' = -7.79 \cdot 10^{-5}$	$B'' = 89.31$
Average of the three measurements in units of $(mc^2)^{-1}$ and mc^2 respectively			$A = (-2.49 \pm 0.70) \cdot 10^{-2}$, $B = 0.12 \pm 0.03$	$A' = (-4.25 \pm 1.50) \cdot 10^{-2}$	$B'' = 0.195 \pm 0.05$

*The coefficients are given everywhere, except in the last row, in units of: A — keV^{-1}, B — keV.

Deviations in the low energy region for In^{114}, Y^{90}, and P^{32} spectra were investigated by Johnson, Johnson, and Langer.[6] The investigations were carried out on a spectrometer with a constant magnetic field. At low energies, deviations from the allowed shape were found in all the β spectra measured. The authors described the form factor by a function of the form 1 + B/E, the experimentally determined value of B lying within the limits $0.2 < B < 0.4$.

Daniel[7] measured the β spectra of Na^{22}, Na^{24}, and P^{32} on an iron-free spectrometer. No deviations were detected for Na^{22}. An indeterminate result was obtained for Na^{24}. A deviation from the Kurie plot was observed in the β spectrum of P^{32}. The form factor was described in this case by a straight line of the form 1 + AE, the experimental value of the coefficient A being $A = (-4.1 \pm 1.3) \times 10^{-2}$ $(mc^2)^{-1}$.

1. MEASUREMENTS

The shape of the P^{32} β spectrum was measured in the present work. The measurements were performed on a β spectrometer with double focusing ($\pi\sqrt{2}$), the mean radius of the trajectory being $r_0 = 22.5$ cm. A stability accurate to 0.01% was obtained for the magnetic field in the apparatus. The magnetic field was measured with an accuracy of 0.03%, and was calibrated with γ lines of ThB, Co^{60}, and Cs^{137}. The magnetometer readings in the 200 — 2000 keV energy range were linear to within 0.17%.

To reduce background, a diaphragm cut out in the shape of the electron beam was set in front of the counter.

An ordinary Geiger counter was used as detector (with a plateau 400 volts long having a slope of 0.5% per 100 volts). The voltage at the working point was maintained within an accuracy of 1%. The stability of the pressure in the counter was maintained within an accuracy of 0.5%. A film of celluloid dissolved in amyl acetate (density of film 0.06 mg/cm²) was fastened to the counter window.

The pressure in the spectrometer chamber was maintained at less than 10^{-4} mm Hg.

Source preparation. All the sources were prepared by the evaporation method. Before the active layer was deposited, the aluminum foil backing was treated in a water solution of insulin. The densities of the active layer and of the aluminum backing for the different sources are given in the table.

Measurement conditions. Before and after measuring the spectrum, the counter characteristics were recorded, the position of the ThB F line was measured, the stability of counter performance was checked, and the background was measured. When making measurements in the energy region up to 1600 keV, not less than 10^4 counts were recorded for each point. In all, 40 — 60 points at approximately equal energy intervals were measured on the spectrum. The spectrum was recorded from low to high-energies and then in the reverse order. Taking into account corrections for decay, the value at a given point at the beginning and at the end of the measurements was the same within the statistical error. The spectrum measurements were made in the energy region between 260 keV and the end of the spectrum. The resolution, determined from the width of the F line of ThB deposited on the same backing that was used for the measurements of the spectrum under investigation, was found to equal 0.45%.

2. RESULTS OF THE MEASUREMENTS

In all, three series of measurements for three different P^{32} sources were analyzed. For the

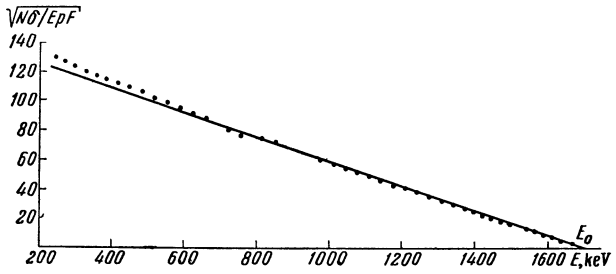

FIG. 1. Kurie plot for the β spectrum of P^{32}.

analysis, Kurie plots were constructed (see Fig. 1) from which the end-point energy of the spectrum E_0 was determined. The points with energy $E > 1000$ keV were used to determine the end-point energy, which was found to be $E_0 = 1712 \pm 2$ keV. Statistical error in the region of energies up to 1500 keV does not exceed 1%.

Due to the smallness of the deviation from the allowed shape, it is impossible to judge its size and form from the Kurie plot. Therefore, in order to investigate more precisely the distribution of the experimental points on the Kurie plot, the form factor was plotted as a function of electron energy:

$$y = N\delta / EpF\,(E_0 - E)^2, \qquad (3)$$

where N is the number of counts in a constant energy interval, F is the Fermi function, E_0 is the end-point energy of the spectrum, p is the electron momentum, and δ is a function which reduces the β spectrum to equal energy intervals. It can be seen from Fig. 2 that the P^{32} form factor slopes towards the energy axis, i.e., the β spectrum deviates from the allowed shape.

The influence of various factors on the deviation of the P^{32} β spectrum from the Kurie plot was examined.

a) The end-point energy of the spectrum E_0 was determined to within 2 keV. Calculations performed for the values E_0 and $E_0 \pm 2$ showed that such an error in determining E_0 has practically no effect on the slope of the form factor.

b) The background constituted less than 1% of the count for a given E, and was itself determined to within 0.3% of the value of the count. Consequently, an error in measuring the background could not displace the points on the form-factor plot by more than 0.3%.

c) We measured, along with the P^{32} spectrum, the β spectrum of In^{114}, which has the same Gamow-Teller transition $1^+ \rightarrow 0^+$ as P^{32} as well as a similar end-point energy ($E_0 = 1978$ keV). The In^{114} form factor at energies $E > 800$ keV was obtained in the form of a straight line parallel to the energy axis, despite the fact that the In^{114} source had a considerably higher density than the P^{32} source. Consequently, the deviation from the allowed shape noted in the P^{32} β spectrum cannot be explained by instrumental error or by the quality of the source.

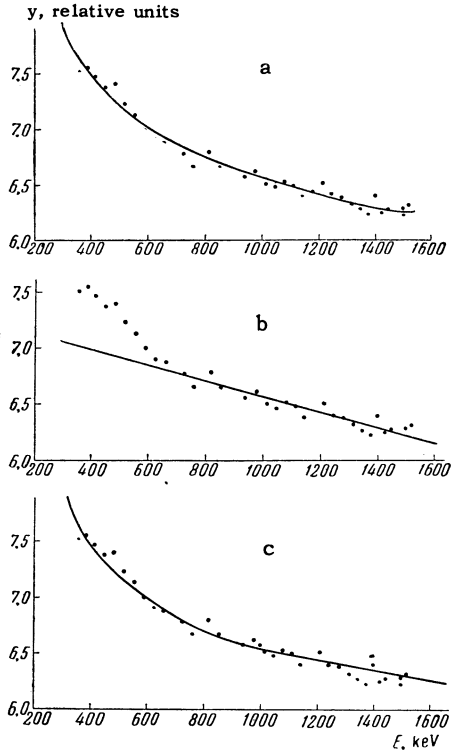

FIG. 2. Form factor for the P^{32} β spectrum. The solid curves are calculated from: a – the formula $y = a(1 + AE + B/E)$, b – $y = a'(1 + A'E)$, c – $y = a''(1 + B''/E)$.

d) Moreover, measurements made with sources of varying thickness showed that the slope of the P^{32} form factor does not depend upon the thickness of the source: comparable results were obtained both for a thick source (2.7 mg/cm^2) and for a carrier-free source in the energy region $E > 700$ keV.

3. ANALYSIS OF THE P^{32} FORM FACTOR

In order to describe the deviation of the β spectrum from the allowed shape in the general case, one may introduce corrections which are proportional to the energy (of the form AE), inversely proportional to the energy (of the form B/E), proportional to the square of the energy (CE^2), and so on.

To describe the P^{32} form factor obtained in our study, a curve was calculated for the 260 – 1700 keV energy region, with account of corrections that were both proportional and inversely proportional to the energy, of the form

$$y = a(1 + AE + B/E). \qquad (4)$$

The coefficients A and B, computed by the method of least squares, are listed in the table. It is evident from Fig. 2a that the P^{32} form factor is described satisfactorily by a curve of form (4).

In addition, form factors of the following types were employed

$$y = a'(1 + A'E), \quad (5)$$
$$y = a''(1 + B''/E), \quad (6)$$

whose coefficients were also computed by the method of least squares.

A calculation based on (5) was performed for the energy region E > 800 kev; Eq. (6) was used for energies ranging from 260 to 1700 keV. The results of these computations are also presented in the table. Expression (5), as can be seen from Fig. 2b, describes well the experimental form factor in the energy region E > 800 keV; in the lower-energy part of the spectrum all of the experimental points lie above this straight line.

Calculation of the form factor (6) over the entire spectrum showed that the experimental shape factor can be described by this curve in the entire energy region considered (see Fig. 2c). However, this curve fits the experimental points less well than the curve described by (4). The β spectrum of P^{32} in the energy region 260 — 1700 keV is therefore best described by Eq. (4), where corrections to the spectrum both directly and inversely proportional to the energy are included.

The calculation made with Eq. (4) yielded the following average values of the coefficients A and B:

$$A = (-2.49 \pm 0.70) \cdot 10^{-2} (mc^2)^{-1},$$
$$B = 0.12 \pm 0.05 mc^2.$$

4. DISCUSSION

In Daniel's article[7] the form factor for the P^{32} spectrum was chosen in the form 1 + AE, where $A = (-4.1 \pm 1.3) \times 10^{-2} (mc^2)^{-1}$. In our case the form factor calculated from the same formula in the energy region E > 800 keV gives approximately the same value: $A = (-4.25 \pm 1.50) \times 10^{-2} (mc^2)^{-1}$.

When the calculation was carried out using formula (4), which in addition considers a correction inversely proportional to the energy (B/E), the contribution from the term AE was reduced, and the coefficient $A = (-2.49 \pm 0.70) \times 10^{-2} (mc^2)^{-1}$.

In [5,6], where the form factor was chosen in the form 1 + B/E, the coefficient B was found to equal $0.05 < B < 0.093$ and $0.2 < B < 0.4$ respectively. Our calculation made from the same equation gave for B a value lying within the limits $0.175 < B < 0.23$ (see table). However, if we use (4), where the contribution of the AE term is taken into account, the magnitude of the coefficient B is somewhat decreased: $B = 0.122 \pm 0.050\ mc^2$.

It can therefore be said that the results of our measurements are in agreement with the results obtained by other authors.[5-7]

Our form factor given by (4) is equivalent to β-spectrum corrections derived by Gell-Mann (see [1]). When deriving this correction, Gell-Mann took into account the principal axial vector matrix element $(\int \sigma)$, the second-order matrix element of the vector variant of the interaction $(\int \alpha \times \mathbf{r})$, and the second-order matrix element of the axial-vector variant of the interaction $(\int \gamma_5 \mathbf{r})$. The terms of higher order, corresponding to the E2 transition, were not considered. These matrix elements are of the form: $\int \mathbf{r}^2$ from the vector variant, $\int \sigma \mathbf{r}^2$ and $\int (\sigma \cdot \mathbf{r})\,\mathbf{r}$ from the axial vector variant of the interaction. If the effect of the Coulomb field on the electron wave function is not taken into account, they are both proportional to $(E_0 - E)(E - 1/E)$. Rough estimates made by Morita[8] show that the contribution from these matrix elements is at least an order of magnitude smaller than that from the matrix elements considered by Gell-Mann.

According to the results of a number of studies (see, for example, [9]), the Coulomb field causes deviations from the allowed spectrum shape of the form $k(E_0 - E)$. In the case of P^{32} these corrections may be as high as 2 — 5%. In view of the fact that the Coulomb field correction and the contribution from the weak magnetism effect are of opposite sign and partially compensate for one another, our value for the coefficient A is possibly the sum of these two effects. In that case the deviation of the spectrum shape due to the weak magnetism effect must be somewhat larger than that obtained in our calculations.

Gell-Mann, examining β transitions of B^{12} and N^{12}, whose spectrum end-point energies lie in the 13 — 16 MeV region, restricted himself to considering a spectrum correction of the form 1 + AE; he discarded the term B/E because of its small contribution in high-energy transitions. The coefficients A and B were considered to be of the same order of magnitude. However in the case of P^{32}, where the principal term of the axial vector variant of the interaction $(\int \sigma)$ is small, interference terms from the axial vector variant can

make a considerable contribution to a deviation from the allowed shape of the β spectrum and, consequently, the coefficient B can be large in absolute magnitude. Moreover, the P^{32} spectrum lies in a lower energy region than the spectra investigated by Gell-Mann, so that the term B/E in the case of P^{32} can make a noticeable contribution.

According to our calculations, the deviations caused by the terms AE and B/E in the energy region near the end of the spectrum are approximately of the same order. It should be noted that the sign of the AE term obtained both in our study and in Daniel's[7] is different from that indicated by Gell-Mann for the case of electron decay.

If we assume that the value for the coefficient A obtained by us is determined exclusively by the weak magnetism contribution, the width of the corresponding γ transition, calculated from formula (2), is $\Gamma_\gamma = 0.05$ eV.

V. V. Kravtsov and V. M. Yadrikhinskaya took part in the present study.

The work was carried out in the nuclear spectroscopy laboratory of the Scientific-Research Institute of Nuclear Physics of the Moscow State University under the direction of V. S. Shpinel'.

The authors thank I. S. Shapiro for his advice and consultation both during the work and in the writing of the present article.

[1] M. Gell-Mann, Phys. Rev. 111, 362 (1958).

[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[3] J. Bernstein and R. R. Lewis, Phys. Rev. 112, 232 (1958).

[4] Pohm, Waddell, and Jansen, Phys. Rev. 101, 1315 (1956).

[5] Porter, Wagner, and Freedman, Phys. Rev. 107, 135 (1957).

[6] Johnson, Johnson, and Langer, Phys. Rev. 112, 2004 (1958).

[7] H. Daniel, Nucl. Phys. 8, 191 (1958).

[8] M. Morita, Phys. Rev. 113, 1584 (1959).

[9] I. Iben, Phys. Rev. 109, 2059 (1958).

Translated by Mrs. J. D. Ullman
57