# *THE INCLUSION OF NEUTRON-PROTON INTERACTION IN THE PAIR CORRELATION MODEL*

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Submitted to JETP editor July 28, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 479-484 (February, 1962)

Quadrupole excitations of even-even spherical nuclei are treated including the p-n interaction. Formulas are derived for the energies of the two $2^+$ levels and for the reduced probabilities of E2 transitions between them and to the ground state.

## INTRODUCTION

In treating the ground and excited states of nuclei on the basis of the pair correlation model, the interaction between protons and neutrons is usually not considered. It is assumed that, because of the large difference in positions of the proton and neutron Fermi levels, the p-n interaction gives a negligibly small contribution to the Cooper pairing, so that including it results only in a change in the self-consistent field of the nucleus and a renormalization of the constants for the residual interaction between quasiparticles (for example, the quadrupole-quadrupole interaction constant). One should, however consider the following points: first, the absence of an influence of the p-n interaction on the pairing effect does not at all mean that it need not be considered in treating the collective excitations, since its contribution to the energy of residual interaction between quasiparticles is of the same order of magnitude as the contributions from the p-p and n-n interactions; second, the difference in location of the Fermi level may be compensated by the Coulomb interaction, so the investigation of the influence of the p-n interaction on the pairing effect ought to be carried out in more detail. In the present paper, using the fact that p-p and n-n pairings occur in states with zero angular momentum and that the p-n interaction does not change the charge of the interacting pair, we show that its presence does not cause any mixing of the proton and neutron amplitudes in the canonical Bogolyubov transformation. For this purpose we use the method developed by Bogolyubov.[1]

The compensation equations have the form

$$A(f_1f_2|F\Phi) = \langle [a_{f_1}a_{f_2};\ \hat{H}]\rangle_0 = 0,$$

$$B(f_1f_2|F\Phi) = \langle [a^+_{f_1}a_{f_2};\ \hat{H}]\rangle_0 = 0, \qquad (1)$$

where

$$H = \sum_{f_1f_2} T(f_1f_2)\, a^+_{f_1}a_{f_2} - \frac{1}{2}\sum_{f_1f_2f_3f_4} U(f_1f_2;\ f_3f_4)\, a^+_{f_1}a^+_{f_2}a_{f_4}a_{f_3},$$

$$\Phi(f_1f_2) = \langle a_{f_1}a_{f_2}\rangle_0 = -\Phi(f_2f_1),$$

$$F(f_1f_2) = \langle a^+_{f_1}a_{f_2}\rangle_0 = F^*(f_2f_1) \qquad (2)$$

and the quantities F and $\Phi$ are related by the following auxiliary conditions:

$$\sum_f [F(f_1f)\,F(ff_2) + \Phi^*(ff_1)\,\Phi(ff_2)] = F(f_1f_2),$$

$$\sum_f [F(f_1f)\,\Phi^*(ff_2) + F(f_2f)\,\Phi^*(ff_1)] = 0. \qquad (3)$$

After solution of system (1), Eqs. (2) determine the coefficients of the canonical transformation

$$a_f = \sum_\nu [u_{f\nu}\alpha_\nu + v_{f\nu}\alpha^+_\nu]. \qquad (4)$$

In the case of spherical nuclei, f = j, m, t, where j and m are the angular momentum and its projection for an individual nucleon, t is the projection of the isotopic spin of the nucleon,

$$T(f_1f_2) = (\varepsilon_{t_1j_1} - \lambda - \mu t_1)\,\delta_{t_1t_2}\delta_{j_1j_2}\delta_{m_1m_2},$$

$$U(f_1f_2;\ f_3f_4) = -U(t_1j_1m_1t_2j_2m_2;\ t_2j_3m_3t_1j_4m_4)\,\delta_{m_1+m_2,\,m_3+m_4}\delta_{t_4t_1}\delta_{t_3t_2}. \qquad (5)$$

We look for a solution of Eqs. (1) of the form

$$\Phi(f_1f_2) = \Phi_1(j_1m_1j_2m_2)\,\delta_{t_1t_2} + \Phi_0(j_1m_1j_2m_2)\,\delta_{t_1,-t_2},$$

$$F(f_1f_2) = F_1(j_1m_1j_2m_2)\,\delta_{t_1t_2} + F_0(j_1m_1j_2m_2)\,\delta_{t_1,-t_2},$$

$$\Phi_1(j_1m_1j_2m_2) = (-1)^{j_1-m_1}\varphi_{t_1j_1}\delta_{j_1j_2}\delta_{m_1,-m_2},$$

$$F_1(j_1m_1j_2m_2) = f_{t_1j_1}\delta_{j_1j_2}\delta_{m_1m_2},$$

$$\Phi_0(j_1m_1j_2m_2) = \sum_J C^{JM}_{j_1m_1j_2m_2}\Phi_{JM}(j_1j_2),$$

$$F_0(j_1m_1j_2m_2) = (-1)^{j_2+m_2}\sum_J C^{JM}_{j_2-m_2j_1m_1}F_{JM}(j_1j_2). \qquad (6)$$

We start with the solution of the equations for $\Phi_0$ and $F_0$. Substituting (6) in (1) and using the relation

$$U(t_1 j_1 m_1 t_2 j_2 m_2;\, t_2 j_3 m_3 t_1 j_4 m_4) = \sum_J C^{JM}_{j_1 m_1 j_2 m_2} U_J(t_1 j_1 t_2 j_2;\, t_2 j_3 t_1 j_4)\, C^{JM}_{j_4 m_4 j_3 m_3},$$

we get

$$A^{\pm}_J(j_1 j_2 \mid F\Phi) = (e_{t_1 j_1} + e_{t_2 j_2})\,\Phi^{\pm}_J(j_1 j_2) + (1 - f_{t_1 j_1} - f_{t_2 j_2})\, S^{\pm}_J(j_1 j_2) - (\Delta_{t_1 j_1} \pm \Delta_{t_2 j_2})\, F^{\pm}_J(j_2 j_1) - (\varphi_{t_1 j_1} \pm \varphi_{t_2 j_2})\, E^{\pm}_J(j_1 j_2) = 0,$$

$$-B^{\pm}_J(j_2 j_1 \mid F\Phi) = (e_{t_2 j_2} - e_{t_1 j_1})\, F^{\pm}_J(j_2 j_1) + (f_{t_2 j_2} - f_{t_1 j_1})\, E^{\pm}_J(j_1 j_2) - (\Delta_{t_1 j_1} \mp \Delta_{t_2 j_2})\, \Phi^{\pm}_J(j_1 j_2) + (\varphi_{t_1 j_1} \mp \varphi_{t_2 j_2})\, S^{\pm}_J(j_1 j_2) = 0, \quad (7)$$

where

$$e_{tj} = \varepsilon_{tj} - \lambda - \mu t - \sum_{Jj'} \frac{2J+1}{2j+1}\,[U_J(tjtj';\, tj'tj) - (-1)^{j+j'-J} U_J(tjtj';\, tjtj')]\, f_{tj'} - \sum_{Jj''} \frac{2J+1}{2j+1}\, U_J(tj - tj'';\, -tj''tj)\, f_{-tj''}, \quad (8)$$

$$\Delta_{tj} = -\sum_{j'} \sqrt{\frac{2j'+1}{2j+1}}\, U_{J=0}(tjtj;\, tj'tj')\, \varphi_{tj'};$$

$$F^{\pm}_J(j_2 j_1) = F^{*}_{JM}(j_2 j_1) \pm (-1)^{J+M} F_{J-M}(j_2 j_1),$$

$$\Phi^{\pm}_J(j_1 j_2) = \Phi_{JM}(j_1 j_2) \pm (-1)^{J+M}\, \Phi^{*}_{J-M}(j_1 j_2),$$

$$E^{\pm}_J(j_1 j_2) = \sum_{j'j''} W_J(tj_1 - tj',\, -tj_2 tj'')\, F^{\pm}_J(j'j''),$$

$$S^{\pm}_J(j_1 j_2) = \sum_{j'j''} U_J(tj_1 - tj_2;\, -tj''tj')\, \Phi^{\pm}_J(j'j''),$$

$$W_J(tj_1 - tj';\, -tj_2 tj'') = \sum_L (2L+1) \begin{Bmatrix} j_1 j_2 J \\ j'' j' L \end{Bmatrix} U_L(tj_1 - tj';\, -tj_2 tj''). \quad (9)$$

The equations (7) form a linear homogeneous system. One can show that the determinant of this system vanishes only for special assumptions concerning the strength of the neutron-proton interaction, namely when the matrix elements $U_J$ and $W_J$ are independent of the angular momentum J of the pair. Since this is not the case in reality, the determinant of the system (7) is different from zero. Consequently Eqs. (7) have only the null solution:

$$\Phi_0 = F_0 = 0. \quad (10)$$

The presence of the conditions (3), which have the form

$$(\varphi_{t_1 j_1} \pm \varphi_{t_2 j_2})\, \Phi^{\mp}_J(j_1 j_2) = (1 - f_{t_1 j_1} - f_{t_2 j_2})\, F^{\mp}_J(j_2 j_1), \quad (3a)$$

does not alter this result, since including them simultaneously reduces both the number of unknowns in (7) and the number of independent equations by a factor of two. When we use (3) and (10), the equations for $F_1$ and $\Phi_1$ take the form

$$2e_{tj}\varphi_{tj} + (1 - 2f_{tj})\,\Delta_{tj} = 0, \quad f^2_{tj} + \varphi^2_{tj} = f_{tj}, \quad (11)$$

from which we find

$$f_{tj} = (E_{tj} - e_{tj})/2E_{tj}, \quad \varphi_{tj} = -\Delta_{tj}/2E_{tj}, \quad E_{tj} = \sqrt{e^2_{tj} + \Delta^2_{tj}},$$

$$\Delta_{tj} = \tfrac{1}{2} \sum_{j'} \sqrt{(2j'+1)/(2j+1)}\, U_{J=0}(tjtj;\, tj'tj')\, \Delta_{tj'}/E_{tj'}. \quad (12)$$

Using (2), (4), (10), and (12), we get the Bogolyubov canonical transformation in the form

$$a_{tjm} = u_{tj}\,\alpha_{tjm} + (-1)^{j-m} v_{tj}\,\alpha^{+}_{tj-m}, \quad v^2_{tj} = f_{tj}, \quad u_{tj}v_{tj} = -\varphi_{tj}. \quad (13)$$

Thus we have found that the p-n interaction causes no mixing of the proton and neutron amplitudes, i.e., the protons and neutrons in the nucleus pair independently. This result is in qualitative agreement with the absence of a gap in the spectrum of excitations of odd-odd nuclei. In fact, if there were proton-neutron pairing, the ground state of odd-odd nuclei would correspond to zero quasiparticles, just as is the case for even-even nuclei.

## QUADRUPOLE OSCILLATIONS OF EVEN-EVEN NUCLEI

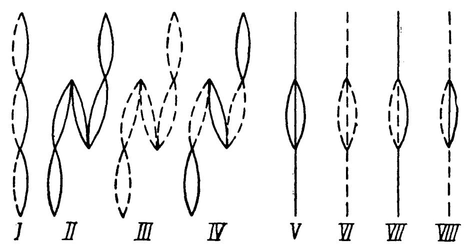
The Hamiltonian for the interaction between quasiparticles is obtained from the initial Hamiltonian by transforming it using (11)—(13). We shall describe the individual terms by graphs of the type used by Baranger,[2] in which the solid line denotes a proton and the dotted lines a neutron (see the figure). One can show that the inclusion of the graphs I is accomplished by solving the equations of the generalized self-consistent field[1]

$$\mathscr{E}_J \Phi^{\pm}_J(j_1 j_2) = A^{\mp}_J(j_1 j_2 \mid F\Phi). \quad (14)$$

The operators for the corresponding excitations have the form

$$O^{+}_{JM} = \sum_{j_1 m_1 j_2 m_2} a(j_1 j_2 J)\, C^{JM}_{j_1 m_1 j_2 m_2}\, \alpha^{+}_{tj_1 m_1}\, \alpha^{+}_{-tj_2 m_2}. \quad (15)$$

In the special case of the "two-level" approximation (see below), the excitation energy is



I II III IV V VI VII VIII

$$\mathscr{E}_J = E_{tj_1} + E_{-tj_2} - U_J(tj_1 - tj_2;\ -tj_2tj_1). \quad (16)$$

As we see from (15) and (16), excitations of this type describe the ground and low-lying states of odd-odd nuclei.

Our problem is to include graphs II—IV, which are known[2,3] to be responsible for the collective excitations. Since we are interested only in qualitative effects associated with the inclusion of the p-n interaction, we shall treat the problem under the same assumptions as are made in treating identical quasiparticles.

1. We start from the simplified model of a spherical nucleus, in which the external protons are in a state with angular momentum $j_p \gg 1$, while the external neutrons are in a state with $j_n \gg 1$ (the "two-level" approximation).

2. We limit ourselves to treating the quadrupole-quadrupole interaction between quasiparticles:

$$U_2(\mathbf{r}_1\mathbf{r}_2) = -g(r_1r_2)\sum_\mu(-1)^\mu Y_{2\mu}(\theta_1\varphi_1)Y_{2-\mu}(\theta_2\varphi_2). \quad (17)$$

3. We shall omit graphs V—VIII, since they give a small contribution compared to graphs II—IV.[4]

Under these assumptions, the Hamiltonian has the form

$$\hat{H} = \sum_{tjm} E_{tj}\alpha^+_{tjm}\alpha_{tjm} - \frac{1}{2}\sum_{\substack{\mu t_1 t_2,\\ jj'j_1j_1'}} \mathscr{L}(Tjj';T_1j_1j_1')\, u_{tj}v_{tj'}u_{t_1j_1}v_{t_1j_1'}[Q^+_\mu(Tjj')\,Q_\mu(T_1j_1j_1') + (-1)^\mu Q^+_{-\mu}(Tjj')\,Q^+_\mu(T_1j_1j_1') + \text{H. c.}], \quad (18)$$

where

$$T = 2t = \pm 1,$$

$$\mathscr{L}(Tjj', T_1j_1j_1') = \frac{1}{5}\iint_0^\infty r_1^2r_2^2dr_1dr_2\varphi^*_l(r_1)\varphi^*_{l_1}(r_2)\,g(r_1r_2)\,\varphi_{l_1'}(r_2)\,\varphi_{l'}(r_1)\langle lj\|Y_2\|l'j'\rangle\langle l_1j_1\|Y_2\|l_1'j_1'\rangle,$$

$$Q^+_\mu(Tjj') = \frac{1}{\sqrt{1+\delta_{jj'}}}\sum_{mm'} C^{2\mu}_{jmj'm'}\alpha^+_{tjm}\alpha^+_{tj'm'}. \quad (19)$$

The Hamiltonian (18) is diagonalized by the transformation

$$R_\mu(s) = \sum_{Tjj'\,(j'\geq j)}[x_s(Tjj')\,Q_\mu(Tjj') + (-1)^\mu y_s(Tjj')\,Q^+_{-\mu}(Tjj')],$$

$$\sum_{Tjj'\,(j'\geq j)}[x_s(Tjj')\,x_{s_1}(Tjj') - y_s(Tjj')\,y_{s_1}(Tjj')] = \delta_{ss_1}. \quad (20)$$

The inverse transformation has the form

$$Q_\mu(Tjj') = \sum_s[x_s(Tjj')\,R_\mu(s) - (-1)^\mu y_s(Tjj')\,R^+_{-\mu}(s)],$$

$$\sum_s[x_s(Tjj')\,x_s(T_1j_1j_1') - y_s(Tjj')\,y_s(T_1j_1j_1')] = \delta_{TT_1}\delta_{jj_1}\delta_{j'j_1'}. \quad (21)$$

The coefficients of the transformation (20) are determined from the condition that the Hamiltonian (18) be diagonal when expressed in terms of the new amplitudes:

$$R_\mu(s)\,\hat{H} - \hat{H}R_\mu(s) = \mathscr{E}_sR_\mu(s). \quad (22)$$

Solving the corresponding secular equation under our assumptions, we get

$$\mathscr{E}_1 = \mathscr{E}_{2+} = \{\tfrac{1}{2}[\mathscr{E}_p^2 + \mathscr{E}_n^2 - 8EV_{pn}\sqrt{1+\delta^2}]\}^{1/2},$$

$$\mathscr{E}_2 = \mathscr{E}_{2'+} = \{\tfrac{1}{2}[\mathscr{E}_p^2 + \mathscr{E}_n^2 + 8EV_{pn}\sqrt{1+\delta^2}]\}^{1/2}, \quad (23)$$

where $\mathscr{E}_p$ and $\mathscr{E}_n$ are the energies of the quadrupole oscillations omitting the p-n interaction:

$$\mathscr{E}_p = 2E_{j_p}\sqrt{1 - V_p/E_{j_p}},\quad \mathscr{E}_n = 2E_{j_n}\sqrt{1 - V_n/E_{j_n}},$$

$$V_p = u^2_{j_p}v^2_{j_p}\mathscr{L}(1j_p^2;\ 1j_p^2),$$

$$V_n = u^2_{j_n}v^2_{j_n}\mathscr{L}(-1j_n^2;\ -1j_n^2),\quad V_{pn} = u_{j_p}v_{j_p}u_{j_n}v_{j_n}\mathscr{L}(1j_p^2,\ -1j_n^2),$$

$$E = \sqrt{E_{j_p}E_{j_n}},\qquad \delta = (\mathscr{E}_n^2 - \mathscr{E}_p^2)/8EV_{pn}. \quad (24)$$

We thus get two levels with spin 2 and positive parity, which have excitation operators $R_\mu(1)$ and $R_\mu(2)$, respectively.

Now let us find the expressions for the reduced probability of E2 transitions from the $2^+$ and $2'^+$ states to the ground state, and for the $2^+ \to 2'^+$ transition. The operator for an E2 transition has the form

$$\mathfrak{M}_E(2\mu) = \frac{1}{\sqrt{5}}\langle l_pj_p\|r^2Y_2\|l_pj_p\rangle \times \sum_{mm'}(-1)^{j_p-m'}C^{2\mu}_{j_pmj_p-m'}a^+_{j_pm}a_{j_pm'}, \quad (25)$$

and simple computations give

$$B(E2)_{2+\to 0} = B^{sp}\frac{2E_{j_p}}{\mathscr{E}_{2+}}\frac{u^2_{j_p}v^2_{j_p}}{1+\delta^2-\delta\sqrt{1+\delta^2}}, \quad (26a)$$

$$B(E2)_{2'+\to 0} = B^{sp}\frac{2E_{j_p}}{\mathscr{E}_{2'+}}\frac{u^2_{j_p}v^2_{j_p}}{1+\delta^2+\delta\sqrt{1+\delta^2}}, \quad (26b)$$

$$B(E2)_{2'+\to 2+} = B^{sp}\frac{(\mathscr{E}_{2+}\mathscr{E}_{2'+} + 4E^2_{j_p})^2}{64\mathscr{E}_{2+}\mathscr{E}_{2'+}E^2_{j_p}}\frac{(u^2_{j_p} - v^2_{j_p})^2}{1+\delta^2}, \quad (26c)$$

where the $B^{sp}$ are the single-particle values of the corresponding reduced probabilities.

To analyze these results it is convenient to go over to the limiting case of $\delta = 0$. From formulas (19) and (24), it follows that this case occurs if the matrix elements for the p-p, n-n, and p-n interactions are all equal and if, in addition, the filling $f_j = n_j/(2j+1)$ (where n is the number of nucleons in a state with given j) is the same for the proton and neutron shells. We note that since, in the region of the mass table in which we are interested, the matrix elements of the p-p, n-n, and p-n interactions are almost the same and the filling of proton and neutron shells is almost identical, $\delta \ll 1$, so that the limiting case of $\delta = 0$ is extremely close to actuality. Using (22), (23), and (26), we easily obtain for this case

$$\mathscr{E}_{2+} = 2E\sqrt{1 - 2V/E} = 2E\gamma,$$
$$R_\mu(2^+) = [(1+\gamma) Q_\mu(1j_p^2) - (-1)^\mu (1-\gamma) Q^+_{-\mu}(1j_p^2) + (1+\gamma) Q_\mu(-1j_n^2) - (-1)^\mu (1-\gamma) Q^+_{-\mu}(-1j_n^2)] / \sqrt{8\gamma}; \quad (27)$$

$$\mathscr{E}_{2'+} = 2E, \quad R_\mu(2'^+) = [Q_\mu(1j_p^2) - Q_\mu(-1j_n^2)]/\sqrt{2}; \quad (28)$$

$$\underset{2+\to 0}{B(E2)} = B^{sp}\frac{1}{\gamma} f(1-f), \ \underset{2'+\to 0}{B(E2)} = B^{sp} f(1-f), \ \underset{2'+\to 2+}{B(E2)} = B^{sp}\frac{(1+\gamma)^2}{16\gamma}(1-2f)^2. \quad (29)$$

As we see from the formulas, the first $2^+$ level is purely collective. This follows both from the form of the excitation operator as well as from the fact that the ratio of the probability of an E2 transition to the ground state to the single-particle value for this quantity contains the ratio of the energy "gap" to the level energy, which results in an enhancement of the transition relative to the single-particle value. Thus we see that the first solution satisfactorily describes the properties of the first $2^+$ level in even-even nuclei. Within the framework of our assumptions, it is a "one-phonon" excitation (in the sense that the operator $R_\mu$, in the framework of the method of approximate second quantization, is a Bose operator).

The situation is different for the interpretation of the second $2^+$ level. As we see from (28)—(29), this excitation does not have collective character. This follows from the fact that: a) the operator $R_\mu(2'^+)$ is a linear combination of excitation operators for proton and neutron pairs; b) the excitation energy coincides with the size of the gap; c) the E2 transitions to the ground state and to the first $2^+$ level have the same intensity as the single-particle transitions (since the factor $1/\gamma$ in the formula for $\underset{2'^+\to 2^+}{B(E2)}$ is compensated by the factor $1/16$). Thus the second of our solutions is a specific state, caused by the presence of the p-n interaction, with properties which are different from the properties of the "two-phonon" excitations (the triplet $0^+2^+4^+$).

Recently $2^+$ levels have been found in the cadmium isotopes, lying above the two-phonon triplet $0^+2^+4^+$ (at 1460 keV in $Cd^{112}$ and 1365 keV in $Cd^{114}$). It is natural to assume that the second solution describes these levels, but to test this hypothesis requires experimental investigation of their electromagnetic properties, in particular the measurement of the probability of E2 transitions to the ground state and the first $2^+$ level.

In conclusion the author thanks L. A. Sliv and Z. B. Shimanskii for discussion of the work.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Translated by M. Hamermesh
75