ON THE INTERFERENCE BETWEEN THE AMPLITUDES OF INELASTIC PROCESSES

I. I. ROĬZEN and D. S. CHERNAVSKII

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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The interference between the one-meson and two-meson amplitudes of high energy inelastic processes is considered. The magnitude of the interference terms depends on the nature of the elastic nucleon interaction. It is shown that such an interference does not occur if the elastic scattering is of diffractional nature.

THE problem of the interference between the one-meson and multi-meson amplitudes of inelastic processes has recently been the subject of many papers.[1-3] The one-meson amplitude corresponds to the diagrams shown in Figs. 1a and b, the multi-meson amplitudes correspond to diagrams with a larger number of intermediate meson lines. This problem is a pressing one, since the one-meson approximation, which has been discussed in many papers,[1-6] is meaningful only if the above-mentioned interference terms are small.

Let us consider this problem for the case of high energy nucleon collisions ($s = 4E^2 \gg m^2$, where E is the nucleon energy in the center of mass system). We note that calculations in the one-meson approximation imply the neglect of two types of interference terms. First, in squaring the amplitude corresponding to the diagram of Fig. 1a, interference terms between the states I_1^1 and I_2^1 might arise. The magnitude of these terms has been discussed in [3]. There it was shown that they are small in most cases.* This has to do with the fact that for $k^2/s \ll 1$ (where k is the four-momentum of the intermediate meson) the angular distribution has two peaks ("cones") which do not overlap (i.e., the particles generated at vertex 1 are emitted into one cone with an angular spread $\sim k^2/s$, and the particles generated at vertex 2 into another).

Second, interference terms between the one-meson and multi-meson amplitudes may occur. These terms will be large only if the angular distribution in the processes of the type 2a or b also has this two-cone character and if the quantum numbers of the final states $I_1^\nu(I_2^\nu)$ and $I_1^1(I_2^1)$ (see the corresponding figures) are identical.

*The general case has not been discussed in [3].

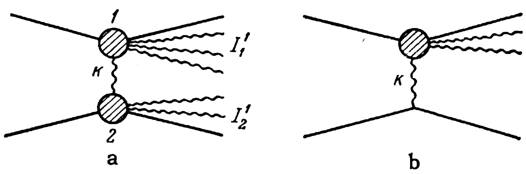

FIG. 1

We note that the problem of the interference between the one-meson and two-meson diagrams (Figs. 1a and 2a) is the most acute. Indeed, in multi-meson processes there is no reason to expect the angular distribution of the secondary particles in the center of mass system to have this two-cone character. Besides, it is rather unlikely in this case that the states $I_1^\nu(I_2^\nu)$ and $I_1^1(I_2^1)$ have the same quantum numbers.[2] On the other hand, the two-meson process (Fig. 2a) is very similar in character to the one-meson process. We shall therefore consider in more detail the interference of the diagrams of Figs. 1a and 2a.*

To this end we now turn to the consideration of the elastic interaction of high energy nucleons. We assume that the imaginary part of the forward

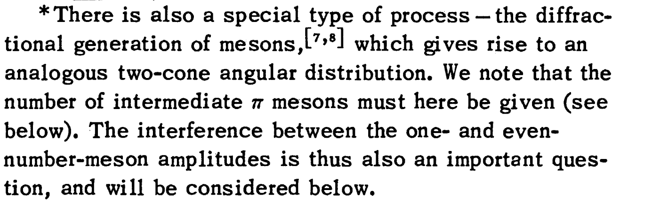

FIG. 2

*There is also a special type of process – the diffractional generation of mesons,[7,8] which gives rise to an analogous two-cone angular distribution. We note that the number of intermediate π mesons must here be given (see below). The interference between the one- and even-number-meson amplitudes is thus also an important question, and will be considered below.

scattering amplitude can be written as a sum of amplitudes with definite numbers of intermediate mesons:

$$\operatorname{Im}\varphi_i^T(s, t=0) = \operatorname{Im}[\varphi_{i(2)}^T + \varphi_{i(3)}^T + \ldots + \varphi_{i(n)}^T]. \quad (1)$$

Here T is the isotopic spin of the system of interacting nucleons. The terms on the right hand side,

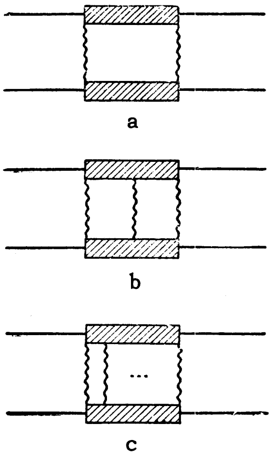

FIG. 3

$\varphi_{i(2)}^T$, $\varphi_{i(3)}^T$, and $\varphi_{i(n)}^T$, correspond to Figs. 3a, b, and c, respectively.* The remainder of the notation is borrowed from the paper of Goldberger et al[9] In this notation the optical theorem is for $s \gg m^2$

$$\operatorname{Im}[\varphi_1^T(s, 0) + \varphi_3^T(s, 0)] = \sum_m \langle i | m\rangle_T \langle m | i\rangle_T = \frac{s^{1/2}}{16\pi}\sigma_{tot}^T(s), \quad (2)$$

where $\sigma_{tot}^T(s)$ is the total cross section for the interaction of two nucleons in a state with isotopic spin T (which is also denoted by the symbol $\langle i|$), and the index m refers to the intermediate state. In analogy with (1) we write

$$\sum_m \langle i | m\rangle\langle m | i\rangle = \sum_{m,\nu,\nu'} \langle i, \nu | m\rangle\langle m | i, \nu'\rangle$$
$$= \sum_{m,\nu=\nu'} \langle i, \nu | m\rangle\langle m | i, \nu\rangle + \sum_{\substack{m,\nu\neq\nu' \\ \nu+\nu'=2r}} \langle i, \nu | m\rangle\langle m | i, \nu'\rangle$$
$$+ \sum_{m,\nu+\nu'=2r+1} \langle i, \nu | m\rangle\langle m | i, \nu'\rangle, \quad (3)$$

where ν and ν' are the number of intermediate mesons in the amplitudes of the type of Fig. 2b. It is easy to show that†

$$\sum_m \langle i, 1 | m\rangle\langle m | i, 1\rangle = \frac{s^{1/2}}{16\pi}\sigma_1(s), \quad (4)$$

where $\sigma_1(s)$ is the cross section computed in the one-meson approximation, and

$$\sum_m \{\langle i, 1 | m\rangle\langle m | i, 2\rangle + \langle i, 2 | m\rangle\langle m | i, 1\rangle\} = s^{1/2}\sigma_{1,2}^{int}(s)/16\pi, \quad (5)$$

where $\sigma_{1,2}^{int}(s)$ is the contribution to the total cross section from the interference of the one- and two-meson amplitudes.

Owing to the assumed absence of interference of the first type, we have at the same time

$$\sum_{m,\nu+\nu'=n} \langle i, \nu | m\rangle_T \langle m | i, \nu'\rangle_T = \operatorname{Im}[\varphi_{1(n)}^T(s, 0) + \varphi_{3(n)}^T(s, 0)]. \quad (6)$$

It follows, in particular, that the term describing the interference between the one- and two-meson diagrams is directly related to the three-meson elastic forward scattering amplitude.

The relative magnitude of the different terms $\varphi_{i(n)}^T$ depends on the character of the nucleon scattering. In many papers, experimental[10] as well as theoretical,[11] it is indicated that the elastic scattering at high energies has diffractional character. This means, in particular, that a) there is no charge exchange scattering, and b) there is no scattering in which the polarization of the nucleons is changed (i.e., no spin flip scattering). It should be noted that these facts are not yet directly established by experiment in the region $s \gg m^2$, although they are very probably true. They will be made the basis of the following discussion.

Condition a) implies the independence of the amplitude of the isotopic spin T:

$$\varphi_i^1 = \varphi_i^0. \quad (7)$$

Using the formalism developed by Goldberger et al,[9] we can easily express the amplitudes $\varphi_i(s, 0)$ in terms of the amplitudes* $\bar{\varphi}_i(\bar{E}, \cos\bar{\vartheta})$ of the crossed $N\bar{N}$ scattering process in the nonphysical region $\bar{E} = 0$, $\cos\bar{\vartheta} = 1 - s/2m^2$:

$$\varphi_1^T(s, 0) = -(\bar{E}/E)[\bar{\varphi}_1^0(0, \cos\bar{\vartheta}) - \bar{\varphi}_3^0(0, \cos\bar{\vartheta})], \quad (8a)$$
$$\varphi_2^T(s, 0) = -(\bar{E}/E)[\bar{\varphi}_3^0(0, \cos\bar{\vartheta}) - \bar{\varphi}_4^0(0, \cos\bar{\vartheta})], \quad (8b)$$
$$\varphi_3^T(s, 0) = -(\bar{E}/E)[\bar{\varphi}_1^0(0, \cos\bar{\vartheta}) + \bar{\varphi}_4^0(0, \cos\bar{\vartheta})], \quad (8c)$$
$$\varphi_4^T(s, 0) = \varphi_5^T(s, 0) \equiv 0 \ . \quad (8d)\dagger$$

Furthermore, the following relation must hold:

*The one-meson elastic scattering (pole term) is omitted here, since it does not contribute to the imaginary part of the amplitude.

†Here it suffices to note that the second part of (2) is true in all orders of perturbation theory.

*Here and in the following all quantities with a bar refer to the corresponding quantity in the crossed $N\bar{N}$ channel, i.e., in the nucleon-antinucleon scattering process.

†Formulas (8a, b, c, d) are true only if $\bar{E} = 0$, and their validity is thus somewhat conditional. They are meaningful in view of the fact that $\bar{\varphi} \approx 1/\bar{E}$ as $\bar{E} \to 0$ as a consequence of normalization.

$$\bar{\varphi}_1 - \bar{\varphi}_2 = \bar{\varphi}_3 - \bar{\varphi}_4. \qquad (9)$$

It follows from (8) that only the amplitudes with $\bar{T} = 0$ remain in the crossed channel, and condition b), together with (8), (9), and formula (4.23a) of Goldberger et al [9] imply that there is only scattering in the state with $\bar{S} = 1$ in the crossed channel. It is clear that the state for which there is $N\bar{N}$ scattering and the set of intermediate π mesons (in the following called the "π cloud" for brevity) have identical quantum numbers. In particular, they have total angular momentum $\bar{J} = 0$.

To show this we note that the system containing the nucleon in the final state and the π cloud must have the same spin and spin projection as the corresponding nucleon in the initial state. The spin of this system is composed of the spin of the nucleon, the spin of the π cloud, and the relative orbital angular momentum between the two. The latter is zero, since $k_\nu = 0$. Therefore the spin of the π cloud must also be equal to zero, since otherwise scattering with nucleon spin flip would occur with a probability of $2/3$. But the spin of the π cloud is just $\bar{J}$.

With such a set of quantum numbers the G parity in the crossed channel is

$$G = (-1)^{\bar{T} + \bar{l} + \bar{S}} = (-1)^n = +1, \qquad (10)$$

so that the number n of intermediate π mesons in the diagrams describing the diffractional forward scattering must be even.

Returning now to formulas (5) and (6), we see that there is no interference between the one- and two-meson diagrams. It is shown in an analogous fashion that generally the diagrams with even and odd numbers of mesons do not interfere. We note that this interference also does not occur in the πN interaction, since different numbers of secondary particles are generated in the $\pi\pi$ vertices of the one- and two-meson diagrams (an even number in the first case, an odd number in the second case; see Fig. 4).

We have seen above that $\bar{J} = 0$ for diffraction scattering in the region of interest $s \gg m^2$, $t = 0$. This means that the amplitudes $\bar{\varphi}_3$, $\bar{\varphi}_4$, and $\bar{\varphi}_5$ vanish in comparison with the amplitudes $\bar{\varphi}_1$ and $\bar{\varphi}_2$, since they describe $N\bar{N}$ scattering in a state with projection of $\bar{J}$ equal to unity. It thus follows from (8a, c) that

$$\varphi_1(s, 0) = \varphi_3(s, 0), \qquad (11)$$

i.e., the forward scattering amplitude does not depend on the projection of the total spin on the direction of motion. This is a natural consequence of the assumption of the diffractional character of

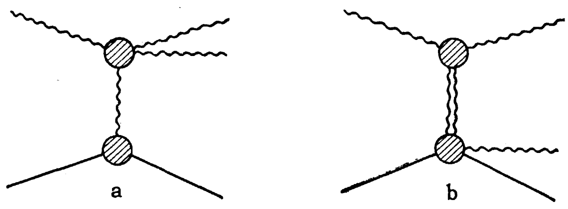

FIG. 4

the elastic NN scattering. According to the optical theorem, the same holds for the total cross section for the scattering of polarized nucleons.

Interference between the inelastic amplitudes of one- and three-meson [and, in general, (2r + 1)-meson] processes is not excluded. One may hope, according to the considerations at the beginning of the paper, that it is not strong. Finally, we note that, for some problems, it is not necessary to require the vanishing of all interference terms in the total cross section. It would suffice to show that the sum of all terms, except the one-meson term, in the cross section cannot become negative. We hope to consider this question in the near future.

In conclusion we take this opportunity to express our gratitude to V. B. Berestetskii, I. Ya. Pomeranchuk, V. Ya. Faĭnberg, and E. L. Feĭnberg for fruitful discussions.

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