desirable to see whether the Mössbauer effect is sensitive to the composition and state of the solvent.

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WIDTH OF CYCLOTRON RESONANCE LINE IN SEMIMETALS AND DETERMINATION OF THE CORRELATION FUNCTION FOR BISMUTH

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THE width of a resonance line is usually defined in terms of the corresponding relaxation time. However, for cyclotron resonance, the Fermi-liquid interaction exerts an appreciable influence on the width of the line. It is evident from previously derived formulas [1] that when $\omega \sim \omega_1$, i.e., when $r \sim \delta_0$, we have $\omega\tau_{\mathrm{eff}} \sim g$ (where $\omega_1 \sim v\omega_0/c$, r —Larmor radius of the electrons, $\delta_0 \sim c/\omega_0$, ω_0 —plasma frequency of the electrons, v —velocity, $g \sim \int G\,dS/v$, $G(\mathbf{p}, \mathbf{p}')$ —correlation function); for $g \sim 1$, resonance is in fact nonexistent. Since Aubrey and Chambers [2] observed in experiments on bismuth (where, $\delta/r \sim 2$ and $\omega\tau \sim 370$ for "holes") that the width of the resonance curve was of the order of unity (and not 10^{-2} which it should have been from $\omega\tau$) then, evidently, appreciable interaction takes place in bismuth, with $g \sim 1$. As far as we know, this is the first instance in which it has been possible to evaluate the order of the Fermi-liquid interaction in metals. It stands to reason that more detailed research would permit the function $G(\mathbf{p}, \mathbf{p}')$ to be clarified in greater detail. [1]

In this connection, it is of interest to determine how the Fermi-liquid interaction manifests itself during resonance over the entire frequency range. Since resonance calls for $\omega\tau \gg 1$, the depth of the skin-layer is $\delta \sim \delta_0 \sim c/\omega_0$ and does not depend on $\omega\tau$ and $r \sim v/\omega$. Consequently, two cases are possible: 1) sufficiently low frequencies, $\omega \ll \omega_1$, where cyclotron resonance [1] takes place (this range may not be attainable for semimetals and semiconductors because of the small ω_1, which corresponds to $\omega_1\tau \ll 1$); 2) sufficiently high frequencies, $\omega \gg \omega_1$, which is the range of diamagnetic resonance.

Let us set down, without derivation, the results in both these frequency ranges. In the cyclotron resonance range [1] we have:

$$\nu_{\mathrm{eff}} \sim \frac{1}{\tau_{\mathrm{eff}}} \sim \begin{cases} \omega_1 (\omega/\omega_1)^2 & \text{(quadratic dispersion)} \\ \omega_1 (\omega/\omega_1)^3 & \text{(non-quadratic dispersion)} \end{cases},$$

and in the diamagnetic resonance range:

$$\nu_{\mathrm{eff}} \sim \frac{1}{\tau_{\mathrm{eff}}} \sim \begin{cases} \omega_1 & \text{(quadratic dispersion)} \\ \omega_1 (\omega_1/\omega) & \text{(non-quadratic dispersion)} \end{cases}.$$

It is clear from the above that the relative resonance width $1/\omega\tau_{\mathrm{eff}}$, due to the Fermi-liquid interaction, has a maximum at $\omega \sim \omega_1$.

Let us explain the cause of the Fermi-liquid suppression of resonance, which has no connection with any real attenuation, and its smallness in almost all cases despite the fact that the interaction itself is not at all small. The point is that the Fermi-liquid interaction leads to an additional spatial dispersion $\omega = \omega(k)$ as compared to a Fermi-gas, where, it is easily seen, $\omega = k\bar{v}_z + q\Omega$ (it is essential to notice that the level spacing $\Delta\Omega$ does not depend on k), where $\bar{v}_z$ —average velocity of electrons within the metal, Ω —Larmor frequency.

The averaging over k, which takes place in the impedance with account of the anomalous skin-effect, leads to a suppression of the resonance. However, the "broadening" of the resonance is not small for kr ~ r/δ ~ 1 alone, since the spatial dispersion is weak in normal skin-effect* as a result of the inequality $kr \ll 1$ and it is therefore possible to expand ω(k); in anomalous skin effect, on the other hand, resonance is possible only in the magnetic field parallel to the metal surface, when $\bar{v}_z = 0$ and ω is finite as $k \to \infty$ (see [1]), so that ω(k) can be expanded in powers of 1/kr.

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*In diamagnetic resonance in an inclined field, the Doppler effect produces a supplementary attenuation with $1/\tau_{eff} \sim \omega_1$ (quadratic dispersion or resonance for nonquadratic dispersion on a section off center) or $1/\tau_{eff} \sim \omega_1(\omega_1/\omega)$ (resonance on a central section).

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THE COLLAPSE OF A SMALL MASS IN THE GENERAL THEORY OF RELATIVITY

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A calculation of the equilibrium of a cold ideal Fermi gas in its own gravitational field made by Volkoff and Oppenheimer [1,2] led to the following result: for a small number of neutrons (N < 0.35 ⊙) there is a single solution, for 0.35 ⊙ < N < 0.750 ⊙ there are two solutions, and for N > 0.75 ⊙ there is no solution at all (the symbol ⊙ here means the number of neutrons in the sun).

It was assumed that the unique solution for N < 0.35 ⊙ is absolutely stable and that for such a value of N collapse is impossible. We shall show that this is not true.

By prescribing a sufficiently large density we can obtain for any given number N of particles a configuration with mass as close to zero as we please, and clearly less than the mass of the static solution. Such a configuration obviously cannot go over into the state of equilibrium (into the static solution), and consequently can only contract without limit.

Let us take an arbitrary spherically symmetrical distribution of motionless matter. We denote the particle density by n and the energy density by ϵ (ϵ includes the rest mass of the particles); n and ϵ are connected by the equation of state.

The metric is given by the expression (we everywhere set c = 1)

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 (\sin^2\theta \, d\varphi^2 + d\theta^2). \qquad (1)$$

As is known from the equation for λ (cf. [3]) it follows that

$$e^{-\lambda(r)} = 1 - \frac{b}{r}\int_0^r \varepsilon(r) r^2 dr, \qquad (2)$$

where b = 8πk. The mass of the star is given by the expression

$$M = 4\pi \int_0^\infty \varepsilon(r) r^2 dr, \qquad (3)$$

and the number of particles by (dω is an invariant volume element)

$$N = \int n d\omega = 4\pi \int_0^\infty n(r) e^{\lambda/2} r^2 dr. \qquad (4)$$

Let us take the distribution of motionless matter given by the formulas

$$\varepsilon = a/r^2, \quad r < R; \quad \varepsilon = 0, \quad r > R. \qquad (5)$$

Then

$$e^{-\lambda} = 1 - ab, \quad r < R, \quad e^{-\lambda} = 1 - abR/r, \quad r > R. \qquad (6)$$

$$M = 4\pi a R, \quad N = \frac{4\pi}{\sqrt{1-ab}} \int_0^R n r^2 dr. \qquad (7)$$

For an ultrarelativistic gas

$$\varepsilon = \hbar (3/\pi^2)^{1/3} n^{4/3}, \quad n = (\pi^2/3)^{1/4} (\varepsilon/\hbar)^{3/4}. \qquad (8)$$

Substituting Eqs. (5) and (8) in Eq. (7), we get

$$N = \text{const}\ a^{3/4} R^{3/2} / \sqrt{1-ab},$$
$$R = \text{const}\ N^{2/3} a^{-1/2} (1-ab)^{1/3}, \quad M = \text{const}\ N^{2/3} a^{1/2} (1-ab)^{1/3}. \qquad (9)$$

It follows from this that $M \to 0$ for $a \to 1/b$, whatever the value of N.* This proves the assertion made above.

For a rough estimate of the energy barrier which separates the equilibrium solution with $M \le Nm$ (m is the mass of the neutron) from the collapsing state, let us find the maximum M from Eq. (9). We get

$$M_{max} \approx N^{2/3} \sqrt{\hbar/k}, \quad M_{max}/Nm \sim N^{-1/3} \sqrt{\hbar/k}/m \approx N/N_{cr}, \qquad (10)$$

where mN_{cr} is of the order of the maximum mass for which a solution exists, i.e., of the order of the mass of the sun. Consequently for systems con-