INVESTIGATION OF MULTIPLE SCATTERING OF 100—200 keV *PROTONS ON CARBON*

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The angular distributions of 100—200 keV protons multiply scattered in polysterene films are obtained by a photographic method. There is a discrepancy of ~ 20—30% between the experimental results and the predictions of Molière's theory. This discrepancy is a consequence of the application of the Thomas-Fermi model in the theory. Satisfactory agreement between the theoretical and experimental data is obtained for target-proton energies up to ~ 90 keV if a carbon-atom potential obtained by the Hartree-Fock method is employed. The discrepancy increases with a further decrease in the energy and the experimental angular distributions become broader than the theoretical.

THE experimental investigation of multiple scattering of charged particles in matter has covered a broad energy range (from ~ 1 to hundreds of MeV).[1] The obtained data, as a rule, are in good agreement with the theoretical predictions in which only elastic collisions of the moving particle with atoms of the medium are taken into account.[2] The question of the applicability of these calculations to the region of lower energies (for protons, tens and hundreds of keV) is open, in view of the lack of experimental data. The purpose of the present experiment was to investigate multiple scattering of 100—200 keV protons on carbon and to obtain in this way information on the limits of applicability of the present theoretical views on the low-energy side.

1. EXPERIMENTAL METHOD

The measurements were made with the electrostatic accelerator of the Moscow State University. A collimated beam of accelerated protons struck a thin target set perpendicularly to the beam axis. The protons experiencing multiple scattering inside the target were recorded in emulsion pellicles set parallel to the target at a distance of ~ 30 mm from it. The duration of the exposure was so chosen that the pattern obtained in the emulsion plates was suitable for photometric measurements. On the basis of the densitometric character and the geometrical conditions of the experiment, we recalculated the darkening density distribution of the emulsion in order to obtain the angular distribution of the scattered particles.

Films of polysterene $(C_8H_8)_n$ were used as the carbon targets; the hydrogen gave only a small contribution to the overall picture of the scattering and it could easily be taken into account. The target thickness was determined by weighing. Measurements were made with five targets of the following thicknesses (in $\mu g/cm^2$): 24 ± 0.6, 40 ± 0.7, 69 ± 0.9, 88 ± 1.1, and 104 ± 1.2. For

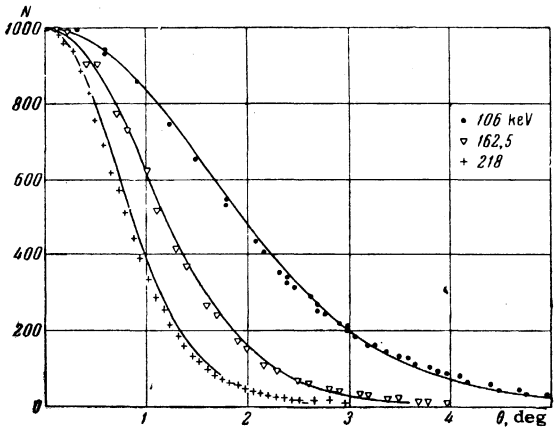

FIG. 1

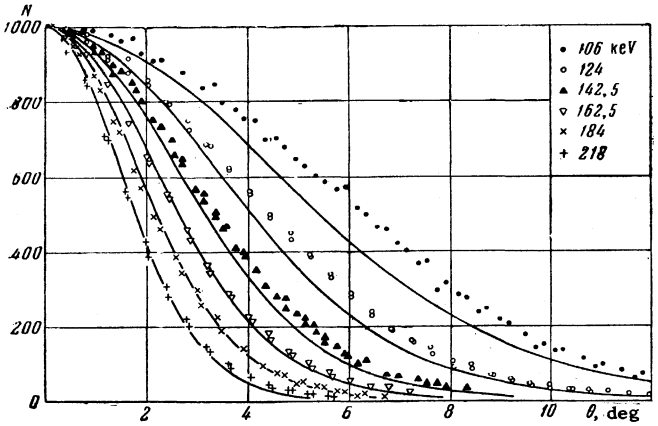

FIG. 2

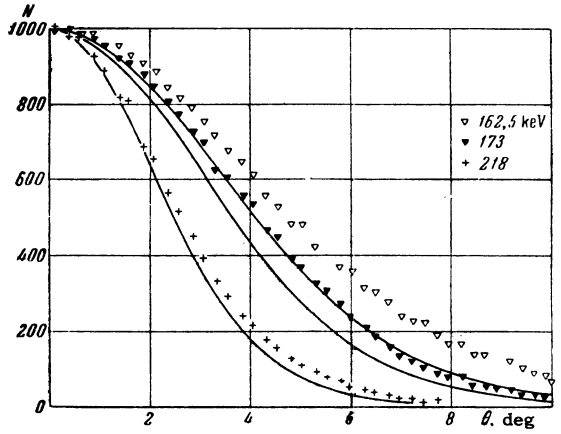

FIG. 3

each target we determined the angular distributions at different proton energies in the interval between 106 and 218 keV. A number of distributions obtained at these proton energies in this interval are shown in Figs. 1—3 (Figs. 1, 2, and 3 represent the results for target thicknesses of 24, 69, and 104 $\mu g/cm^2$, respectively.) The experimental error, as a rule, did not exceed 3.5% close to 50% of the maximum; in the region of the maximum of the distribution (at 0°) it decreased to 1.5—2%. A more detailed description of the method is given elsewhere.[3]

2. DISCUSSION OF RESULTS

There exist, at present, several variants of the theory of multiple scattering,[4-7] differing primarily in their mathematical approach. Among them, we may mention the theory of Molière,[4] which, as Bethe has shown,[2] possesses a great degree of generality and is formulated mathematically in a form convenient for its practical application.

In this report, we compare the experimental results with Molière's theory. According to this theory, the angular distribution of the multiply scattered particles is represented in the form of a series

$$f(\vartheta)\,\vartheta d\vartheta = \vartheta d\vartheta\left[e^{-\vartheta^2} + \frac{1}{B}f^1(\vartheta) + \frac{1}{B^2}f^2(\vartheta) + \ldots\right]. \quad (1)$$

The functions $f^{(1)}(\vartheta)$ and $f^{(2)}(\vartheta)$ have been tabulated by Bethe[2] and Molière.[4] The quantity ϑ is connected with the scattering angle θ by the relation $\vartheta = \theta/\chi_c\sqrt{B}$, while the parameter B is determined from the equation

$$B - \ln B = 2\ln(\chi_c/\chi_a) + 1 - 2C, \quad (2)$$

where C is the Euler constant. The angle χ_c is determined by the condition that the probability for a single scattering by an angle $\chi > \chi_c$ is 1. Its value is connected with the number of scattering centers N in a unit volume, the target thickness t, the charge of the moving particle Z_1e, its energy E, and the nuclear charge of the scattering atom Z_2e by the relation $\chi_c^2 = \pi N t Z_1^2 Z_2^2 e^4 E^{-2}$. The angle χ_a, called the screening angle, contains the features of single scattering. Molière showed that for a scattering potential calculated from the Thomas-Fermi model it is possible to express χ_a with good accuracy in the following simple form[8] obtained with the aid of the WKB approximation:

$$\chi_a = \chi_0\sqrt{1.13 + 3.76\alpha^2}, \quad (3)$$

where $\chi_0 = \lambda\!\!\!^{-}/a$ and $\alpha = Z_1Z_2e^2/\hbar v$; v and $\lambda\!\!\!^{-}$ are the velocity and wavelength of the moving particle and a is the Fermi radius of the atom ($a = 0.885\times a_0 Z^{-1/3}$, a_0 is the radius of the Bohr orbit).

Molière's theory is constructed on the assumption that the angles are small ($\sin\theta \approx \theta$). Moreover, it assumes that the conditions $\alpha\chi_0 \ll 1$ and $B \gtrsim 4.5$ are fulfilled. The breaking off of series (1) at the third term then leads to an error no greater than 2%. In our case, the conditions formulated above are fulfilled.

Since in the described experiment the protons lost an appreciable part of their energy in the target, we used the averaged values of the angles χ_a and χ_c for the calculation of B and ϑ. The averaging was carried out by means of the formulas

$$\bar{\chi}_c^2 = \int_{E_0}^{E_f}[(\chi_c^2)_C + (\chi_c^2)_H]\frac{dE}{(dE/dx)}, \quad (4)$$

$$\bar{\chi}_c^2\ln\bar{\chi}_a^2 = \int_{E_0}^{E_f}[(\chi_c^2\ln\chi_a^2)_C + (\chi_c^2\ln\chi_a^2)_H]\frac{dE}{(dE/dx)}. \quad (5)$$

The subscripts C and H refer to the carbon and hydrogen atoms, E_0 is the initial proton energy, $E_f = E_0 - \Delta E$ is the final energy, ΔE is the total energy lost in the target. The dependence of the specific energy loss of the proton dE/dx in polysterene on the proton energy was taken from Lorentz and Zimmerman.[9]

Strictly speaking, the calculation of the potential of the hydrogen atom from the Thomas-Fermi model is without meaning, but it did not contribute an appreciable error to the calculation of $\bar{\chi}_a$, since the contribution of the second term in expression (5) is < 3%. In the calculation, we used the value 1.055 g/cm^3 as the specific weight of polysterene. The quantities E_f, χ_c, and B calculated for a number of values of t and E_0 are shown in Table I.

Table I

t μ/cm²	E_0 keV	E_f keV	$(\theta_{1/2})_e$ min	χ_c min	V(r) from Thomas-Fermi		$V_H(r)$ exact, $V_C(r)$ from Hartree-Fock	
					B	$(\theta_{1/2})_e/(\theta_{1/2})_t$	B	$(\theta_{1/2})_e/(\theta_{1/2})_t$
24.0	106	82.4	114	60	4.60	1.25	6.67	0.98
	162.5	142	69	37.5	4.60	1.23	6.65	0.97
	218	200	49	26.8	4.60	1.21	6.63	0.94
40.0	106	66.5	178	85	5.25	1.26	7.28	1.03
	162.5	128	100	51.3	5.25	1.21	7.26	0.99
	218	188.5	69	35.6	5.25	1.17	7.23	0.97
69	106	38.4	382	151	5.90	1.42	7.95	1.19
	124	56.4	266	114	5.90	1.31	7.93	1.10
	142.5	76.7	198	88	5.90	1.25	7.91	1.05
	162.5	100.5	155	74	5.90	1.17	7.89	0.99
	184	126	129	62.2	5.90	1.15	7.88	0.99
	218	165.5	104	50	5.90	1.17	7.87	0.99
88	124	38.1	430	157	6.20	1.43	8.19	1.25
	162.5	81.5	213	92	6.20	1.26	8.18	1.07
	173	94.5	193	83.2	6.20	1.26	8.17	1.06
	218	150	126	59.2	6.20	1.17	8.15	0.99
104	162.5	65.5	287	111	6.45	1.38	8.36	1.17
	173	78.7	249	99	6.45	1.33	8.35	1.15
	218	136	156	67.2	6.45	1.23	8.33	1.06

Attention is drawn to the absence of a dependence of the parameter B on the energy. This is a consequence of the specific relation between the quantities χ_a and E obtained as a result of the use of the Thomas-Fermi model.

For all the cases considered, we calculated the angular distributions and determined the values of their half-widths $(\theta_{1/2})_t$. Comparison of the theoretical and experimental values of the half-widths shows that the experimental values $(\theta_{1/2})_e$ are systematically ~20—30% greater than the theoretical values $(\theta_{1/2})_t$. The reason for such a significant difference should be sought, above all, in the fact that the Thomas-Fermi model gives too rough an approximation for the electron distribution in light atoms such as carbon. In order to remove this shortcoming of the theory, we took the electron distribution in the atom calculated from the Hartree-Fock model. The radial dependence of the scattering potential in the case of the carbon atom was found with the aid of numerical integration of the relations

$$V_C(r) = \frac{Z_1 e^2}{a_0} \int_{r/a_0}^{\infty} \frac{Z_{eff}(y)\,dy}{y^2}, \quad Z_{eff}(y) = Z_2 - \int_0^y U(y)\,dy, \quad (6)$$

where Z_1 and Z_2 are the charges of the incident particle and the scattering atom. The radial distribution of the electron density U(y) was taken from Landolt-Börnstein.[10] In the case of the hydrogen atom, we used the exact expression for the potential:[11]

$$V_H(r) = \frac{Z_1 e^2}{r}\left(1 + \frac{r}{a_0}\right) e^{-2r/a_0}. \quad (7)$$

For the calculation of χ_a, we took the functions $V_C(r)$ and $V_H(r)$ in the form

$$V(r) = (Z_1 Z_2 e^2 / r)\,\omega(r/a)$$

and the function $\omega(r/a)$ was approximated (see [4]) by a polynomial of the form

$$\omega(r/a) = \sum_{i=1}^{3} a_i e^{-b_i r/a}. \quad (8)$$

The parameters a_i and b_i obtained in this way for carbon and hydrogen are shown in Table II. Of course, they turn out to be considerably different from the corresponding values of a_i^0 and b_i^0 occurring in Molière's theory. The functions $\omega(r/a)$ and the curves approximating them are shown in Fig. 4. The upper curve was calculated from the Thomas-Fermi model.

Using the values of a_i and b_i shown in Table II and applying Molière's procedure for the approximation of the exact dependence of χ_a on the energy by expression (3), we obtain

$$\chi_a = \chi_0 \sqrt{0.26 + 0.71\alpha^2} \quad \text{for C,} \quad (9)$$

$$\chi_a = \chi_0 \sqrt{0.45 + 1.30\alpha^2} \quad \text{for H.} \quad (9')$$

From Fig. 5 it is seen that the approximate functions (9) and (9′), represented by the dashed and

Table II

i	Carbon		Hydrogen		From Thomas-Fermi Model	
	a_i	b_i	a_i	b_i	a_i^0	b_i^0
1	0.1	5.0	0.75	1.3	0.1	6.0
2	—0.4	1.3	0.27	0.9	0.55	1.2
3	1.3	0.85	—0.02	0.3	0.35	0.3

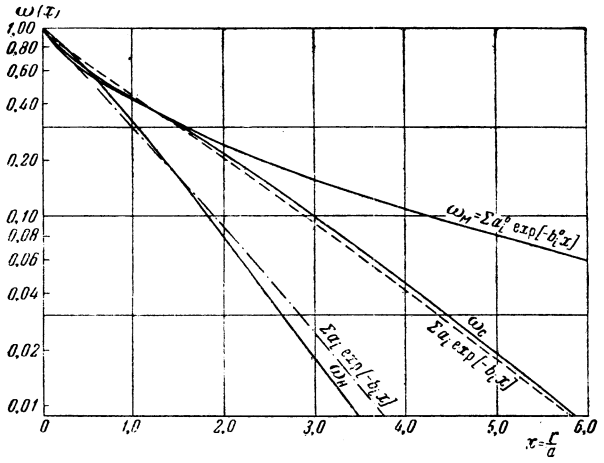

FIG. 4

dash-dotted lines, are in good agreement with the results of the direct calculation of the value χ_a (the points for $_6C^{12}$ are denoted by circles and the calculated points for $_1H^1$ are denoted by crosses). The solid line represents the function (3) used by Molière. The functions (9) and (9′) were used to calculate the new values of the parameter B. They are shown in Table I. The theoretical angular distributions calculated with their aid are shown in Figs. 1—3 by solid lines.

The experimental and theoretical values of the half widths $\theta_{1/2}$ are compared in the last column of Table I.

As seen from the figures and the table, the experimental and theoretical results are in good agreement for all energies at thicknesses of 24 and 40 $\mu g/cm^2$. With thicker targets, agreement is observed only for larger initial energies.

The foregoing behavior indicates that the deviation of the experimental distribution from the theoretical one occurs after the particle energy drops below some critical value as a result of the slowing down in the target. The existence and the magnitude of this critical energy can be seen in Fig. 6, where the ratio $(\theta_{1/2})_e/(\theta_{1/2})_t$ is shown as a function of the energy E_f; the data for all thicknesses are shown in one figure.

From the analysis of the results, we can conclude the following. Multiple scattering of protons in carbon is described well by Molière's theory down to an energy 80—90 keV if the Hartree-Fock model is used only for the calculation of the scattering potential. For proton energies below this value, the present theory does not give a correct description of the experimental results. In order to explain the reason for this difference, it is necessary to accumulate more experimental material. There is, however, a preliminary indication of at least two factors which are not taken into account in the theory and which can affect the experimental results.

1) Inelastic processes, in particular, charge exchange of the moving ions. As is known,[12] for proton energies of the order of several tens of keV, this effect is very important. It is particularly important to take into account inelastic processes in calculations of multiple scattering of nuclei heavier than protons. For them, the inelastic processes, as well as the partial screening of the charge will play a role even at much greater energies. There are no experimental data for such multiply charged particles. However, they are of interest, in particular, in connection with the use of recoil nuclei in nuclear investigations.[13,14]

2) Deformation of the electron shells in the case of solid targets due to the interaction between neighboring atoms. Here, it is of great interest to compare the experimental data obtained for solid and gaseous targets.

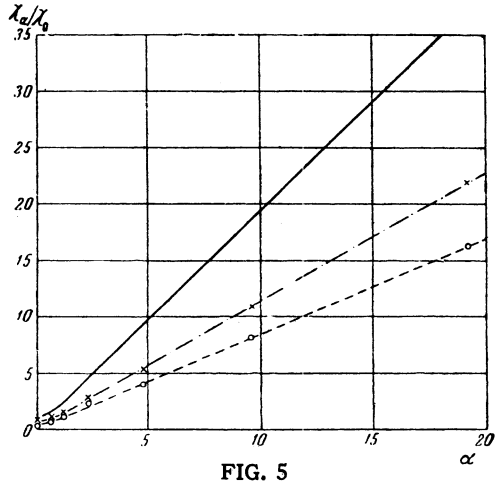

FIG. 5

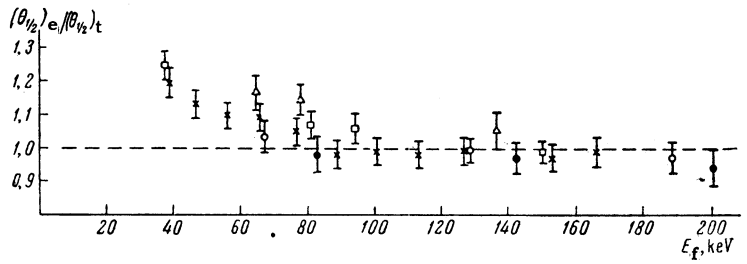

FIG. 6. Comparison of experimental and theoretical values of the half-widths of the distributions for different thicknesses (in $\mu g/cm^2$); ●—24, o—40, ×—69, □—88, Δ—104.

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