THE EQUILIBRIUM SHAPE OF ATOMIC NUCLEI

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Submitted to JETP editor June 6, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 777-778 (March, 1962)

Single-particle energy levels in an infinitely deep nonspherical rectangular well are calculated by diagonalizing the energy matrix with account of spin-orbit coupling. The dependence of the total energy on the nonsphericity parameters is determined for different nucleon configurations. The equilibrium shape of the nucleus in such a model is considered.

RECENTLY the theory of nonaxial nuclei has made great progress.[1] It is therefore of interest to consider the behavior of nucleons in a potential field that has no axial symmetry, and to determine the equilibrium shape of the nuclei, i.e., to ascertain what shape of the potential well achieves a minimum energy of the system of nucleons. Such calculations have been carried out for the oscillator potential by Geĭlikman,[2] Volkov and Inopin,[3] and Newton.[4] The case of an infinitely deep potential well with vertical walls was considered by Zaikin[5] on the basis of perturbation theory, without account of spin-orbit coupling. We also consider this case but with account of spin-orbit interaction, and obtain a much more accurate solution.

Let us consider a nucleon in an infinitely deep potential well with vertical sides, having in space the shape of an ellipsoid with semi-axes $a_x r_0$, $a_y r_0$, $a_z r_0$, where r_0 is the radius of the sphere of equal size. If we change to new coordinates which transform the ellipsoid to a sphere of radius r_0

$$x = a_x x', \quad y = a_y y', \quad z = a_z z', \tag{1}$$

then the equation which determines the state of the nucleons will have the form

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 - k(\hat{s}\,\hat{l}) + \hat{V}_1 + \hat{V}_2\right\}\psi_i = E_i\psi_i, \tag{2}$$

where k is the spin-orbit coupling constant. The operators $\hat{V}_1$ and $\hat{V}_2$, which appear as a consequence of the nonsphericity have, with accuracy up to quadratic terms in the deformation, the form

$$\hat{V}_1 = \rho\left[\cos\gamma - \tfrac{1}{4}\rho(1 + 2\cos^2\gamma)\right](\hat{p}^2 - 3\hat{p}_z^2)/2m - \sqrt{3}\,\rho\sin\gamma\left(1 + \tfrac{1}{2}\rho\cos\gamma\right)(\hat{p}_x^2 - \hat{p}_y^2)/2m + \rho^2\hat{\mathbf{p}}^2/2m,$$

$$\hat{V}_2 = -\tfrac{1}{2}\rho\left(\cos\gamma - \tfrac{1}{2}\rho\sin^2\gamma\right)(\hat{s}\,\hat{l} - 3\hat{s}_z\hat{l}_z) + \tfrac{1}{2}\sqrt{3}\,\rho\sin\gamma(1 - \rho\cos\gamma)(\hat{s}_x\hat{l}_x - \hat{s}_y\hat{l}_y) + \tfrac{1}{4}\rho^2(\hat{s}\,\hat{l}), \tag{3}$$

where $\rho \approx \sqrt{5/4\pi\beta}$, β, and γ are the deformation parameters introduced by A. Bohr.[6]

With the aid of the eigenfunctions for the spherical potential, we construct the energy matrix for all possible states with indices n, l, j, Ω; diagonalizing this matrix, we obtain the energy of the nucleons in the field under consideration as a function of β and γ. The total energy of the system of nucleons can be determined by summing the energies of all filled levels. In that manner we find the equilibrium shape of the nucleus which corresponds to the minimum value of the total energy as a function of the shape parameters of the nucleus.

The numerical calculations were carried out on the BÉSM electronic computer, using matrices of 76th order, corresponding to 76 particles of one kind. The calculations were made for spin-orbit coupling constants k = 0, 2, 3 (for k = 2 and 3, the order of the levels in the case of a spherically symmetric potential is close to the order of the levels in the scheme of Klinkenberg[7]).

The calculations have shown that the configurations of nucleons of a single type have axial symmetry; closed shells correspond to spherical symmetry. Configurations of nucleons of a different type, which correspond to nuclei with Z, N < 76 also have axial symmetry in most cases. However, for some configurations, the equilibrium shape does not have axial symmetry; for example, Kr^{92}_{36}($k = 0$, $\beta_0 = 0.08$, $\gamma_0 = 25°$); Ti^{46}_{22} ($k = 2$ $\beta_0 = 0.08$, $\gamma_0 = 25°$); Ge^{76}_{32} ($k = 3$, $\beta_0 = 0.11$, $\gamma_0 = 20°$).

From the considerations set forth above, it follows that the independent particle model for the case of an infinitely deep potential well allows a departure of the equilibrium shape of the nuclei from the axially symmetric.

It should be noted that the results of the present work differ somewhat from the results of [3-5], in which the result was obtained that the configurations of nucleons of different type can have non-axial equilibrium shape.

We note that the present calculation has a model character; for example, consideration of the pair forces can have an effect on the results.

The author thanks D. A. Zaikin for valuable advice and discussion of the results.

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Translated by R. T. Beyer
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