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# THEORY OF NONLINEAR GALVANOMAGNETIC PHENOMENA IN SEMICONDUCTORS

R. F. KAZARINOV and V. G. SKOBOV

Leningrad Physico-technical Institute, Academy of Sciences, U.S.S.R.

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The current in a semiconductor located in crossed electric and magnetic fields is investigated theoretically. The energy the electrons receive from the electric field upon scattering is greater than that they can impart to the lattice. This leads to an increase in the temperature of the electron gas. It is shown that the electrons have a Boltzmann distribution but with an effective temperature proportional to the square of the electric field. The dependence of the semiconductor resistance on the magnetic field strength and on the electric current is derived. In the limiting quantum case, when all electrons are on the lower Landau level, the resistance decreases as the cube of the current.

As shown earlier by Davydov, [1] deviations from Ohm's law can be observed in semiconductors even for comparatively weak electric fields E. This effect is associated with the fact that the electrons very slowly lose the energy acquired from the electric field. Collisions of the electron with impurities are elastic, while, in the emission of a phonon, the electron gives up only an insignificant (of the order of s/v) amount of its energy (s is the velocity of sound in the crystal, v is the velocity of the electron). Therefore, the energy $eEl$ acquired by the electron in the mean free path $l$ can be shown to be larger than the energy given up to the lattice in the same time.

As a consequence of this, the electron distribution function $F(\mathbf{p})$ undergoes significant changes. Its part $f(\mathbf{p})$ that is nonsymmetric in the quasi-momentum of the electron $\mathbf{p}$ remains much smaller than the symmetric part $F_0$ just as in the case of a weak electric field. This is connected with the rapid relaxation in the momentum: in the collision the momentum of the electron changes by an amount of its own order of magnitude. However, in contrast with the linear approximation in E, the function $F_0$ becomes essentially nonequilibrium, since the electron gas is heated and the mean kinetic energy of the electrons is changed. This heating takes place up to the temperature at which the energy of the radiated phonons becomes equal to the energy acquired by the electrons from the electric field. The symmetric part of the distribution function $F_0$ is determined from the condition of conservation of the number of electrons with a given kinetic energy. The $F_0$ found from this condition depends on E, which also determines the nonlinear effect.

The present research is devoted to the study of the current which flows through a semiconductor located in strong crossed electric and magnetic fields. Here we consider both the classical case $\hbar\Omega \ll T$ and the limiting quantum case $\hbar\Omega \gg T$, where $\Omega = eH/mc$, H is the magnetic field intensity, m is the effective mass of the conduction electron, c is the velocity of light, and T is the temperature in energy units.

The considerations given above relative to the nature of the nonlinearity of the current as a function of the electric field remain in force also for the case of crossed fields, except that the mean free path of the electron $l$ is replaced by its Larmor radius R. The latter is connected with the fact that, as the result of the collision, the electron in this case is displaced by a distance of the order of R in the direction of the electric field. Moreover, the degree of heating of the electron gas will depend on the value not only of the electric, but also of the magnetic field.

**1.** In classical theory, the condition for finding $F_0$ can be obtained by averaging the kinetic equation over the states with a given energy $\epsilon$. The kinetic energy has the form*

$$e\left\{\mathbf{E} + \frac{1}{c}\,[\mathbf{vH}]\right\}\frac{\partial F(\mathbf{p})}{\partial \mathbf{p}} + I_{\mathbf{p}}(F) = 0, \qquad (1)$$

where

$$F(\mathbf{p}) = F_0(\varepsilon_p) + f(\mathbf{p}), \quad f(-\mathbf{p}) = -f(\mathbf{p});$$

$$I_{\mathbf{p}}(F) = \sum_{\mathbf{p}'}\{F(\mathbf{p})\,W_{\mathbf{pp}'} - F(\mathbf{p}')\,W_{\mathbf{p}'\mathbf{p}}\}$$

is the collision integral, $W_{\mathbf{pp}'}$ is the transition probability of the electron per unit time from the

*$[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

state with momentum $\mathbf{p}$ to the state with momentum $\mathbf{p}'$; $\mathbf{v} = \mathbf{p}/m$; $\epsilon_p = p^2/2m$.

Equation (1) can be rewritten in the form

$$e\mathbf{E}\mathbf{v}\,\frac{\partial F_0}{\partial \varepsilon_p} + e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} - \Omega\,\frac{\partial f}{\partial \varphi} + I_{\mathbf{p}}(F_0) + I_{\mathbf{p}}(f) = 0, \quad (2)$$

where $\mathbf{H}$ is directed along the z axis and $\varphi$ is the polar angle in the $p_xp_y$ plane; f is a periodic function of $\varphi$ with period $2\pi$.

We average Eq. (2) over the states with energy $\epsilon$. For this purpose, we multiply (2) by $\delta(\epsilon_p - \epsilon)$ and sum over $\mathbf{p}$. The equation thus obtained for $F_0$ is fully equivalent to the equation of Davydov, [1] but is more descriptive. In the summation, the first and fifth terms on the left side of (2) vanish because of odd parity, while the third vanishes because of the periodicity of f in $\varphi$. As a result, we get

$$e\mathbf{E}\sum_{\mathbf{p}}\frac{\partial f}{\partial \mathbf{p}}\,\delta(\varepsilon_p - \varepsilon) = \sum_{\mathbf{p}\mathbf{p}'} F_0(\varepsilon_p)\,W_{\mathbf{p}\mathbf{p}'}[\delta(\varepsilon_{p'} - \varepsilon) - \delta(\varepsilon_p - \varepsilon)], \quad (3)$$

where we have relabelled the summation indices $\mathbf{p}$ and $\mathbf{p}'$ in the first term on the right side of (3). It is evident that only the inelastic scattering of the electrons gives a contribution to the right side of (3) when $\epsilon_{p'} \neq \epsilon_p$. Integrating the left side of Eq. (3) by parts, and keeping in mind the relation

$$\frac{\partial}{\partial \mathbf{p}}\,\delta(\varepsilon_p - \varepsilon) = -\frac{\partial}{\partial \varepsilon}\,\mathbf{v}\delta(\varepsilon_p - \varepsilon),$$

we put it in the form $\partial w(\epsilon)/\partial\epsilon$, where

$$w(\varepsilon) = \sum_{\mathbf{p}} e\mathbf{E}\mathbf{v}f(\mathbf{p})\delta(\varepsilon_p - \varepsilon)$$

is the power absorbed by electrons with energy $\epsilon$ from the electric field.

Let us first consider the case in which the scattering of the electrons is brought about by their interaction with acoustic phonons. Then, in the Born approximation,

$$W_{\mathbf{p}\mathbf{p}'}^{(\mathrm{ph})} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}}|C_{\mathbf{q}}|^2\{(N_{\mathbf{q}} + 1)\,\delta_{\mathbf{p}'+\mathbf{q},\mathbf{p}}\,\delta(\varepsilon_p - \varepsilon_{p'} - \hbar\omega_q) + N_{\mathbf{q}}\,\delta_{\mathbf{p}',\mathbf{p}+\mathbf{q}}\,\delta(\varepsilon_p + \hbar\omega_q - \varepsilon_{p'})\}. \quad (4)$$

Here $N_{\mathbf{q}}$ is the phonon distribution function; $|C_{\mathbf{q}}|^2$ for longitudinal acoustic phonons has the form

$$|C_{\mathbf{q}}|^2 = C^2\hbar q^2/2\rho\omega_q V_0,$$

where C is the constant of the deformation potential, $\rho$ is the density of the crystal, $V_0$ is its volume, $\omega_q = sq$ is the frequency of a phonon with wave vector $\mathbf{q}$.

We substitute expression (4) in (3), and in the term corresponding to phonon absorption we relabel the summation indices $\mathbf{p}$ and $\mathbf{p}'$. As a result (3) takes on the form

$$\frac{\partial w(\varepsilon)}{\partial \varepsilon} = \frac{2\pi}{\hbar}\sum_{\mathbf{p}\mathbf{p}'\mathbf{q}}|C_{\mathbf{q}}|^2\delta_{\mathbf{p}'+\mathbf{q},\mathbf{p}}\,\delta(\varepsilon_p - \varepsilon_{p'} - \hbar\omega_q) \times [\delta(\varepsilon_{p'} - \varepsilon) - \delta(\varepsilon_p - \varepsilon)]\{(N_{\mathbf{q}} + 1)F_0(\varepsilon_p) - N_{\mathbf{q}}F_0(\varepsilon_{p'})\}. \quad (5)$$

Expanding Eq. (5) in powers of the small parameter $\hbar\omega_q/\epsilon \sim s/v$, we get $\partial\omega(\epsilon)/\partial\epsilon = \partial Q(\epsilon)/\partial\epsilon$, where

$$Q(\varepsilon) \approx 2\pi\sum_{\mathbf{p}\mathbf{p}'\mathbf{q}}\omega_q|C_{\mathbf{q}}|^2\delta_{\mathbf{q}'+\mathbf{q},\mathbf{p}}\,\delta(\varepsilon_p - \varepsilon_{p'})\,\delta(\varepsilon_p - \varepsilon) \times \left\{1 + \hbar\omega_{\mathbf{q}}N_{\mathbf{q}}\frac{\partial}{\partial\varepsilon_p}\right\}F_0(\varepsilon_p) \quad (6)$$

is the power radiated by electrons with energy $\epsilon$ in the form of phonons. Taking it into account that in the stationary state all the energy received by the electrons from the electric field is radiated in the form of phonons, we get the relation

$$w(\varepsilon) = Q(\varepsilon), \quad (7)$$

which represents the differential energy balance.

2. From the equation for the density matrix, we can also derive Eq. (7) from quantum theory. The difference is that for this equation to be satisfied in the quantum limit $\hbar\Omega \gg (\epsilon - \hbar\Omega/2)$ the small quantity must be

$$s\sqrt{m\hbar\Omega}/(\varepsilon - \hbar\Omega/2),$$

and not s/v as in the classical case $\hbar\Omega \ll \epsilon$. This is connected with the fact that the phonons interacting with the electrons have in this case a momentum of the order of the transverse momentum of the electron $\sqrt{m\hbar\Omega}$. Moreover, one can expect that the current created by electrons with the given energy and the power radiated by them are expressed in terms of the symmetric part of the electron distribution function $F_0$ just as in the theory that is linear in E. This is a consequence of the fact that the work done by the electric field over a distance equal to the Larmor radius R is small in comparison with the characteristic energy of the electrons $\epsilon$, so that the effect of the electric field on the state of the electron can be neglected in the absence of scattering.

Let us expand the condition (7) in the quantum case. For simplicity, we limit ourselves to consideration of the isotropic quadratic spectrum of conduction electrons and do not consider their spin. The states of the electron in a magnetic field are characterized by the magnetic quantum number n, the projection of the quasimomentum in the direction of the magnetic field $\hbar p_z$, and the coordinate of the center of rotation X, while the energy eigenvalues have the form

$$\varepsilon_\alpha = \hbar\Omega(n + 1/2) + \hbar^2 p_z^2/2m,$$

where $\alpha$ denotes the choice of the quantum numbers n, $p_z$, and X.

We have already noted that $j_x(\epsilon)$, the current of electrons with energy $\epsilon$ in the direction of E (x axis), is expressed in terms of $F_0$ in the same fashion as in the linear theory by means of the equilibrium distribution function. Therefore, in accord with Titeica, [2] we have in the case of a strong magnetic field, $\Omega\tau \gg 1$ ($\tau$ is the relaxation time of the electrons)

$$j_x(\varepsilon) = \frac{1}{2} e^2 E \sum_{\alpha\beta} (X_\beta - X_\alpha)^2 W_{\alpha\beta} \frac{\partial F_0(\varepsilon_\alpha)}{\partial \varepsilon_\alpha} \delta(\varepsilon_\alpha - \varepsilon), \quad (8)$$

where $W_{\alpha\beta}$ is the probability per unit time of an electron transition from state $\alpha$ to state $\beta$. In the case of phonon scattering, $W_{\alpha\beta}^{(ph)}$ is obtained by replacement of p and p′ in Eq. (4) by the states $\alpha$ and $\beta$. Here $\delta_{\mathbf{p}',\mathbf{p}+\mathbf{q}}$ must be replaced by the square of the modulus of the matrix element of $e^{i\mathbf{q}\cdot\mathbf{r}}$ between the wave functions of the electron $\alpha$ and $\beta$:

$$|(\beta|e^{i\mathbf{qr}}|\alpha)|^2 = \delta_{p_z', p_z+q_z} \delta_{X', X+q_y/\gamma} [Q_n^{(n'-n)}(q_\perp^2/2\gamma)]^2,$$

$$Q_n^{(m)}(u) = u^{m/2} e^{-u/2} L_n^{(m)}(u), \quad (9)$$

$L_n^{(m)}(u)$ is the associated Laguerre polynomial, normalized to unity; $q_\perp^2 = q_x^2 + q_y^2$; $\gamma = eH/\hbar c$ is the square of the inverse of the "magnetic length."

The phonon distribution function $N_\mathbf{q}$, generally speaking, is not an equilibrium one. However, if the relaxation time of the phonons is smaller than the mean time between two successive acts of radiation of phonons by electrons, then it can be assumed that

$$N_\mathbf{q} = [\exp(\hbar\omega_q/T) - 1]^{-1}.$$

It follows from Eq. (9) that the momenta of the phonons interacting with the electrons are equal in order of magnitude to the momentum of the electron. In the quantum case $\epsilon - \hbar\Omega/2 \ll \hbar\Omega$, the longitudinal momentum of the electron is much less than the transverse one, so that the momenta of the radiated phonons lie principally in the xy plane. Therefore the maximum energy of the phonons emitted or absorbed by electrons with energy $\epsilon$ is equal in order of magnitude to $s\sqrt{m\epsilon}$ in the classical case and $s\sqrt{m\hbar\Omega}$ in the limiting quantum case. Assuming that in both cases this energy is small in comparison with T, we get

$$N_\mathbf{q} = T/\hbar\omega_q.$$

The power absorbed by electrons with energy $\epsilon$ from the electric field is $w(\epsilon) = Ej_x(\epsilon)$; therefore, Eq. (7) for $F_0(\epsilon)$ can be written in the form

$$-T\left(\frac{cE}{sH}\right)^2 \eta(\varepsilon) \frac{\partial F_0(\varepsilon)}{\partial \varepsilon} = \left(1 + T\frac{\partial}{\partial\varepsilon}\right) F_0(\varepsilon),$$

$$\eta(\varepsilon) = \sum_\mathbf{q} (q_y/q)^2 \mu(\mathbf{q}, \varepsilon) \Big/ \sum_\mathbf{q} \mu(\mathbf{q}, \varepsilon), \quad (10)$$

where

$$\mu(\mathbf{q}, \varepsilon) = \sum_{\alpha\beta} \omega_q |(\beta|e^{i\mathbf{qr}}|\alpha)|^2 \delta(\varepsilon_\alpha - \varepsilon)\delta(\varepsilon_\beta - \varepsilon),$$

while in Eqs. (8) for $j_x(\epsilon)$ we neglected the inelasticity. In the classical case $\hbar\Omega \ll \epsilon$, the quantity $\eta(\epsilon)$ is equal to $1/3$, while in the limiting quantum case it is $1/2$. This corresponds to the fact that in the first case the phonons are emitted uniformly in all three directions, while in the second they are emitted principally in the plane perpendicular to the magnetic field.

Thus, in both limiting cases $F_0(\epsilon)$ has the form of a Boltzmann distribution with effective temperature $T_{eff}$:

$$F_0(\varepsilon) = Z^{-1} \exp(-\varepsilon/T_{eff}), \qquad Z = \sum_\alpha \exp(-\varepsilon_\alpha/T_{eff}). \quad (11)$$

$$T_{eff} = T\{1 + \eta(cE/sH)^2\}. \quad (12)$$

It follows from (12) that the heating of the electron gas and the nonlinearity associated with it becomes important for

$$cE/sH > 1. \quad (13)$$

This condition has a rather simple physical meaning. Let the electron pass from state $\alpha$ to state $\beta$ with emission of the phonon q and acquire thereby from the electric field an energy

$$eE(X_\beta - X_\alpha) = eEq_y/\gamma = \hbar c q_y E/H. \quad (14)$$

The condition that this energy exceed the energy of the emitted phonon $\hbar\omega_q$ is identical with (13).

As the magnetic field H decreases and the Larmor radius R becomes larger than the electron free path $l$, one must replace R by $l$ in the parameter $cE/sH \sim cER/\hbar\omega_q$ which determines the heating of the electron gas. In this case, Eq. (10) transforms into the equation of Davydov [1] for a weak magnetic field, and $F_0(\epsilon)$ ceases to be a Boltzmann function.

**3.** In the case in which the scattering of the electrons is principally determined by the interaction with impurities, the transition probability $W_{\alpha\beta}$ in Eq. (8) contains a component $W_{\alpha\beta}^{(i)}$ which is due to scattering on impurities. It is not difficult to show that Eq. (10) for $F_0(\epsilon)$ takes the form

$$F_0(\varepsilon) + T\left\{1 + \eta(\varepsilon)\left(\frac{cE}{sH}\right)^2 [1 + \nu_i(\varepsilon)/\nu_{ph}(\varepsilon)]\right\} \frac{\partial F_0}{\partial\varepsilon} = 0 \quad (15)$$

in this case, where $\nu_{ph}(\epsilon)$ and $\nu_i(\epsilon)$ are the collision frequencies of electrons with energy $\epsilon$ with phonons and impurities, respectively. In the case of neutral impurities, the ratio $\nu_i/\nu_{ph}$, both in the classical and quantum limits, does not depend on the energy $\epsilon$ and the value of H. If the impurities are ionized, then, in the Born approximation,

$$\nu_i/\nu_{ph} \sim H^0\varepsilon^{-2} \qquad \varepsilon \gg \hbar\Omega,$$

$$\nu_i/\nu_{ph} \sim H^{-2}\varepsilon^0 \qquad (\varepsilon - \hbar\Omega/2) \ll \hbar\Omega.$$

Thus, in the quantum limit, for all scattering mechanisms considered, the distribution function is shown to be Boltzmann but with an effective temperature proportional to the square of the electric field.

**4.** Now it is not difficult to find the dependence of the transverse current density $j_x$ on the electric and magnetic fields. As was noted above, to obtain $j_x$ it suffices to replace the equilibrium distribution function in the linear approximation formula by the nonequilibrium but symmetric $F_0$. Inasmuch as the latter differs from the Boltzmann distribution by the replacement of the temperature T with the effective temperature $T_{eff}$, to calculate the current produced by the scattering of electrons on the impurities it is necessary to put $T_{eff}$ in place of T in the linear approximation formula for $j_x^{(0)}$. In the case of the scattering of electrons on phonons, it is necessary in the formula for $j_x^{(0)}$ to put $T_{eff}$ in place of all factors T except the one which arises from the phonon distribution function. The dependence of the transverse current $j_x^{(0)}$ on the electron temperature is different in the quantum and classical limits. For scattering of electrons on acoustic phonons or on neutral impurities,[3] we have in the case of a constant concentration of the conduction electrons, $n_e$ = const,

$$j_x^{(0)} \sim ET^{1/2}(\hbar\Omega)^{-2} \qquad \hbar\Omega \ll T,$$
$$j_x^{(0)} \sim ET^{-3/2} \qquad \hbar\Omega \gg T. \tag{16}$$

Substituting $T_{eff}$ for T in (16), we obtain the dependence of $j_x$ on E and H:

$$j_x = \frac{e^2 n_e E}{m\Omega^2\tau}\left(\frac{2T}{\hbar\Omega}\right)^{k-1/2}\left[1 + \eta\left(\frac{cE}{sH}\right)^2(1 + \nu_i/\nu_{ph})\right]^k, \tag{17}$$

where $k = 1/2$ for $\hbar\Omega \ll T_{eff}$ and $k = -3/2$ for $\hbar\Omega \gg T_{eff}$. It follows from this expression that for $T_{eff} \gg T$

$$j_x \sim E^2/H^3 \qquad \hbar\Omega \ll T_{eff}, \tag{18}$$
$$j_x \sim H^3/E^2 \qquad \hbar\Omega \gg T_{eff}. \tag{19}$$

We note that the dependence of $j_x$ on E and H is determined by Eq. (18) even in the case $\hbar\Omega \gg T$, if the condition $\hbar\Omega \ll T_{eff}$ is fulfilled.

The Hall current $j_y$ has the form

$$j_y = cen_eE/H. \tag{20}$$

We note that Eqs. (17) and (20) for $j_x$ and $j_y$ are valid only for $j_x \ll j_y$.

In the experiment, the resistivity of the crystal $\rho_{ik}$ is determined by the relation

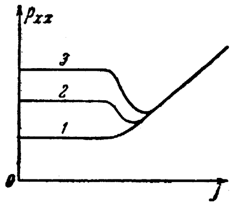
$$\rho_{ik} = E_i j_k / j^2. \tag{21}$$

In the case $j_x \ll j_y$ the angle between the vectors **j** and **E** is close to $\pi/2$ and $\rho_{xy} = \rho_{xy}^{(0)} = H/cen_e$. By using Eqs. (17) and (20), it is not difficult to show that in the case under considerations,

$$\rho_{xx}(j) = \sigma_0^{-1}\left[1 + \eta\left(\frac{j}{sen_e}\right)^2(1 + \nu_i/\nu_{ph})\right]^k\left(\frac{2T}{\hbar\Omega}\right)^{k-1/2} \tag{22}$$

where $\sigma_0$ is the conductivity of the crystal for H = 0 and E → 0.

The dependence of $\rho_{xx}$ on the current density j for $n_e$ = const is shown in the drawing. Curve 1




corresponds to the classical limit $\hbar\Omega \ll T$, while curves 2 and 3 correspond to the quantum case $\hbar\Omega \gg T$. In the region of small j, Ohm's law is valid, and $\rho_{xx}$ does not depend on j. In the classical case, $\rho_{xx}^{(0)}$ does not depend on the magnetic field and in the limiting quantum case it is proportional to $H^2$ (curve 3 corresponds to a larger value of the magnetic field than curve 2). With increase in j the second component in the square brackets in (22) is increased. Curve 1, which corresponds to $k = 1/2$, deviates in the direction of large values of $\rho_{xx}$ and for $T_{eff} \gg T$, the value $\rho_{xx} \sim j$. Curves 2 and 3 in the region $T \ll T_{eff} \ll \hbar\Omega$ $(k = -3/2)$ deviate from a straight line in the direction of lower values of $\rho_{xx}$, and coalesce into curve 1 in the region of $\hbar\Omega \ll T_{eff}$, $(k = 1/2)$.

Such is the dependence of $\rho_{xx}$ on j for scattering of electrons on phonons and neutral impurities. If the scattering is determined chiefly by the interaction with ionized impurities, then the dependence of $\rho_{xx}$ on j and H is different. However, in the region $T \ll T_{eff} \ll \hbar\Omega$, the dependence of the resistance $\rho_{xx}$ on the current remains as before: $\rho_{xx} \sim j^{-3}$.

**5.** Everything pointed out above refers to the case of a constant concentration of conduction electrons. This case can be realized, for example, in InSb. In this semiconductor, all the impurities are ionized and $n_e$ = const even at low temperatures. At the same time, the quantum limit in InSb can be achieved at a comparatively high temperature thanks to the small effective mass.

In certain cases, the dependence of $n_e$ on the electric and magnetic fields can easily be established. Impurity semiconductors can serve as an example in those cases in which impact ionization of the atoms of the impurity and the reverse process—triple impact—predominate over multiphonon ionization and recombination. Under these conditions, the electrons of the impurity atoms and the conduction electrons are in statistical equilibrium and the concentration of conduction electrons is determined by the usual formula, in which we have $T_{eff}$ in place of $v_i$.

$$n_e = (n_a Z)^{1/2} \exp(-V_i/2T_{eff}), \qquad T_{eff} \ll V_i. \qquad (23)$$

**6.** We now estimate to what degree the angle $\vartheta$ between the directions of the electric and magnetic fields can depart from a right angle, without changing the results obtained in the research.

Let **H** ∥ Oz; $E_x = E \sin\vartheta$; $E_y = 0$; $E_z = E\cos\vartheta$, whence $\tan\vartheta \gg 1$. In the case when

$$j_z E_z \ll j_x E_x \qquad (24)$$

heating of the electron gas, which is connected with the longitudinal current $j_z$, is not important and this current can be computed in linear approximation. The effect of $E_x$ on the effective temperature of the electrons can be taken into account as above.

In the classical case $\hbar\Omega \ll T_{eff}$, in scattering of electrons on phonons or neutral impurities,

$$j_z/j_x \approx (\Omega\tau_{eff})^2 \operatorname{ctg}\vartheta = (\Omega\tau)^2 (T/T_{eff}) \operatorname{ctg}\vartheta, \qquad (25)^*$$

where $\tau'$ is the relaxation time of the electrons for E = H = 0.

Substituting (25) in (24) and setting $\vartheta = \pi/2 - \Delta\vartheta$, we put the condition (24) in the form

$$(\Delta\vartheta)^2 \ll T_{eff}/T\,(\Omega\tau)^2. \qquad (26)$$

It can be shown that in the quantum limit $\hbar\Omega \gg T_{eff}$

$$j_z/j_x \sim (T/\hbar\Omega)(\tau T_{eff}/\hbar)^2 \operatorname{ctg}\vartheta, \qquad (27)$$

and the condition (24) has the form

$$(\Delta\vartheta)^2 \ll (\hbar\Omega/T)(\hbar/\tau T_{eff})^2. \qquad (28)$$

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*ctg = cot.

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