RECONSTRUCTION OF THE SCATTERING MATRIX NEAR THRESHOLD

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Submitted to JETP editor December 1, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 1249-1251 (May, 1962)

The information on the scattering matrix which can be obtained from measurements of the energy dependence of polarization quantities relating to inelastic scattering near a reaction threshold is analyzed from the point of view of a "complete experiment." It is shown that such measurements significantly modify and simplify the problem of phase analysis at the threshold energy, and in some cases even make the phase analysis superfluous.

In the present paper we study, from the point of view of a complete experiment, the information concerning nuclear scattering which one can derive from the energy dependence of the phases near threshold, which has been considered by Wigner,[1] one of the present authors,[2] and by Breit.[3]

1. THE CASE OF SPINLESS PARTICLES

We shall first carry out the analysis of scattering of spinless particles near a reaction threshold, following the method described in [2]. In the case of spin zero particles, the reaction goes via the s-state of the initial system, and the s-phase for elastic scattering suffers the largest change. Let us expand it in powers of the wave number k_1 of the created particles and restrict ourselves to the leading terms in the expansion:

$$e^{2i\delta_0} = e^{2i\delta_0(\mathrm{th})}\,[1 + ak_1]. \tag{1}$$

Below threshold, the phase δ_0 is real, k_1 is pure imaginary, and the quantity a must be real. Above threshold, the condition that the phase be real is replaced by the condition of unitarity of the matrix

$$\begin{pmatrix} e^{2i\delta_0} & mk_1^{1/2} \\ mk_1^{1/2} & x \end{pmatrix},$$

from which it follows, in particular, that

$$|e^{2i\delta_0}|^2 + |m|^2 k_1 = 1.$$

Substituting the expansion of the phase shift, we find that $\mathrm{Re}\, a = -|m|^2/2$ or, since a is real,

$$a = -|m|^2/2.$$

Thus the scattering amplitude near threshold has the following form:

$$f(\vartheta) = f_{\mathrm{th}}(\vartheta) + \tfrac{1}{4} i |m|^2 k_1 k^{-1} e^{2i\delta_0(\mathrm{th})} \tag{2}$$

(where k is the wave number in the elastic channel).

We now consider the differential cross section. Below threshold

$$\sigma(\vartheta) = \sigma_{\mathrm{th}}(\vartheta) - \tfrac{1}{2}|m|^2 |k_1| k^{-1}\, \mathrm{Re}\,\{e^{-2i\delta_0(\mathrm{th})} f_{\mathrm{th}}(\vartheta)\}.$$

Above threshold

$$\sigma(\vartheta) = \sigma_{\mathrm{th}}(\vartheta) + \tfrac{1}{2}|m|^2 k_1 k^{-1}\, \mathrm{Im}\,\{e^{-2i\delta_0(\mathrm{th})} f_{\mathrm{th}}(\vartheta).\}$$

From this it follows that by studying the angular distribution above and below threshold, we obtain, except for a factor, the scattering amplitude itself, and not its square modulus, as one does in the usual cross section measurements.

The coefficients multiplying the amplitude are easily found. The quantity $|m|^2$ is determined from the reaction cross section ($\sigma_r = \pi k^{-2}|m|^2 k_1$) and the phase $\delta_0(\mathrm{th})$ can be found from the total elastic cross section:

$$\sigma = \sigma_{\mathrm{th}} - \frac{\pi}{k^2}|m|^2|k_1| \times \begin{cases} \sin 2\delta_0\,(\mathrm{th}) & \text{(below threshold)} \\ 2\sin^2\delta_0\,(\mathrm{th}) & \text{(above threshold)} \end{cases}$$

2. THE ROLE OF THE THRESHOLD IN THE SCATTERING OF SPIN-½ PARTICLES BY SPIN-0 PARTICLES

This case has been treated in detail in a paper of Okun' and one of the authors.[4] We shall restrict ourselves here to the simplest case, where the particles resulting from the reaction have the same spins as the initial particles and the parity is unchanged. If the relative parities change, the formulas become somewhat more complicated, but the conclusions are the same.

Under these assumptions, the reaction proceeds via a single channel, the state with orbital angular momentum 0 and total angular momentum ½. The corresponding elastic scattering phase $\delta_{\frac{1}{2}0}$ is real below threshold, and the derivation of the relation

(2) in this case coincides verbatim with that given in the preceding section. The elastic scattering matrix near threshold is the following:

$$M = a_{\text{th}}\ (\vartheta) + b_{\text{th}}\ (\vartheta)(\sigma\,\mathbf{n}) + \tfrac{1}{4}\,i\,|\,m\,|^2 k_1 k^{-1} \exp\{2i\delta_{1/2\,0}\ (\text{th})\},$$

i.e., only the coefficient a changes, while b is unchanged. From this it is easy to obtain the expressions for the cross section and polarization. Below threshold we have

$$\sigma(\vartheta) = \sigma_{\text{th}}\ (\vartheta) - \tfrac{1}{2}|\,m|^2\,|\,k_1\,|\,k^{-1}\,\mathrm{Re}\,\{e^{-2i\delta_{1/2\,0}\,(\text{th})}\ a_{\text{th}}\ (\vartheta)\},$$

$$\sigma\,(\vartheta)P\,(\vartheta) = [\sigma\,(\vartheta)\,P\,(\vartheta)]_{\text{th}} - \tfrac{1}{2}\,|m\,|^2\,|\,k_1\,|\,k^{-1}\,\mathrm{Re}\,\{e^{-2i\delta_{1/2\,0}\,(\text{th})}\ b_{\text{th}}\ (\vartheta)\}.$$

Above threshold

$$\sigma(\vartheta) = \sigma_{\text{th}}\ (\vartheta) + \tfrac{1}{2}\,|\,m|^2\,k_1 k^{-1}\,\mathrm{Im}\{e^{-2i\delta_0\,(\text{th})}\ a_{\text{th}}\ (\vartheta)\},$$

$$\sigma\,(\vartheta)\,P\,(\vartheta) = [\sigma\,(\vartheta)\,P\,(\vartheta)]_{\text{th}} + \tfrac{1}{2}\,|\,m\,|^2\,k_1 k^{-1}\,\mathrm{Im}\,\{e^{-2i\delta_0\,(\text{th})}\ b_{\text{th}}\ (\vartheta)\}.$$

As in the spinless case, measurements of the cross section and polarization as a function of energy enable one to determine the reaction matrix completely.

Thus in these cases the phase analysis is superfluous for the reconstruction of the scattering matrix. However, even if one measures only the cross section near threshold, the phase analysis is significantly changed. Whereas usually one has to analyze the two quadratic quantities $|\,a\,|^2 + |\,b\,|^2$ and Re ab*, here, in addition to $|\,a\,|^2 + |\,b\,|^2$, one knows one of the amplitudes from experiment.

3. p-p SCATTERING NEAR THE THRESHOLD FOR THE REACTION $p + p \to \pi + d$

Let us consider the special features which appear in proton-proton scattering near the threshold for creation of a pion and a deuteron. Here the relative parities of the particles before and after the reaction are different, and the reaction near threshold occurs only through the state of the proton-proton system for which the orbital angular momentum, total angular momentum and spin are all equal to unity. The corresponding elastic scattering phase $\delta^1_{11;11}$ is real, and the derivation of a relation of the type of (2) is the same as in the spinless case. However, since the reaction goes through a p state of the initial system, the addition to the scattering matrix depends on angles and spins. Near threshold

$$M = \{\alpha + \beta\,(\sigma_1\mathbf{n})\,(\sigma_2\mathbf{n}) + \gamma\,(\sigma_1 + \sigma_2,\mathbf{n}) + \delta\,(\sigma_1\mathbf{m})\,(\sigma_2\mathbf{m}) + \varepsilon\,(\sigma_1\mathbf{l})\,(\sigma_2\mathbf{l})\} + \eta k_1 \Big\{\cos\vartheta - \frac{1}{2}\,i\sin\vartheta\,(\sigma_1 + \sigma_2,\mathbf{n}) - \sin^2\frac{\vartheta}{2}\,(\sigma_1\mathbf{m})\,(\sigma_2\mathbf{m}) + \cos^2\frac{\vartheta}{2}\,(\sigma_1\mathbf{l})\ (\sigma_2\mathbf{l})\Big\},$$

$$\eta = -\frac{3\pi}{8ik}\,|\,m\,|^2\exp\{2i\delta^1_{11,11}\ (\text{th})\}.$$

If we go over from $\alpha, \beta, \ldots$ to their combinations $a = \alpha + \beta$, $b = \alpha - \beta$, $c = \delta + \epsilon$, $d = \delta - \epsilon$, $e = 2\gamma$, then near threshold

$$a = a_{\text{th}} + \eta k_1\cos\vartheta, \qquad b = b_{\text{th}} + \eta k_1\cos\vartheta,$$
$$c = c_{\text{th}} + \eta k_1 \cos\vartheta,$$
$$d = d_{\text{th}} - \eta k_1, \quad e = e_{\text{th}} - i\eta k_1 \sin\vartheta.$$

If we now use the familiar expressions for measured quantities in terms of a, b, ... (cf., for example, [5]), we can easily obtain the near-threshold corrections. Here we give expressions only for the cross section and polarization in the scattering of unpolarized particles. Below threshold we have

$$\sigma\,(\vartheta) = \sigma_{\text{th}}\ (\vartheta) + |\,k_1|\ \ \mathrm{Im}\,\eta^*\ [(a + b + c)_{\text{th}}\ \cos\vartheta - d_{\text{th}} + ie_{\text{th}}\ \sin\vartheta],$$

$$\sigma\,(\vartheta)\,P\,(\vartheta) = [\sigma\,(\vartheta)\,P\,(\vartheta)]_{\text{th}} + |\,k_1|\,\mathrm{Im}\,\eta^*\ [ia_{\text{th}}\ \sin\vartheta + e_{\text{th}}\ \cos\vartheta].$$

Above threshold

$$\sigma\,(\vartheta) = \sigma_{\text{th}}\ (\vartheta) + k_1\ \mathrm{Re}\,\eta^*\ [(a + b + c)_{\text{th}}\ \cos\vartheta - d_{\text{th}} + ie_{\text{th}}\ \sin\vartheta],$$

$$\sigma\,(\vartheta)\,P\,(\vartheta) = [\sigma\,(\vartheta)\,P\,(\vartheta)]_{\text{th}} + k_1\ \mathrm{Re}\,\eta^*\ [ia_{\text{th}}\ \sin\vartheta + e_{\text{th}}\ \cos\vartheta].$$

As we see, the situation is the same as in the cases considered earlier—the measurement of the energy dependence of any quantity enables us to find directly some definite linear combination of coefficients of the scattering matrix. The computations show that if one could make measurements of the threshold dependence for any five quantities, for example the quantities specially selected in [5], this would make it possible to find the scattering matrix without resorting to phase analysis or the equivalent use of the unitarity relation. But even in incomplete form, threshold measurements are an important aid for the phase analysis; measurements of just the cross sections and polarizations give six equations and permit one to omit the very difficult experiments on triple scattering. It is true that this has its own difficulties, since one has to get sufficient energy resolution and must measure the quantities with sufficient accuracy for separating out the relatively small, energy-dependent terms.

[1] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).
[2] A. I. Baz', JETP **33**, 923 (1957), Soviet Phys. JETP **6**, 709 (1958).
[3] G. Breit, Phys. Rev. **107**, 1612 (1957).
[4] A. I. Baz' and L. B. Okun', JETP **35**, 757 (1958), Soviet Phys. JETP **8**, 526 (1959).
[5] Puzikov, Ryndin, and Smorodinskiĭ, JETP **32**, 592 (1957), Soviet Phys. JETP **5**, 489 (1957).

Translated by M. Hamermesh
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