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ANALYTIC PROPERTIES OF THE PARTIAL WAVE AMPLITUDES AND THE ASYMPTOTIC BEHAVIOR OF THE SCATTERING AMPLITUDE

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It is shown that if the simplest inelastic channels are taken into account in the unitarity condition, the partial wave amplitudes can have as a function of the angular momentum as singularities only poles and singularities whose position does not depend on the energy. If one assumes that this result holds generally then it follows that the asymptotic behavior of the scattering amplitudes at high energies can lead either to a slowly decreasing cross section [2] or it has the character described earlier. [3]

1. INTRODUCTION

In a previous paper [1] it was shown that the asymptotic behavior of the scattering amplitude for high energies s and fixed momentum transfer t are given by the position and character of the singularities in the l-dependence of the partial wave amplitude $f_l(t)$ in the channel in which t corresponds to the energy. It was shown that the asymptotic behavior varies appreciably, depending on whether the positions of the singularities $f_l(t)$ depend on t or not. If they do not depend on t the asymptotic behavior corresponds to a slowly decreasing cross section. [2] If the positions of the singularities of $f_l(t)$ depend on t and if their character is unknown, then the asymptotic behavior can be very involved and it may not exhibit a connection between the physical and nonphysical regions.

Earlier [3] we have discussed the properties of the asymptotic behavior of the scattering amplitude, obtained if one assumes that $f_l(t)$ is a meromorphic function of l as in the nonrelativistic case, [4] or if its first singularities from the side of high l are poles.

In the present paper we shall show that by investigating $f_l(t)$ as a function of t on the nonphysical sheets [5] one can arrive at the conclusion that $f_l(t)$ cannot have any t-dependent singularities in l other than poles. This statement will be proved here with account of the simplest inelastic channels in the unitarity condition only. In the next paper we shall show that it remains true also if one takes into account a number of more complicated inelastic channels (a three-particle intermediate state). This leads one to think that the result is true in general. The difficulty of the proof for the general case lies in our inability to formulate in a lucid manner the analytic properties of production amplitudes known from perturbation theory.

If we assume this result, then it follows that there are only two possible types of asymptotic behavior of the scattering amplitude in the channel where s is the energy: 1) the asymptotic behavior leads to a slowly decreasing total cross section; 2) the asymptotic behavior is as discussed in [3] whereby the total cross section may be constant while the elastic scattering cross section approaches zero and the interaction radius increases with increasing energy.

The essential feature on which are based all discussions of this and future papers is the assumption that for the scattering amplitudes there hold dispersion relations over the momentum transfer at arbitrary energies with a finite number of subtractions.

2. PARTIAL WAVE AMPLITUDES WITH COMPLEX l

We consider the invariant scattering amplitude A(s, t) for spinless particles in the channel where t is the energy, and we represent it in the form of a series of partial waves

$$A(s,t)=\sum_{n=0}^{\infty}(2n+1)f_n(t)P_n(z),$$

$$f_n(t)=\frac{1}{2}\int_{-1}^{1}P_n(z)A(s,t)\,dz; \qquad (1)$$

$$s=m_1^2+m_2^2-2p_{10}p_{20}+2p_1p_2z; \qquad (2)$$

here p_{10}, p_{20}, p_1, p_2 are the energies and momenta in the center-of-mass system. We assume that A(s, t) obeys the usual dispersion relations in s with fixed t

$$A(s,t)=\frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'-s}A_1(s',t)+\frac{1}{\pi}\int_{u_0}^{\infty}\frac{du'}{u'-u}A_2(u',t). \quad (3)$$

We consider this relation to be fulfilled in the sense that a sufficient number of subtractions has been performed.

From (1) and (3) follows that

$$f_n(t)=\frac{1}{\pi}\int_{z_0}^{\infty}Q_n(z)A_1(s,t)\,dz+(-1)^n\frac{1}{\pi}\int_{z'_0}^{\infty}Q_n(z)A_2(u,t)\,dz, \quad (4)$$

where $Q_n(z)$ is the Legendre function of the second kind, $n > n_0$, and n_0 is given by the number of subtractions; z is determined in the first integral by (2) and in the second by an analogous connection with u. It is convenient to consider separately the parts of A(s, t) that are symmetric and antisymmetric with respect to the replacement of z by $-z$:

$$A(s,t)=A^+(s,t)+A^-(s,t) \quad (5)$$

and correspondingly

$$f_n=\tfrac{1}{2}(1+(-1)^n)f_n^+ +\tfrac{1}{2}(1-(-1)^n)f_n^-, \quad (6)$$

$$f_n^{\pm}(t)=\frac{1}{\pi}\int_{z_0}^{\infty}Q_n(z)A_1(s,t)\,dz\pm\frac{1}{\pi}\int_{z_0'}^{\infty}Q_n(z)A_2(u,t)\,dz. \quad (7)$$

The quantities $f_n^{\pm}$ can be easily generalized to include complex angular momenta. However, if no additional conditions are imposed, this can be done in a multitude of ways. Thus, for example, we can determine $f_l^{\pm}$ by inserting in (7) Q_l instead of Q_n and considering l to be noninteger, or we can replace in (1) P_n by P_l, where P_l and Q_l are the Legendre functions of the first and second kind. This way we obtain different analytical functions $f_l^{\pm}$ and $f_l^{\pm\prime}$.

We assume the following conditions which will define uniquely the functions $f_l^{\pm}$: (a) $f_l^{\pm}$ has to coincide with $f_n^{\pm}$ for either even or odd l; (b) $f_l^{\pm}$ must be analytic functions in the half-plane Re $l \geq l_0$, $l_0 \leq n_0$; (c) $f_l^{\pm}$ must be bounded if $l \to \infty$ along an arbitrary direction in the half-plane Re $l > l_0$.

Evidently these conditions define $f_l^{\pm}$ uniquely: if there would exist two such functions their difference would be an analytic function in the half-plane, vanishing at infinity and having zeros at all even or odd points. According to Blaschke's theorem modified for a half-plane,[6] such a function vanishes identically. All these conditions are fulfilled by the function obtained by exchanging Q_l for Q_n in (7). As has been shown in [1], the singularities of this function determine the asymptotic behavior of the scattering amplitude as $s \to \infty$ by means of the relation

$$A^{\pm}(s,t)=\frac{1}{2}\sum_{n=0}^{n_0}(2n+1)(1\pm(-1)^n)f_n(t)P_n(z)$$
$$+\frac{i}{4}\int_{l_0-i\infty}^{l_0+i\infty}\frac{(2l+1)}{\sin l\pi}f_l^{\pm}(t)\{P_l(-z)\pm P_l(z)\}\,dl. \quad (8)$$

We have treated the uniqueness of the quantities $f_l^{\pm}$ introduced in [1] in some detail because the clarification of this question allows one to write down without further calculation the unitarity condition which the functions $f_l^{\pm}$ have to fulfill. For simplicity we omit from now on the ±, remembering that the symmetric and antisymmetric parts have to be treated separately. We now consider the problem of unitarity. At an energy t below the threshold for inelastic processes, $f_n(t)$ obeys the unitarity condition

$$\frac{1}{2i}\left(f_n^r-f_n^a\right)=\frac{k}{\omega}f_n^r f_n^a,\quad f_n^r\equiv f_n,\quad f_n^a=f_n^*.$$

If we now construct from the functions f_n^r and f_n^a, which are defined for integer n, the functions f_l^r and f_l^a fulfilling the above enumerated conditions, then we can show, repeating the arguments used above to prove uniqueness, that the unitarity condition has to be fulfilled for arbitrary complex l:

$$\frac{1}{2i}(f_l^r-f_l^a)=\frac{k}{\omega}f_l^r f_l^a,\qquad f_l^a=(f_{l^*}^r)^*. \quad (9)$$

3. ANALYTIC PROPERTIES OF THE PARTIAL WAVES

Before embarking on the investigation of the singularities of $f_l(t)$ as a function of l we consider the analytic properties of $f_l(t)$ as a function of t. We assume here that the Mandelstam representation applies and that for arbitrary t we can thus limit ourselves in the dispersion relation (3) to a finite number of subtractions. This means that one can choose an l_0 defined by the following requirements: $f_l(t)$ is analytic in the half-plane Re $l \geq l_0$, where l_0 does not depend on t. For reasons of simplicity we take the masses of the particles to be equal and $A_1 = A_2$. Then

$$f_l(t)=\frac{4}{\pi}\int_{s_0}^{\infty}Q_l\left(1+\frac{2s}{t-4m^2}\right)A_1(s,t)\frac{ds}{t-4m^2}. \quad (10)$$

For Re $l > l_0$ the integral then converges for arbitrary t. Since $A_1(s, t)$ for fixed s does not have complex singularities in t and $1 + 2s/(t - 4m^2) \neq \pm 1$ for complex t, the function $f_l(t)$ also does not have complex singularities in t. We now turn to the singularities of $f_l(t)$ on the real axis.

To begin with, $f_l(t)$ has singularities at $t = (\Sigma\mu_i)^2$. These correspond to the thresholds of the inelastic processes which arise because of $A_1(s, t)$. Furthermore, for $t - 4m^2 \to 0$

$$Q_l\left(1 + \frac{2s}{t - 4m^2}\right) \sim \left(\frac{t - 4m^2}{2s}\right)^{l+1}$$

and thus $f_l(t) \sim (t - 4m^2)^l$. It thus has singularities which differ from the usual singularities $(t - 4m^2)^{1/2}$ of the partial waves with integer l. However, if in place of $f_l(t)$ we consider the functions $\varphi_l(t) = f_l(t)(t - 4m^2)^l$, then $\varphi_l(t)$ have only the usual singularity.

If $t < 4m^2$, then $\varphi_l(t)$ and $f_l(t)$ have singularities at values of t such that $1 + 2s/(t - 4m^2) \geq -1$, i.e., for $t \leq 4m^2 - s$. Since the minimum $s = s_0$, $A_1(s, t)$ has singularities also for $t < 4m^2$ (the left cut), given by the condition $4m^2 - s - t = u_1$ when $t = t_0 = 4m^2 - s_0$ (if $s_0 = 4m^2$ then $t_0 = 0$). These lead to singularities of $f(t)$ at

$$t = t_1 = 4m^2 - s_0 - u_1.$$

We evaluate the discontinuity across the left cut of the function $\varphi_l(t)$. For $t_1 < t < t_0$ we have

$$\Delta\varphi_l = \frac{1}{2i}[\varphi_l(t + i\varepsilon) - \varphi_l(t - i\varepsilon)] = 2\int_{s_0}^{4m^2 - t} P_l\left(\frac{2s}{4m^2 - t} - 1\right) \times A_1(s, t)(4m^2 - t)^{-l-1}\, ds. \tag{11}$$

For $t < t_1$ we have

$$\Delta\varphi_l = 4\left\{-\frac{1}{2}\int_{s_0}^{4m^2 - t} P_l\left(\frac{2s}{4m^2 - t} - 1\right) A_1(s, t - i\varepsilon)\, ds + \frac{1}{\pi} \times \int_{s_0}^{4m^2 - t - u_1} Q_l\left(\frac{2s}{4m^2 - t} - 1 + i\varepsilon\right) \operatorname{Im} A_1(s, t)\, ds\right\}(4m^2 - t)^{-l-1}. \tag{12}$$

These expressions are obtained in an elementary way if one recognizes that $[Q_l(z + i\epsilon) - Q_l(z - i\epsilon)]/2i = -\pi P_l(z)/2$. We have written them out in order to emphasize the fact that $\Delta\varphi_l$ is determined by integrals over a finite domain and is therefore an analytic function of l for all complex l, with the exception of negative integer values at which Q_l has simple poles. This fact will be of importance in what follows.

4. SINGULARITIES OF THE PARTIAL WAVE AMPLITUDES AS FUNCTIONS OF THE ANGULAR MOMENTUM

In this section we consider the singularities of $f_l(t)$ as functions of l. All possible singularities of $f_l(t)$ can be put into two classes: 1) singularities whose position depends on t (moving singularities) and 2) singularities whose position does not depend on t (stationary singularities).

In this paper we are interested only in the moving singularities and we show that when one neglects the inelastic processes these singularities can be only simple poles due to resonances. In order to show this we have to study the properties of $f_l(t)$ as a function of t on the nonphysical sheets, as was done by Gunson and Taylor [5] for integer l. This is so because if $f_l(t)$ has a singularity at $l = l(t)$ as a function of l then, generally speaking, it has the same singularity as a function of t at $t = t(l)$. Since $f_l(t)$ is not a single-valued function of t, the singularity $t = t(l)$ can occur on nonphysical sheets.

We start from the fact proved above by using the Mandelstam representation that when $\operatorname{Re} l \geq l_0$ there are no singularities $t = t(l)$ on the physical sheet of the t plane (see Fig. 1). We now vary l in the region $\operatorname{Re} l < l_0$. If we then find the first singularity at some $l = l'$ and some $t = t'$,

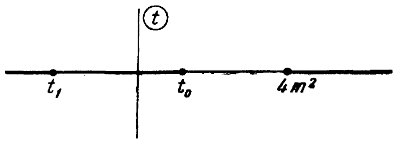

FIG. 1

this implies that for $l = l'$ the first singularity appears on the physical sheet of the t plane at $t = t'$. We use the word "first" in this connection in the following meaning that $\operatorname{Re} l' > \operatorname{Re} l$ for any other singularity of $f_l(t)$ that depends on t. From continuity considerations it is clear that t′ can lie only on the cuts $t > 4m^2$ and $t < t_0$ of the function $f_l(t)$, i.e., the singularity can come only from the other sheets and cannot simply appear at an arbitrary t.

In order to determine the character of the singularity at $t = t'$ it suffices to find out what kind of singularities existed on the other sheets in the vicinity of the real axis for $\operatorname{Re} l > \operatorname{Re} l'$.

We first consider the left cut $t < t_0$. In order to continue the function to the other sheet we express $\varphi_l^r = \varphi_l(t + i\epsilon)$ in terms of $\varphi_l^a = \varphi_l(t - i\epsilon)$ and $\Delta\varphi_l$, namely $\varphi_l^r = \varphi_l^a + 2i\Delta\varphi_l$. If the right-hand side can be continued into the lower half-plane, then this will yield the continuation of φ_l^r to the second sheet and φ_l^a can by definition be continued to the lower half-plane. The analytic continuation of $\Delta\varphi_l$ is determined by the analytic continuation of $A_1(s, t)$. However, independently of the analytic continuation of $A_1(s, t)$ the function $\Delta\varphi_l$ will not have l-dependent singularities since it is given by an integral with respect to the analytic functions P_l and Q_l over a finite interval. There are therefore no singularities which depend on l in the nonphysical sheets associated with the left cut. This fact can be explained also in a different way. By virtue of (11) and (12), the discontinuity $\Delta\varphi_l$, being defined for $\operatorname{Re} l > l_0$, is uniquely determined and finite for all values of l (except for negative integers). Therefore the contribution from the left cut does not have singularities in l.

We now turn to the right cut. Here we obtain from the unitarity condition

$$\frac{1}{2i}(\varphi_l^r - \varphi_l^a) = \rho_l \varphi_l^r \varphi_l^a, \quad \rho_l = (k/\omega)(t-4m^2)^l$$

and therefore

$$\varphi_l^r = \varphi_l^a / (1 - 2i\rho_l\varphi_l^a). \tag{13}$$

It follows from this that for $\operatorname{Re} l > l_0$ there can exist on a nonphysical sheet only poles which are given by the condition $\rho_l\varphi_l = 1/2i$ and which can emerge to the physical sheet at $l = l'$. For integer l these poles correspond to the resonances discussed in [5].

We have thus shown that if we neglect inelastic processes the moving singularities of $f_l(t)$ can be only poles.

In the next section we consider the simplest inelastic processes and show that they do not change the above conclusions.

5. THE CASE OF SEVERAL CHANNELS

We consider the case where there exist two types of particles, A and B, with masses m_1 and m_2 ($m_2 > m_1$) so that the following reactions are possible:

$$A + A \to A + A, \quad A + A \to B + B,$$
$$B + B \to B + B. \tag{14}$$

We designate the amplitudes for the partial waves of these reactions by f_l, g_l, and h_l respectively, and the corresponding invariant amplitudes by $A^f(s, t)$, $A^g(s, t)$, and $A^h(s, t)$. To begin with we assume that m_2 is not large so that none of the amplitudes A^f, A^g, and A^h have anomalous thresholds. Then we can introduce, in complete analogy to the previous sections, partial wave amplitudes with complex l. They can be written in the following form:

$$f_l^\pm = \frac{1}{\pi}\int_{z_{11}^0}^{\infty} Q_l(z_{11}) A_1^f(s_{11}, t)\, dz_{11} \pm (A_1^f \to A_2^f),$$

$$g_l^\pm = \frac{1}{\pi}\int_{z_{12}^0}^{\infty} Q_l(z_{12}) A_1^g(s_{12}, t)\, dz_{12} \pm (A_1^g \to A_2^g),$$

$$h_l^\pm = \frac{1}{\pi}\int_{z_{22}^0}^{\infty} Q_l(z_{22}) A_1^h(s_{22}, t)\, dz_{22} \pm (A_1^h \to A_2^h), \tag{15}$$

$$z_{11} = 1 + \frac{2s_{11}}{t - 4m_1^2}, \quad z_{12} = \frac{t + 2s_{22} - 2m_1^2 - 2m_2^2}{\left[(t-4m_1^2)(t-4m_2^2)\right]^{1/2}},$$

$$z_{22} = 1 + \frac{2s_{22}}{t - 4m_2^2}. \tag{16}$$

These formulae are valid for $\operatorname{Re} l > l_1$, $\operatorname{Re} l > l_2$, and $\operatorname{Re} l > l_3$ respectively.

To determine the character of the moving singularities, we are interested in the singularities on the other sheets for $\operatorname{Re} l$ larger than the largest of the numbers l_1, l_2, and l_3. Evaluating the discontinuities Δf_l, Δg_l, and Δh_l on the left cuts we again conclude that there are no l-dependent singularities on the nonphysical sheets associated with these cuts since the discontinuities are given by integrals with respect to P_l and Q_l over finite intervals.

The analytic continuations into the nonphysical sheets associated with the right-hand cuts $t \geq 4m_1^2$ and $t \geq 4m_2^2$ are given by the unitarity conditions

$$\frac{1}{2i}(f_l^r - f_l^a) = \frac{k_1}{\omega} f_l^r f_l^a + \frac{k_2}{\omega} g_l^r g_l^a,$$

$$\frac{1}{2i}(g_l^r - g_l^a) = \frac{k_1}{\omega} f_l^r g_l^a + \frac{k_2}{\omega} g_l^r h_l^a,$$

$$\frac{1}{2i}(h_l^r - h_l^a) = \frac{k_1}{\omega} g_l^r g_l^a + \frac{k_2}{\omega} h_l^r h_l^a. \tag{17}$$

Strictly speaking, the values of the analytic function on the two boundaries of the cut are functions of the type φ_l^r and φ_l^a. For simplicity, however, we do not consider the complications of the unitarity conditions associated with this circumstance, since they have no influence on the result. The continuation to the first nonphysical sheet associated with the singular point $t = 4m_1^2$ is obtained by omitting the second term on the right-hand side and solving the equations for f_l^r, g_l^r, and h_l^r. This way we get

$$f_l' = \frac{f_l^a}{1 - 2if_l^a k_1/\omega}, \quad g_l' = \frac{g_l^a}{1 - 2if_l^a k_1/\omega},$$

$$h_l' = h_l^a + 2i\frac{k_1}{\omega}\frac{g_l^a g_l^a}{1 - 2if_l^a k_1/\omega}. \tag{18}$$

The continuation to the second nonphysical sheet associated with the singular point $t = 4m_2^2$ can most easily be performed by noting that according to (17) and (18)

$$\frac{1}{2i}[f_l^r - f_l'] = \frac{k_2}{\omega} g_l^r g_l', \quad \frac{1}{2i}[g_l^r - g_l'] = \frac{k_2}{\omega} g_l^r h_l',$$
$$\frac{1}{2i}[h_l^r - h_l'] = \frac{k_2}{\omega} h_l^r h_l'; \tag{19}$$

from this we obtain

$$f_l'' = f_l' + 2i\frac{k_2}{\omega}\frac{g_l' g_l'}{1 - 2ih_l' k_2/\omega}, \qquad g_l'' = \frac{g_l'}{1 - 2ih_l' k_2/\omega},$$
$$h_l'' = \frac{h_l'}{1 - 2ih_l' k_2/\omega}. \tag{20}$$

It follows from (18) and (20) that on both unknown sheets there are no l-dependent singularities except poles. Furthermore, it follows from (18) that g_l' and h_l' cannot have poles which are not contained in f_l', but g_l' and h_l' may not have a pole in the point where f_l' has a pole if $g_l^a = 0$ in that point. On the second sheet f_l and h_l exchange places. This indicates that the numbers l_1, l_2, and l_3, which determine the value of Re l at which the singularities emerge onto the physical sheet, have to satisfy the inequalities $l_2 \le l_1$, $l_2 \le l_3$. If one excludes the special case in which a zero of g_l^a coincides with a pole of f_l' or h_l'' then all three amplitudes will have poles at the same $l = l(t)$ or $t = t(l)$.

We now turn to the question whether the situation will change when we increase the mass m_2 so that anomalous thresholds appear in the amplitudes $A^g(s, t)$ and $A^h(s, t)$. The appearance of anomalous thresholds leads, first, to the occurrence of additional regions of integration over z_{12} and z_{22} in the formulae (15) for g_l and h_l. In these $A_1^{g,h}(s, t)$ have in general complex singularities in t. However, these additions cannot change the analytic properties of g_l and h_l as functions of l, since they are finite integrals involving Q_l and thus are analytic functions of l. Furthermore, even with an unchanged region of integration over z_{12} and z_{22} additional singularities appear in $g_l(t)$ and $h_l(t)$, connected with the anomalous thresholds of A_1^g and A_1^h in t.

It is convenient to consider these anomalous thresholds from the point of view of the behavior of the singularities on the nonphysical sheet, as was done in [7]. With m_2 close to m_1 and no anomalous thresholds, the singularities and cuts of g_l and h_l are shown in Figs. 2 and 3 with heavy lines. As m_2^2 becomes larger the left cut approaches

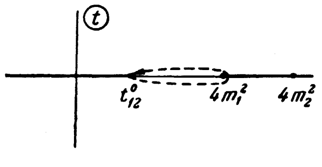

FIG. 2

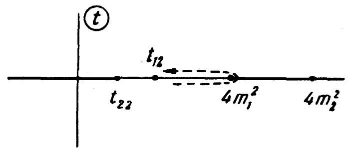

FIG. 3

the point $t = 4m_1^2$ and reaches it for $m_2^2 = m_1^2 + s_{12}^0$. Here t_{12}^0 reaches its maximum and decreases with further increase in m_2. One can easily verify that actually the singularity at t_{12}^0 moves off to another sheet. However, because of the second equation of (18) a similar singularity exists on the second sheet (a singularity in g_l^a) which moves with increasing m_2 from the second sheet to the first and leads to an anomalous addition to the dispersion relation for g_l. Simultaneously there appears an anomalous contribution to the dispersion relations for h_l, since according to the last equation of (18) h_l also has a singularity on the second sheet at $t = t_{12}^0$. This singularity also emerges onto the physical sheet at $m_2^2 = m_1^2 + s_{12}^0$.

One can see from the above description that the discontinuities on the cuts associated with the anomalous thresholds are proportional to the discontinuity Δg_0^a on the left cut (the left cut leads to the second sheet) which does not contain singularities in l and therefore there are likewise no l-dependent singularities in the nonphysical sheet connected with the anomalous singularity.

If the mass m_2 becomes unstable, then t_{12}^0 either reaches the singularities of g_l and h_l on the left cut, or moves into the complex plane (if the inelastic process results in particles with different masses). In either case this cannot lead to the appearance of l-dependent singularities.

Thus the appearance of anomalous thresholds or of unstable particles has no influence on the above-obtained result that the only moving singularities can be poles. In a following paper we shall show, under rather general assumptions concerning the analytic properties of the production amplitudes, that this result does not change if one takes into account the inelastic channel of the process in which two particles are changed into three.

6. TWO TYPES OF ASYMPTOTIC BEHAVIOR OF THE SCATTERING AMPLITUDE

If one accepts the above considerations then $f_l(t)$ can have either t-independent singularities or poles.

If the first (from the side of large l) singularities turn out to be stationary singularities, then the asymptotic behavior of $A_1(s, t)$ for large s has the form $s^{l_0}B(\xi, t)$,[2] where l_0 is the position of the singularity, $\xi = \ln s$, $B(\xi, t) < C/\xi$, which leads to a decreasing total cross section ($l_0 \leq 1$, see [8]). If the first singularity is a pole, then $A_1(s, t) \sim s^{l_0(t)}$, which corresponds to the previously[8] described picture of the scattering, according to which the interaction radius grows with energy as $\xi^{1/2}$, the elastic scattering cross section decreases as $1/\xi$, while the total cross section may approach a constant. Such a picture evidently does not contradict the presently available experimental data, since the elastic scattering cross section is considerably smaller than the total cross section for all reactions, and the diffraction peak in $d\sigma/dt$ evidently becomes narrower with increasing energy s.[9]

In conclusion I express my deep gratitude to L. D. Landau, I. Ya. Pomeranchuk, and I. T. Dyatlov for useful discussions and to I. M. Shmushkevich who pointed out a number of errors in the work.

[1] V. N. Gribov, JETP **41**, 1962 (1961), Soviet Phys. JETP **14**, 1395 (1962).

[2] V. N. Gribov, Nucl. Phys. **22**, 249 (1961).

[3] V. N. Gribov, JETP **41**, 667 (1961), Soviet Phys. JETP **14**, 478 (1962).

[4] T. Regge, Nuovo cimento **14**, 951 (1959); **18**, 947 (1960).

[5] J. Gunson and J. G. Taylor, Phys. Rev. **119**, 1121 (1960).

[6] N. N. Meĭman and N. G. Chebotarev, Proc. Math. Inst. Acad. Sciences (U.S.S.R.) **26**, 154 (1949).

[7] R. Oehme, Phys. Rev. **121**, 1840 (1961); Blankenbeckler, Goldberger, MacDowell, and Treiman, Phys. Rev. **123**, 692 (1961).

[8] M. Froissart, Phys. Rev. **123**, 1053 (1961).

[9] G. Cocconi, Proc. of the Int. Conf. on Theoret. Aspects of Very High Energy Phenomena, CERN (1961).

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208