THE MAGNETIC MOMENT OF THIN SUPERCONDUCTING FILMS

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An expression is derived for the magnetic moment of a thin superconducting film of finite dimensions located in a magnetic field perpendicular to its surface.

IN the experimental study of the magnetic properties of superconducting films, one usually tries to arrange the film so that the external magnetic field is as nearly parallel to the surface as possible. The experimental results obtained in this way are interpreted with the help of theoretical formulas which apply to the case of a film of infinite extent located in a magnetic field that is strictly parallel to the surface of the film (see, for example, the review of Ginzburg [1]).

However, there are practical difficulties involved in getting the film strictly parallel to the field, and one is always dealing experimentally with films set at some small angle to the field. One can say that the film has two components in this case—parallel ($H_{\parallel}$) and perpendicular ($H_{\perp}$) to the surface of the film, and the resulting magnetic moment of the film also has two parts—parallel ($M_{\parallel}$) and perpendicular ($M_{\perp}$) to the film surface.

In some cases, the perpendicular component of the magnetic moment is measured directly by experiment (see, for example, the experiments of Sevast'yanov and Sokolina [2,3]). However, a theoretical expression for $M_{\perp}$ has not yet been derived.

In the present work, an expression is deduced for the magnetic moment of a thin film which has the shape of a flat ellipsoid of revolution, and which is located in a field parallel to the axis of rotation. The analysis is carried out within the framework of the local London theory of superconductivity. The phenomenological Ginzburg-Landau theory [1] gives the same result for the moment if the quasi-wave function Ψ = const is introduced in the theory. Thus the result that is obtained applies to London superconductors close to the critical temperature T_{cr}.

2. Let the superconducting sample be located in an external magnetic field $\mathbf{H}_0$. The London equation for the determination of the field inside a superconductor has the form

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = -\delta^{-2}\mathbf{A}, \tag{1}^*$$

where $\mathbf{A}$ is the vector potential and δ is the penetration depth of the field. Outside the superconductor we have

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = 0, \tag{2}$$

where $\mathbf{H} = \mathbf{H}_0$ at large distances from the sample. The specimen is assumed to have the shape of a flat ellipsoid of revolution, with minor semiaxis b and major semiaxis a; the field $\mathbf{H}_0$ is directed along the axis of rotation, i.e., the major semiaxis a is perpendicular to the field.

Equations (1) and (2) are expressed in oblate spherical coordinates (see, for example, Stratton, [4] page 60) and the usual boundary problem is solved with the condition that $\mathbf{A}$ be continuous on the boundary of the sample. The solution is expressed in terms of a series in a small parameter $\sim ab/\delta^2$. As the result of simple but rather cumbersome calculations, five terms of such an expansion have been found.

The magnetic moment of the sample is determined from the formula

$$\mathbf{M} = \frac{1}{2c}\int [\mathbf{jr}]\, d^3\mathbf{r}, \tag{3}^\dagger$$

where c is the velocity of light, $\mathbf{j} = (c/4\pi\delta^2)\mathbf{A}$; the integration is carried out over the volume of the specimen. We write out the final expression for the magnetic moment, valid in the limiting case $a \gg b$:

$$M_{\perp} = \frac{1}{5}M_0\left[x - \frac{2}{7}x^2 + \frac{5}{7\cdot 2^3}x^3 - \frac{71}{7\cdot 11\cdot 2^5}x^4 + \frac{38639}{7\cdot 11\cdot 13\cdot 2^{12}}x^5 - \ldots\right] \equiv M_0 f(x); \tag{4}$$

here $M_0 = -(2a^3/3\pi)H_0$ is the magnetic moment of a bulk superconducting, flat ellipsoid of revolution

*rot = curl.

†$[\mathbf{jr}] = \mathbf{j}\times\mathbf{r}$.

(see [5]), and $x = \pi ab/4\delta^2$ is a parameter.

Formally, the expansion parameter x must be considered to be a small quantity ($x \ll 1$). In practice, however, (4) is an asymptotic series which can be used even for $x \gtrsim 1$ (a similar circumstance exists in the expansion of the moment of a sphere, cylinder, etc [1]). It can be shown that the series given in (4) allows us to consider values of $x \lesssim 2$—3. Calculation of the further terms of the expansion in (4) is extremely involved; however, the behavior of the moment M_1 is obvious: upon increase in x, we have $M_\perp \to M_0$.

The following interesting consequence results from the considerations just given; even a very thin film (for example, $\sim 10^{-6}$ cm and less) behaves in a magnetic field perpendicular to its surface as if it were a bulk specimen, provided only that the transverse dimensions of the film are sufficiently large (large parameter s). This circumstance is well confirmed by experiment.[2,3]

In the experiments of Sevast'yanov and Sokolina[2,3] the film had in a number of cases the shape of a disc of thickness d and diameter D. If these films are approximated by an ellipsoid of revolution with $b = d/2$ and $a = D/2$, then the films used in the experiment have the parameter $x \gtrsim 3$—4. The experimental and theoretical dependences of the moment on the parameter x go over smoothly into one another [see Fig. 7 in [3]], which makes it possible to speak of qualitative agreement between experiment and theory.

It is also of interest to make a comparison of the quantities $M_\parallel$ and $M_\perp$. For this purpose, we use the well-known expression (see, for example, [1])

$$M_\parallel = -H_0 \frac{V}{4\pi}\left[1 - \frac{\delta}{b}\,\mathrm{th}\,\frac{b}{\delta}\right], \qquad (5)^*$$

which corresponds to a film of half thickness b in a parallel field, where we have written the volume of the film V in the form $V = \frac{4}{3}\pi a^2 b$. Assuming $b/\delta \ll 1$, we find from (4), (5)

$$M_\parallel / M_\perp \approx b^3/\delta^2 a f(x). \qquad (6)$$

Taking as an example a typical experimental film with $b = 10^{-6}$ cm, $a = 10^{-1}$ cm, $\delta = 3 \times 10^{-5}$ cm, we get an estimate of $M_\parallel / M_\perp \sim 10^{-8}$ from (6) (here $x \sim 100$ and $f(x) \sim 1$). It is then clear that $M_\perp$ can exceed $M_\parallel$ by a large amount, and that account of the perpendicular component of the magnetic field can be very important in a number of cases.

In conclusion, I thank B. K. Sevast'yanov for useful discussions.

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*th = tanh.