UNITARY SYMMETRY AND THE UNIVERSAL WEAK INTERACTION

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A hypothesis is examined according to which the constant G_Λ for the leptonic decays of strange particles is approximately one fourth of the universal constant $G = 10^{-5}\,m^{-2}$ and the matrix elements for these decays satisfy the conditions (3) to (5) imposed by the unitary symmetry of the strong interactions. This hypothesis is in agreement with the experimental data available at present and allows one to make a number of predictions. In particular, the relations (13), (14), (16), and (18) should be valid. Moreover, the spectra and polarization of the particles emitted in $K_{\mu 3}$ decay and in the leptonic decays of the Λ hyperon are predicted. A verification of these predictions may serve as a test of the validity of the hypothesis.

IT is known[1-3] that the unitary symmetry of the strong interactions in the Sakata model implies the equality of the strong interaction constants and the masses of the proton, neutron, and Λ hyperon. The consequences of this symmetry for the systematics of the strongly interacting particles have been considered by a number of authors.[1-5] In this paper we shall discuss the consequences of the unitary symmetry for the weak interactions, and in particular, for the leptonic decays of the strange particles.

In the Sakata model, the strangeness conserving leptonic decays are due to the interaction of the nucleon current with the leptons:

$$\frac{G_N}{\sqrt{2}}(\bar{p}O_\alpha n)(\bar{l}O_\alpha\nu) + \text{H.c.}; \tag{1}$$

$$G_N = G = 10^{-5}m^{-2}, \qquad O_\alpha = \gamma_\alpha(1+\gamma_5), \qquad l = e \text{ or } \mu.$$

The leptonic decays in which the strangeness changes are due to the interaction of the strangeness-carrying current

$$\frac{G_\Lambda}{\sqrt{2}}(\bar{p}O_\alpha\Lambda)(\bar{l}O_\alpha\nu) + \text{H.c.} \tag{2}$$

It follows from (1) and (2) that the following relations between matrix elements hold with the same accuracy with which the strong interactions satisfy the requirements of unitary symmetry:

$$M(\Lambda\to p+e(\mu)+\bar{\nu}) : M(n\to p+e(\mu)+\bar{\nu}) = G_\Lambda : G_N, \tag{3}$$

$$M(K\to e(\mu)+\nu) : M(\pi\to e(\mu)+\nu) = G_\Lambda : G_N, \tag{4}$$

$$2M(K^+\to\pi^0+e(\mu)+\nu) : M(\pi^+\to\pi^0+e(\mu)+\nu) = G_\Lambda : G_N, \tag{5}$$

$$M(K^+\to\sigma_1^0+e(\mu)+\nu) : M(K^+\to\pi^0+e(\mu)+\nu) = \sqrt{3}. \tag{5'}$$

The factor 2 in relation (5) is due to the fact that the transition $\pi^+\to\pi^0$ can go through the decay $p\to n$ as well as through the decay $\bar{n}\to\bar{p}$, whereas the transition $K^+\to\pi^0$ can go only through the decay $\bar{\Lambda}\to\bar{p}$. [We recall that in the Sakata model $\pi^+ = p\bar{n}$, $\pi^0 = (p\bar{p}-n\bar{n})/\sqrt{2}$, $K^+ = p\bar{\Lambda}$.] The value $\sqrt{3}$ in relation (5′) is obtained in a similar way by taking into account the relation

$$\sigma_1^0 = (p\bar{p}+n\bar{n}-2\Lambda\bar{\Lambda})/\sqrt{6}.$$

The relations (5) and (5′) were first obtained by Ikeda, Miyachi, and Ogawa.[6] The analogs of relations (3) to (5′) in the "eightfold" scheme of Gell-Mann[7] have recently been obtained by Cabibbo and Gatto.[8]

The corresponding decay rates are equal to

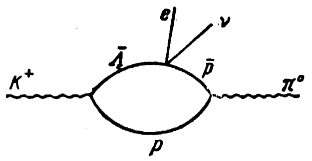
$$w_i = 2\pi M_i^2\rho_i, \tag{6}$$

where ρ_i is the Lorentz invariant density of final states.

Comparing the decay rates calculated according to (6) with the corresponding experimental data and thus determining the quantities M_i^2, we find easily that every one of the relations (3), (4), and (5) gives

$$(G_\Lambda/G_N)^2 \approx 1/10 - 1/20. \tag{7}$$

There are now two alternatives. If we assume that the unitary symmetry is not strongly violated in the leptonic decays of strange particles, relation (7) tells us to abandon the idea of a universal weak coupling constant G. If, on the other hand, we hold on to the concept of universal weak coupling ($G_\Lambda = G_N = G$), we are forced to conclude that unitary symmetry is strongly violated in the leptonic decays of the strange particles.



The second alternative is favored by the circumstance that we already know a number of physical effects in which the unitary symmetry is very strongly violated: it is sufficient, for example, to compare the production cross sections for π and K mesons in the region of energies up to about one BeV. It is possible that the matrix elements for the leptonic decays are related to this class of effects. The particular attractiveness of this alternative lies, in our opinion, in the fact that it leaves open the possibility of constructing a theory of elementary particles in which each of the fundamental interactions (weak, strong, electromagnetic, and "anomalous") is universal and characterized by a single coupling constant. (An attempt to construct such a theory is contained in a paper by the authors.[9]) The possibility is not excluded, however, that the experimental verification of the first alternative does not imply a violation of the universality of the weak interaction. The constant G_Λ introduced by us might not be the "true" constant for the strangeness-carrying current, but a phenomenological constant which arises, for example, on account of the renormalization of the Λp current by the "anomalous" interaction discussed in [9]. With this in mind we shall below investigate the first alternative in more detail.

The presently available experimental data are not in disagreement with the unitary relations (3), (4), and (5), if one assumes a limited universality of the weak interaction, i.e., if one assumes that all leptonic decays of the strange particles are described by the single constant G_Λ which is approximately one fourth as large as the constant G. This hypothesis of limited universality and unitary symmetry is essentially contained in the papers of Ikeda, Miyachi, and Ogawa[6] and Ohnuki.[2] In addition to what has been done by these authors, we include in our discussion not only the weak vector interaction, but also the axial vector interaction, and analyze possible ways of testing this hypothesis by experiment. Below we shall indicate a number of experiments which may serve as a test of the validity of the hypothesis of the limited universality of the weak interaction.

For the determination of the ratio G_Λ/G_N we can use the rates of the $K_{\mu 2}$ and $\pi_{\mu 2}$ decays, which have been measured with the highest accuracy. Writing the amplitude for the $K_{\mu 2}$ decay in the form

$$\frac{G_\Lambda}{\sqrt{2}} f_K \varphi_K \varphi_\pi k_\alpha (\bar{\nu} O_\alpha \mu), \tag{8}$$

and the amplitude for the $\pi_{\mu 2}$ decay in the form

$$\frac{G_N}{\sqrt{2}} f_\pi \varphi_\pi k_\alpha (\bar{\nu} O_\alpha \mu), \tag{9}$$

we find, according to (4),

$$f_K = f_\pi. \tag{10}$$

The comparison with the corresponding experimental rates gives[10]

$$G_\Lambda^2/G_N^2 \approx 1/_{14}. \tag{11}$$

For the other decays, this ratio is known with much less accuracy.

In the unitary approximation the amplitudes for the K_{e3} and $K_{\mu 3}$ decays are uniquely determined:

$$M = \tfrac{1}{2} G_\Lambda \varphi_K \varphi_\pi (p_K + p_\pi)_\alpha (\bar{\nu} O_\alpha l) = \tfrac{1}{2} G_\Lambda \varphi_K \varphi_\pi (2p_K - q)_\alpha (\bar{\nu} O_\alpha l), \tag{12}$$

where q is the total four-momentum of the leptons. This allows us to predict the absolute rates of the K_{e3} and $K_{\mu 3}$ decays as well as all their differential characteristics. The probability for the K_{e3} decay is equal to

$$w(K_{e3}) = G_\Lambda^2 m_K^5 \cdot 0.6/1536\pi^3 = 5{,}1 \cdot 10^6 \text{ sec}^{-1} \tag{13}$$

(the experiments give two different values for this probability: 3.4×10^6 and 6.2×10^6 sec^{-1}).

The probability for the $K_{\mu 3}$ decay is equal to

$$w(K_{\mu 3}) = 0.4 \cdot G_\Lambda^2 m_K^5/1536\pi^3 = 3{,}4 \cdot 10^6 \text{ sec}^{-1} \tag{14}$$

(experiment gives approximately 3.3×10^6 sec^{-1}).

In the general case the amplitude (12) is proportional to $gp_K - fq$, and the ratio of the rates of the $K_{\mu 3}$ and K_{e3} decays is equal to

$$w(K_{\mu 3})/w(K_{e3}) = (0{,}5g^2 - 0{,}2fg + 0{,}05f^2)/0{,}6g^2. \tag{15}$$

In the unitary limit g = 2, f = 1, and

$$w(K_{e3})/w(K_{\mu 3}) = 1.5. \tag{16}$$

It follows from (15) that in the interval

$$-0{,}5 \leqslant f/g \leqslant 4.5 \tag{17}$$

the inequality $w(K_{e3})/w(K_{\mu 3}) > 1$ obtains.

The fact that this inequality obtains for a rather large range of values of the quantity f/g makes its experimental verification particularly interesting. If it turns out that $w(K_{e3}) = w(K_{\mu 3})$, as experiment seems to indicate, this will imply that our hypothesis is wrong. The ratio f/g can also be determined by measuring the spectrum of the

π mesons and μ mesons, the angular distribution, and the polarization of the latter. The corresponding formulas for arbitrary values of f/g are given in the review article [10].

It should be emphasized that even if experiment yields $f/g = 1/2$ in accordance with the hypothesis of limited universality, this does not yet prove the validity of this hypothesis. Indeed, consideration of the simplest graph of perturbation theory (see the figure) shows that the matrix element for the $K_{\mu3}$ and K_{e3} decays should be proportional to $p_K + p_\pi$ whatever the value of the constant $g_{\Lambda NK}$, if large virtual momenta are important. This property of the matrix element is therefore not connected with the unitary symmetry in a one-to-one correspondence.

Of great interest as a test of the validity of the hypothesis of limited universality and unitary symmetry is the precise measurement of the absolute rate of the leptonic decays of the Λ hyperon: $\Lambda \rightarrow p + e(\mu) + \bar{\nu}$. According to our hypothesis, the β decay probability for the Λ hyperon should be equal to

$$w(\Lambda \rightarrow p + e + \bar{\nu}) \approx 6 \cdot 10^6 \text{ sec}^{-1} \qquad (18)$$

and should amount to about 0.15% of the total decay rate of the Λ hyperon. Experiment [11] gives a value close to 0.15%, but with a large error. It would also be very interesting to investigate experimentally whether the ratio of the axial vector and vector constants in the β decay of the Λ hyperon is indeed equal to 1.25, as required by relation (3).

It is not clear at the present time to which unitary multiplets the known hyperons belong. If some of them belong to one and the same multiplet, one can write down relations between the matrix elements for their leptonic decays which are the analogs of (3). The crudest and most feasible test of the validity of these relations would be the measurement of the rates of the decays

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}, \qquad \Xi^- \rightarrow \Lambda^0 + e^- + \bar{\nu},$$
$$\Xi^- \rightarrow \Sigma^0 + e^- + \bar{\nu}, \qquad \Xi^0 \rightarrow \Sigma^+ + e^- + \bar{\nu}$$

and the corresponding muonic decays.

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